MATHEMATICAL DISCOURSES

AND

DEMONSTRATIONS,

TOUCHING

Two *NEW SCIENCES*; pertaining to

THE

MECHANICKS

AND

LOCAL MOTION:

BY

GALILÆUS GALILÆUS LYNCEUS,

Chiefe *Phylosopher* and *Mathematitian* to the most Serene *GRAND DUKE* of *TUSCANY*.

WITH

AN APPENDIX OF THE

Centre of Gravity

Of some *SOLIDS*.

Englished from the Originall *Latine* and *Italian*,

By THOMAS SALUSBURY, *Esq*;

LONDON,

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GALILEUS, HIS DIALOGUES OF MOTION.

The First Dialogue.

INTERLOCUTORS,

SALVIATUS, SAGREDUS, and SIMPLICIUS.

SALVIATUS.

THE frequent resort (Gentlemen) to your Famous Arsenal of *Venice*, presenteth, in my thinking, to your Speculative Wits, a large field to Philosophate in; and more particularly, as to that part which is called the *Mechanicks*: in regard that there all kinds of Engines, and Machines are continually put in use, by a huge number of Artificers of all sorts; amongst whom, as well through the observations of their Predecessors, as those, which through their own care they continually are making, it's probable, that there are some very learned, and bravely discours'd Men.

A Description of the Arsenal of Venice.

It is a large field for Wits to Philosophate in.

SAGR. Sir, you are not therein mistaken: and I my self, out of

a natural Curiositie, do frequentlie for my Recreation visit that place, and confer with these persons; which for a certain preheminence that they have above the rest we call * *Overseers*: whose discourse hath oft helped me in the investigation of not onsy wonderful, but abstruce, and incredible Effects: and indeed I have been at a losse sometimes, and despaired to penetrate how that could possibly come to passe, which far from all expectation my senses demonstrated to be true; and yet that which not long since that good Old-man told us, is a saying and proposition, though common enough, yet in my opinion wholly vain, as are many others, often in the mouths of unskilful persons; introduced by them, as I suppose, to shew that they understand how to speak something about that, of which neverthelesse they are incapable.

* Proti.

The Opinion of Common Artificers are often false.

SALV. It may be Sir, you speak of that last proposition which he affirmed, when we desired to understand, why they made so much greater provision of supporters, and other provisions, and reinforcements about that Galeasse, which was to be launcht than is made about lesser Vessels, and he answered us, that they did so to avoid the peril of breaking its Keel, through the mighty weight of its vast bulk, an inconvenience to which lesser ships are not subject.

Great Ships apter than others to break their Keels in Launching, according to some.

SAGR. I do intend the same, and chiefly that last conclusion, which he added to his others, and which I alwaies esteemed a vain conceit of the Vulgar, namely, That in these and other Machines we must not argue from the lesse to the greater, because many Mechanical Inventions take in little, which hold not in great. But being that all the Reasons of the Mechanicks, have their foundations from Geometry; in which I see not that greatnesse and smalnesse make Circles, Triangles, Cilinders, Cones, or any other solid Figures subject to different passions; when the great Machine is conformed in all its members to the proportions of the lesse that is useful, and fit for exercise to which it is designed; I cannot see why it also should not be exempt from the unlucky, sinister, and destructive accidents that may befall it.

SALV. The saying of the Vulgar is absolutely vain, and so false, that its contrary may be affirmed with equal truth, saying, That many Machines may be made more perfect in great than little: As for instance, a Clock that shews and strikes the Houres, may be made more exact in one certain size, than in another lesse. With better ground is that same conclusion usurped by other more intelligent persons, who refer the cause of such effects in these great Machines different from what is collected from the pure, and abstracted Demonstrations of Geometry, to the imperfection of the matter, which is subject to many alterations, and defects. But here, I know not whether I may without contracting some suspition

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ſuſpition of Arrogance ſay, that thither alſo doth the recourſe to the defects of the matter (able to blemiſh the perfecteſt Mathematical Demonſtrations) ſuffice to excuſe the diſobedience of Machines in concrete, to the ſame abſtracted and Ideal: yet notwithſtanding I will ſpeak it, affirming, That abſtracting all imperfections from the Matter, and ſuppoſing it moſt perfect, and unalterable, and from all accidental mutation exempt, yet neverthelesſe its only being Material, cauſeth, that the greater Machine, made of the ſame matter, and with the ſame proportions, as the leſſer; ſhall anſwer in all other conditions to the leſſer in exact Symetry, except in ſtrength, and reſiſtance againſt violent invaſions: but the greater it is, ſo much in proportion ſhall it be weaker. And becauſe I ſuppoſe the Matter to be unalterable, that is alwaies the ſame, it is manifeſt, that one may produce Demonſtrations of it, no leſſe ſimply and purely Mathematical, then of eternal, and neceſſary Affections: Therefore, *Sagredus*, Revoke the opinion which you, and it may be, all the reſt hold, that have ſtudied the Mechanicks; that Machines, and Frames compoſed of the ſame Matter, with punctual obſervation of the ſelf-ſame proportion between their parts, ought to be equally, or to ſay better, proportionally diſpoſed to Reſiſt; and to yield to External injuries and aſſaults: For if it may be Geometrically demonſtrated, that the greater are alwaies in proportion leſs able to reſiſt, than the leſſe; ſo that in fine there is not only in all Machines & Fabricks Artificial, but Natural alſo, a term neceſſarily aſcribed, beyond which neither Art, nor Nature may paſſe; may paſſe, I ſay, alwaies obſerving the ſame proportions with the Identity of the Matter.

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weight, there could not be another such-a-one in the World: So that if its length (for example) were Centuple to its thicknesse, there cannot be found another Launce of the same Matter, that being in length Centuple to its thicknesse, shall be able to sustain it self precisely, as that did, and no more: for all that are bigger shall break, and the lesser shall be able, besides their own, to sustain some additional weight. And this that I say of the *State of bearing it self*, I would have understood to be spoken of every other Constitution, and thus if one Transome bear or sustain the force of ten Transomes equal to it, such another Beam cannot bear the weight of ten that are equal to it. Now be pleased, Sir, and you *Simplicius* to observe, how true Conclusions, though at the first sight they seem improbable, yet never so little glanced at, do depose the Vailes which obscure them, and make a voluntary shew of their secrets nakedly, and simply. Who sees not, that a Horse falling from a height of three or four yards, will break his bones, but a Dog falling so many yards, or a Cat eight or ten, will receive no hurt; nor likewise a Grashopper from a Tower, nor an Ant thrown from the Orbe of the Moon? Little Children escape all harm in their falls, whereas persons grown up break either their shins or faces. And as lesser Animals are in proportion more robustious, and strong than greater, so the lesser Plants better support themselves: and I already believe, that both of you think, that an Oake two hundred foot high could not support its branches spread like one of an indifferent size; and that Nature could not have made an Horse as big as twenty Horses, nor a Giant ten times as tall as a Man, unlesse she did it either miraculously, or else by much altering the proportion of the Members, and particularly of the Bones, enlarging them very much above the Symetry of common Bones. To suppose likewise, that in Artificial Machines, the greater and lesser are with equal facility made, and preserved, is a manifest Errour: and thus for instance, small Spires, Pillars, and other solid figures may be safely moved, laid along, and reared upright, without danger of breaking them; but the very great upon every sinister accident fall in pieces, and for no other reason but their own weight. And here it is necessary that I relate an accident, worthy of notice, as are all those events that occur unexpectedly, especially when the means used to prevent an inconvenience, proveth in fine the most potent cause of the disorder. There was a very great Pillar of Marble laid along, and two Rowlers were put under the same neer to the ends of it; it came into the mind of a certain Ingineer some time after, that it would be expedient, the better to secure it from breaking in the midst through its own weight, to put under it in that part yet another Rowler: the counsel seemed generally very seasonable, but the successe demonstrated it to be wholly

Truth upon a little Courting, throweth off her Vail, and shews her Secrets naked.

Great Animals receive more harm by a fall than lesser.

Nature could not have made of meaner Horses bigger, and have retained the same strength, but by altering their Symetry.

A great Marble Pillar broken by its own weight, and why.

wholly contrary: for many moneths had not past, before the Pillar crackt, and broke in the middle just upon the new Rowler.

SIMP. This was an accident truly strange, and indeed *preter spem*, especially if it were derived from the addition of new support in the middle.

SALV. From that doubtless it did proceed; and the known cause of the Effect removeth the wonder: for the two pieces of the Pillar being taken from off the Rowlers, one of those bearers on which one end of the Column had rested, was by length of time rotten, and sunk away; and that in the midst continuing sound, and strong, occasioned that half the Column lay hollow in the air without any support at the end; so that its own unweildy weight, made it do that, which it would not have done, if it had rested only upon the two first Bearers, for as they had shrunk away it would have followed. And here none can think that this would have faln out in a little Column, though of the same stone, and of a length answerable to its thicknesse, in the very same proportion as the thickness, and length of the great Pillar.

SAGR. I am now assured of the effect, but do not yet comprehend the cause, how in the augmentation of Matter, the Resistance and Strength ought not also to multiply at the same rate. And I admire at it so much the more, in regard I see, on the contrary, in other cases the strength of Resistance against Fraction to encrease much more than the enlargement of the matter encreaseth. For if (for example) there be two Nailes fastned in a Wall, the one twice as thick as the other, that shall bear a weight not only double to this, but triple, and quadruple.

SALV. You may say octuple, and not be wide of the truth: nor is this effect contrary to the former, though in appearance it seemeth so different.

A Naile double in thicknesse to another being fastned in a Wall, sustains a Weight octuple to that of the lesser.

SAGR. Therefore *Salviatus*, explain unto us these Riddles, and level us these Rocks, if you can do it: for indeed I guesse this matter of Resistance to be a field replenished with rare, and useful Contemplations, and if you be content that this be the subject of our this-daies discourse, it will be to me, and I believe to *Simplicius*, very acceptable.

SALV. I cannot refuse to serve you, since my Memory serveth me, in minding me of that which I formerly learnt of our *Accademick*, who hath made many Speculations on this subject, and all conformable (as his manner is) to Geometrical Demonstration: insomuch that, not without reason, this of his may be called a *New Science*; for though some of the Conclusions have been observed by others, and in the first place by *Aristotle*, yet neverthelesse are they not any of the most curious, or (which more importeth) proved by necessary Demonstrations deduced from their primary, and

By Accademick here, as in his Dialogues of the Systeme, Galileus meaneth himself.

Aristotle the first Observer of Mechanical Conclusions, but they neither not the most curious nor solidly demonstrated.

and indubitable fundamentals. And becauſe, as I ſay, I deſire demonſtratively to aſſure you, and not with only probable diſcourſes to perſwade you; preſuppoſing, that you have ſo much knowledge of the Mechanical Concluſions, by others heretofore fundamentally handled, as ſufficeth for our purpoſe; it is requiſite, that before we proceed any further, we conſider what effect that is which opperates in the Fraction of a Beam of Wood, or other Solid, whoſe parts are firmly connected; becauſe this is the firſt *Notion*, whereon the firſt and ſimple principle dependeth, which as familiarly known, we may take for granted. For the clearer explanation whereof; let us take the Cilinder, or Priſme, *A. B.* of Wood, or other ſolid and coherent matter, faſtned above in *A*, and hanging perpendicular; to which, at the other end *B*, let there hang the Weight *C*: It is manifeſt, that how great ſoever the Tenacity and coherence of the parts of the ſaid Solid to one another be, ſo it be not infinite, it may be overcome by the Force of the drawing Weight *C*: whoſe Gravity I ſuppoſe may be encreaſed as much as we pleaſe; by the encreaſe whereof the ſaid Solid in fine ſhall break, like as if it had been a Cord. And, as in a Cord, we underſtand its reſiſtance to proceed from the multitude of the ſtrings or threads in the Hemp that compoſe it, ſo in Wood we ſee its veins, and grain diſtended lengthwaies, that render it far more reſiſting againſt Fraction, then any Rope would be, of the ſame thickneſſe: but in a Cylinder of ſtone or Metal the Tenacity of its parts, (which yet ſeemeth greater) dependeth on another kind of Cement, than of ſtrings, or grains, and yet they alſo being drawn with equivalent force, break.

SIMP. If the thing ſucceed as you ſay, I underſtand well enough, that the thread or grain of the Wood which is as long as the ſaid Wood may make it ſtrong and able to Reſiſt a great violence done to it to break it: But a Cord compoſed of ſtrings of Hemp, no longer than two, or three foot a piece, how can it be ſo ſtrong when it is ſpun out, it may be, to a hundred times that length? Now I would farther underſtand your opinion concerning the Connection of the parts of Metals, Stones, and other matters deprived of ſuch Ligatures, which neverthleſſe, if I be not deceived, are yet more tenacious.

SALV. We muſt be neceſſitated to digreſſe into new Speculations, and not much to our purpoſe, if we ſhould reſolve thoſe difficulties you ſtart.

SAGR.

Sagr. But if Digressions may lead us to the knowledge of new Truths, what prejudice is it to us, that are not obliged to a strict and concise method, but that make our Congressions only for our divertisement to digresse sometimes, lest we let slip those Notions, which perhaps the offered occasion being past, may never meet with another opportunity of remembrance? Nay, who knows not, that many times curiosity may thereby discover hints of more worth, than the primarily intended Conclusions? Therefore I entreat you to give satisfaction to *Simplicius*, and my self also, no lesse curious than he, and desirous to understand what that Cement is, that holdeth the parts of those Solids so tenaciously conjoyned, which yet neverthelesse in conclusion are dissoluble: a knowledge which furthermore is necessary for the understanding of the coherence of the parts of those very ligaments, whereof some Solids are composed.

What that Cement is that Connecteth the parts of Solids.

Salv. Well, since it is your pleasure, I will herein serve you. And the first difficulty is, how the threads of a Cord or Rope an hundred foot long should so closely connect together (none of them exceeding two or three foot) that it requireth a great violence to break them. But tell me, *Simplicius*, cannot you hold one single string of Hemp so fast between your fingers by one end, that I pulling by the other end should break it sooner than get it from you? Questionlesse you might: when then, those threads are not only at the end, but also in every part of their length, held fast with much strength by him that graspeth them, is it not apparent, that it is a much harder matter to pluck them from him that holds them, then to break them? Now in the Cord, the same act of twisting, binds the threads mutually within one another, in such sort, that pulling the Cord with great force, the threads of it break asunder, but separate and part not from one another; as is plainly seen by viewing the short ends of the said threads in the broken place, that are not a span long; as they would be, if the division of the Cord had been made by the sole seperating of them in drawing the Cord, and not by breaking them.

How a Rope or Cord resisteth Fraction.

In breaking a Rope the parts are not separated, but broken.

Sagr. For confirmation of this, let me add, that the Cord is sometimes seen to break, not by pulling it length-waies, but by over-twisting it: an argument, in my judgment, concluding that the threads are so enterchangeably compresst by one another, that those compressings permit not the compressed to slip so very little, as is requisite to lengthen it out that it wind about the Cord, which in the twining breaketh, and consequently in some small measure swels in thicknesse.

Salv. You say very well; but consider by the way, how one truth draweth on another. That thread, which griped between the fingers

fingers, did not yield to follow him that would have forceably drawn it from between them, resisted, because it was stayed by a double compression, since the upper finger prest no lesse against the nether, than it pressed against that. And there is no question, that if of these two pressures, one alone might be retained, there would remain half of that Resistance, which depended conjunctively on them both: but because you cannot with removing, *v.g.* the upper finger take away its pression, without taking away the other part also; it will be necessary by some new Artifice to retain one of them, and to find a way that the same thread may compresse it self against the finger or other solid body upon which it is put; and this is done by winding the same thread about the Solid. For the better understanding whereof, I will briefly give it you in Figure; and let *A B* and *CD* be two Cilinders, and between them let there be distended the thread *E F*, which for greater plainnesse I will represent to be a small Cord: there is no doubt but that the two Cylinders being pressed hard one against the other, the Cord *E F* pulled by the end *F* will Resist no smal force before it will slip from between the two Solids compressing it: but if we remove one of them, though the Cord continue touching the other, yet shall it not by such contact be hindered from slipping away. But if holding it fast, though but gently in the point A, towards the top of the Cylinder, we wind, or belay it about the same spirally in A F L O T R, and pull it by the end R: it is manifest, that it will begin to presse the Cylinder, and if the windings and wreathes be many, it shall in its effectual drawing alwaies presse it so much the straiter about the Cylinder: and by multiplying the wreathes if you make the contact longer, and consequently more invincible, the more difficult still shall it be to withdraw the Cord, and make it yield to the force that pulls it. Now who seeth not, that the same Resistance is in the threads, which with many thousand such twinings spin the thick Cord? Yea, the stresse of such twisting bindeth with such Tenacity, that a few Rushes, and of no great length, (so that the wreaths and windings are but few wherewith they entertwine) make very strong bands, called, as I take it, * Thum-ropes.

* Fusta.

SAGR. Your Discourse hath removed the wonder out of my mind at two effects, whereof I did not well understand the reason; One was to see, how two, or at the most three twines of the

Rope

Rope about the Axis of a Crane did not only hold it, that being drawn by the immense force of the weight, which it held, it slipt nor shrunk not; but that moreover turning the Crane about, the said Axis with the sole touch of the Rope which begirteth it, did in the after turnings, draw and raise up vast stones, whilst the strength of a little Boy sufficed to hold and stay the other end of the same Cord. The other is at a plain, but cunning, Instrument found out by a young Kinsman of mine, by which with a Cord he could let himself down from a window without much gauling the palmes of his hands, as to his great smart not long before he had done. For the better understanding whereof, take this Scheame: About such a Cylinder of Wood as A B, two Inches thick, and six or eight Inches long, he cut a hollow notch spirally, for one turn and a half and no more, and of widenesse fit for the Cord he would use; which he made to enter through the notch at the end A, and to come out at the other B, incircling afterwards the Cylinder in a barrel or socket of Wood, or rather Tin, but divided lengthwaies, and made with Claspes or Hinges to open and shut at pleasure: and then grasping and holding the said Barrel or Case with both his hands, the rope being made fast above, he hung by his arms; and such was the compression of the Cord between the moving Socket and the Cylinder, that at pleasure griping his hands closer he could stay himself without descending, and slacking his hold a little, he could let himself down as he pleased.

An Hand-Pully or Instrument invented by an amarous person to let himself down from any great height with a Cord without gauling his hands.

SALV. An ingenious invention verily, and for a full explanation of its nature, me-thinks I discover, as it were by a shadow, the light of some other additional discoveries: but I will not at this time deviate any more from my purpose upon this particular: and the rather in regard you are desirous to hear my opinion of the Resistance of other Bodies against fraction, whose texture is not with threads, and fibrous strings, as is that of Ropes, and most kinds of Wood; but the connection of their parts seem to depend on other Causes; which in my judgment may be reduced to two heads: one is the much talked-of Repugnance that Nature hath against the admission of Vacuity: for another (this of Vacuity not sufficing) there must be introduced some glue, viscous matter, or Cement, that tenaciously connecteth the Corpuscles of which the said Body is compacted.

Why such Bodies resist Fraction that are not framed with Fibrous filaments.

I will first speak of *Vacuity*, shewing by plain experiments,

The first Cause of the Coherence of Bodies is their Repugnance to Vacuity.

This is proved by the Coherence of two polished Marbles.

what and how great its virtue is. And first of all the seeing at pleasure two flat pieces of either Marble, Metal, or Glasse, exquisitely planed, slickt, and polished, that being laid upon one the other, without any difficulty slide along upon each other, if drawn sidewaies, (a certain argument that no glue connects them,) but that going about to seperate them, keeping them equidistant, there is found such repugnance, that the uppermost will be lifted up, and will draw the other after it, and keep it perpetually raised, though it be pretty thick, and heavy, evidently proveth to us, how much Nature abhorreth to admit, though for a short moment of time, the void space, that would be between them, till the concourse of the parts of the Circum-Ambient Air should have possest, and repleated it. We see likewise, that if those two Plates be not exactly polished, and consequently their contact not every where exquisite; in going about to separate them gently, there will be found no Renitence more than that of their meer weight, but in the sudden raising, the nether Stone will rise, and instantly fall down again, following the upper only for that very small time which serveth for the expansion of that little Air which interposeth betwixt the Plates, that did not every where touch, and for the ingression of the other circumfused. The like Resistance, which so sensibly discovers it self betwixt the two Plates, cannot be doubted to reside also between the parts of a Solid, and that it entereth into their connection, at least in part, and as their Concomitant Cause.

SAGR. Hold, I pray you, and permit me to impart unto you a particular Consideration, just now come into my Mind, and this it is; That seeing how the lower Plate followeth the upper, and is by a speedy motion raised, we are thereby ascertained that (contrary to the saying of many Philosophers, and perchance of *Aristotle* himself) the Motion in *Vacuity* would not be Instantaneous; for should it be such, the proposed Plates without the least repugnance would Seperate; since the self-same instant of time would suffice for their separation, and for the concourse of the Ambient Air to repleat that *Vacuity*, which might remain between them. By the Inferiour Plates following the Superiour therefore may be gathered, that in the *Vacuity* the Motion would not be Instantaneous. And also it may be inferred, that even betwixt those Plates there resteth some *Vacuity*, at least for some very short time, that is, for so long as the Ambient Air is moving whilst it concurreth to replete the *Vacuum*: for if there did no *Vacuity* remain, there would be no need either of the Concourse, or Motion of the Ambient. We must therefore say that *Vacuity* sometimes is admitted, though by Violence or against Nature, (albeit it is my opinion, that nothing is contrary to Nature, but that which is impossible

Vacuity partly the cause of the Coherence between the parts of Solids.

possible

possible, which again never is.) But here starts up another difficulty, and it is, That though Experience assures me of the truth of the Conclusion, yet my Judgment is not thorowly satisfied of the Cause, to which such an effect may be ascribed. For as much as the effect of the Seperation of the two Plates, is in time before the Vacuity which should succeed by consequence upon the Separation. And because, in my opinion, the Cause ought, if not in Time, at least in Nature, to precede the Effect: and that of a Positive Effect, the Cause ought also to be Positive; I cannot conceive, how the Cause of the Adhesion of the two Plates, and of their Repugnance to Separation, (Effects that are already in Act) should be assigned to Vacuity, which yet is not, but should follow. And of things that are not in being, there can be no Operation; according to the infallible Maxime of Philosophy.

Of a Positive Effect the Cause is Positive.

Non-entity is attended with Non-operation.

SIMP. But since you grant *Aristotle* this Axiome, I do not think you will deny another that is most excellent, and true; to wit, That Nature doth not attempt Impossibilities: Upon which Axiom I think the Solution of our doubt depends: because therefore a void space is of it self impossible, Nature forbids the doing that, in consequence of which Vacuity would necessarily succeed; and such an act is the separation of the two Plates.

Nature doth not attempt Impossibilities.

SAGR. Now, (admitting this which *Simplicius* alledgeth is a sufficient Solution of my Doubt) in persuance of the discourse with which I began, it seemeth to me, that this same Repugnance to Vacuity should be a sufficient Cement in the parts of a Solid of Stone, Metal, or what other substance is more firmly conjoyned, and averse to Division. For if a single Effect, hath but one sole Cause, as I understand, and think; or if many be assigned, they are reducible to one alone: why should not this of Vacuity, which certainly is one, be sufficient to answer all Resistances?

SALV. I will not at this time enter upon this contest, whether Vacuity, without other Cement, be in it self alone sufficient to keep together the separable parts of firm Bodies; but yet this I say, that the Reason of the Vacuity, which is of force, and concluding in the two Plates, sufficeth not of it self alone for the firm connection of the parts of a solid Cylinder of Marble, or Metal, the which forced with great violence, pulling them streight out, in fine, divide and separate. And in case I have found a way to distinguish this already-known Resistance dependent on Vacuity, from all others whatsoever that may concur with it in strengthening the Connection, and make you see how that it alone is not neer sufficient for such an Effect, would not you grant that it would be necessary to introduce some other? Help him out, *Simplicius*, for he stands studying what to answer.

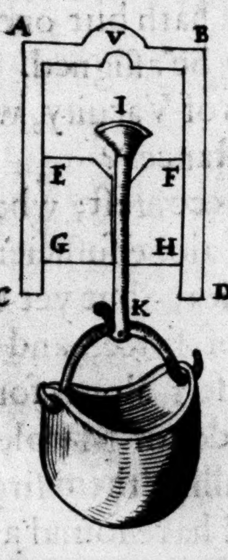
SIMP. The Suspension of *Sagredus* must needs be upon ano-

ther account, there being no place left for doubting of so clear, and necessary a Consequence.

SAGR. You Divine *Simplicius*, I was thinking if a Million of Gold *per annum*, coming from *Spaine*, not being sufficient to pay the Army, whether it was necessary to make any other provision than of Money to pay the Souldiers. But proceed, *Salviatus*, and supposing that I admit of your Consequence, shew us how to separate the opperation of Vacuity from the other, that measuring it we may see how it's insufficient for the Effect of which we speak.

SALV. Your Genius hath prompted you. Well, I will tell you the way to part the Virtue of Vacuity from the rest, and then how to measure it. And to sever it, we will take a continuate matter, whose parts are destitute of all other Resistance to Separation, save only that of Vacuity, such as Water at large hath been demonstrated to be in a certain Tractate of our *Accademick*. So that when ever a Cylinder of Water is so disposed, that being drawn we find a Resistance against the separation of its parts, this must be acknowledged to proceed from no other cause, but from repugnance to Vacuity. But to make such an experiment, I have imagined a device, which with the help of a small Diagram, may be better exprest than by my bare words. Let this Figure C A B D be the Profile of a Cylinder of Metal, or of Glass, which must be made hollow within, but turned exactly round; into whose Concave must enter a Cylinder of Wood, exquisitely fitted to touch every where, whose Profile is noted by E G H F, which Cylinder may be thrust upwards, and downwards: and this I would have bored in the middle, so that there may a rod of Iron pass thorow, hooked in the end K, and the other end I, shall grow thicker in fashion of a Cone, or Top; and let the hole made for the same thorow the Cylinder of Wood be also cut hollow in the upper part, like a Conical Superficies, and exactly fitted to receive the Conick end I, of the Iron I K, as oft as it is drawn down by the part K. Then I put the Cylinder of Wood E H into the Concave Cylinder A D, and would not have it come to touch the uppermost Superficies of the said hollow Cylinder, but that it stay two or three fingers breadth from it: and I would have that space filled with Water; which should be put therein, holding the Vessel with the mouth C D upwards; and thereupon press down the Stopper E H, holding the Conical part I somewhat distant from the hollow that was made

How to measure the Virtue of Vacuity in Solids distinct from other convenient Causes of their Coherence. Water a Continuate Matter, and void of all other aversion to separation, save that of Vacuity.



for

for it in the Wood, to leave way for the Air to go out, which in thrusting down the Stopper will issue out by the hole of the Wood, which therefore should be made a little wider than the thickness of the Hook of Iron I K. The Air being let out, and the Iron pull'd back, which close stoppeth the wood with its Conick part I, then turn the vessel with its mouth downwards, and fasten to the hook K a Bucket that may receive into it sand, or other weighty matter, and you may hang so much weight thereat, that at length the Superiour surface of the Stopper E F will separate and forsake the inferiour part of the Water; to which nothing else held it connected but the Repugnance against Vacuity: afterwards weighing the Stopper with the Iron, the Bucket, and all that was in it, you will have the quantity of the Force of the Vacuity. And if affixing to a Cylinder of Marble, or Christal, as thick as the Cylinder of Water, such a weight, that together with the proper weight of the Marble or Christal it self, equalleth the gravity of all those fore-named things, a Rupture follow thereupon; we may without doubt affirm, that the only reason of Vacuity holdeth the parts of Marble and Christal conjoyned: but not sufficing; and seeing that to break it there must be added four times as much weight, it must be confessed, that the Resistance of Vacuity is one part of five, and that the other Resistance is quadruple to that of Vacuity.

SIMP. It cannot be denied, but that the Invention is Ingenious: but I hold it to be subject to many difficulties, which makes me question it; for who shall assure us, that the Air cannot penetrate between the Glass, and the Stopper, though it be close stopt with Flax, or other pliant matter? And also it's a Question, whether Wax or Turpentine will serve to make the Cone I, stop the hole close: Again, Why may not the parts of the Water withdraw and rarefie themselves? Why may not the Air, or Exhalations, or other more subtil Substances penetrate through the Porosities of the Wood, or Glass it self?

SALV. *Simplicius* is very nimble at raising doubts, and, in part, helping us to resolve them, as to the Penetration of the Air through the Wood, or between the Wood and Glass. But I moreover observe, that we may at the same time secure our selves, and withall acquire new Notions, if the fore-named doubts take place; for if the Water be by Nature, howbeit with violence, capable of extention, as it falleth out in Air, you shall see the Stopper to descend: and if in the upper part of the Glass we make a small prominent Boss, as this V; in case any Air, or other more Tenuous or Spirituous Matter should penetrate thorow the Substance, or Porosity of the Glass, or Wood, it would be seen to reunite (the water giving place) in the eminence V: which things not being perceived, we rest assured that the Experiment was made with due caution:

caution: and see that the Water is not capable of extension, nor the Glass permeable by any matter, though never so subtil.

The Nature of the attraction of Water by Pumps.

SAGR. And I, by means of these Discourses have found the Cause of an Effect, that hath for a long time puzled my mind with wonder, and kept it in Ignorance. I have heretofore observed a Cistern, wherein, for the drawing thence of Water, there was made a Pump, by some one that thought, perhaps, (but in vain) to be thereby able to draw, with less labour, the same, or greater quantity of Water, than with the ordinary Buckets; and this Pump had its Sucker and Value on high, so that the Water was made to ascend by Attraction, and not by Impulse, as do the Pumps that work below. This, whilst there is any Water in the Cistern to such a determinate height, will draw it plentifully; but when the Water ebbeth below a certain Mark, the Pump will work no more. I conceited, the first time that I observed this accident, that the Engine —— had been spoyled, and looking for the Workman, that he might amend it; he told me, that there was no defect at all, other than what was in the Water, which being fallen too low, permitted not it self to be raised to such a height; and farther said, that neither Pump, or other Machine, that raiseth the water by Attraction, was possibly able to make it rise a hair more than eighteen Braces, and be the Pumps wide or narrow, this is the utmost limited measure of their height. And I have hitherto been so dull of apprehension, that though I knew that a Rope, a Stick, and a Rod of Iron might be so and so lengthened, that at last, holding it up on high in the Air, its own weight would break it, yet I never remembred, that the same would much more easily happen in a Rope, or Thread of Water. And what other is that which is attracted in the Pump than a Cylinder of Water, which having its contraction above, prolonged more and more, in the end arriveth to that term, beyond which being drawn, it breaketh by its foregoing over-weight, just as if it was a Rope.

Water raised or attracted by a Pump riseth not above eleven yards.

SALV. It is even so as you say; and because the said height of eighteen Braces is the prefixed term of the Elevation, to which any quantity of Water, be it (that is to say, be the Pump) broad, narrow, or even, so narrow as to the thickness of a straw, can sustain it self; when ever we weigh the water contained in eighteen Braces of Pipe, be it broad or narrow, we have the value of Resistance of Vacuity in Cylinders of whatsoever solid matter, of the thickness of the proposed Pipes. And since I have said so much, we will shew, that a man may easily find in all Metals, Stones, Timbers, Glasses, &c. How far one may lengthen out Cylinders, strings, or rods of any thickness, beyond which, being opprest with their own weight, they can no longer hold, but break in pieces. Take for example a Brass wyer of any certain thickness, and length, and

To what length Cylinders or Ropes of any Matter may be prolonged, beyond which being charged they break by their own weight

and fixing one of its ends on high, add gradually more and more weight to the other, till at last it break, and let the greatest weight that it can bear be *v. gr.* fifty pounds. It is manifest that fifty pound of Brass more than its own weight, which let us suppose, for example, to be one eighth of an Ounce, drawn out into a Wyer of the like thickness, would be the greatest length of the Wyer that could bear it self. Then measure how long the Wyer was which brake, and let it be for instance a yard; and because it weighed one eighth of an Ounce; and poised, or bore it self, and fifty pounds more; which are Four Thousand Eight Hundred eighths of Ounces; we say, that all Wyers of Brass, whatever thickness they be of, can hold, at the length of Four Thousand Eight Hundred and one yards, and no more: and so, a Brass Wyer being able to hold to the length of 4801 yards; the Resistance it findeth dependent on Vacuity, in respect of the remainder, is as much as is equivalent to the weight of a Rope of Water eighteen Braces long, and of the same thickness with the said Brass Wyer: and finding Brass to be *v. gr.* nine times heavier than Water, in any Wyer of Brass, the Resistance against Fraction dependent on the reason of Vacuity, importeth as much as two Braces of the same Wyer weigheth. And thus arguing, and operating, we may find the length of the Wyers, or Threads of all Solid Matters reduced to the utmost length that they can subsist of, and also what part Vacuity hath in their Resistance.

SAGR. It resteth now, that you declare to us wherein consists the remainder of that Tenacity, that is, what that Glue or Renitence is, which connecteth together the parts of a Solid, besides that which is derived from Vacuity; because I cannot imagine what that Cement is, that cannot be burnt, or consumed in a very hot Furnace in two, three, or four Moneths, nor ten, nor an hundred; and yet Gold, Silver, and Glass, standing so long Liquified, when it is taken out, its parts return, upon cooling, to reunite, and conjoyn, as before. And again, because the same difficulty which I meet with in the Connection of the parts of the Glass, I find also in the parts of the Cement, that is, what thing that should be which maketh them cleave so close together.

SALV. I told you but even now, that your Genius prompted you: I am also in the same strait: and also whereas I have in general told you, how that Repugnance against Vacuity is unquestionably that which permits not, unless with great violence, the separation of the two Plates, and moreover of the two great pieces of the Pillar of Marble, or Brass, I cannot see why it should not also take place, and be likewise the Cause of the Coherence of the lesser parts, and even of the very least and last, of the same Matters: and being that of one sole Effect, there is but one only true, and most

There is but one sole Cause of one sole Effect.

most potent Cause; if I can find no other Cement, why may I not try whether this of Vacuity, which I have already found, may be sufficient?

SIMP. But when you have already demonstrated the Resistance of the great Vacuity in the separation of the two great parts of a Solid to be very small in comparison of that which connecteth, and consolidates the little Particles, or Atomes, why will you not still hold, for certain, that this is extreamly differing from that?

SALV. To this *Sagredus* answereth, That every particular Souldier is still paid with money collected by the general Impositions of Shillings and Pence, although a Million of Gold sufficeth not to pay the whole Army. And who knows, but that other exceeding small Vacuities may operate amongst those small Atomes, (even like as that was of the self-same money) wherewith all the parts are connected? I will tell you what I have sometimes fancied: and I give it you, not as an unquestionable Truth, but as a kind of Conjecture very undigested, submitting it to exacter considerations: Pick out of it what pleaseth you, and judge of the rest as you think fit. Considering sometimes how the Fire, penetrating and insinuating between the small Atomes of this or that Metal, which were before so closely consolidated, in the end separates, and disunites them; and how, the Fire being gone, they return with the same Tenacity as before to Consolidation, without diminishing in quantity, (at all in Gold, and very little in other Metals,) though they continue a long time melted; I have thought that that might happen, by reason the extream small parts of the Fire, penetrating through the narrow pores of the Metal (through which the least parts of Air, or of many other Fluids, could not for their closeness perforate) by repleating the small interposing Vacuities might free the minute parts of the same from the violence, wherewith the said Vacuities attract them one to another, prohibiting their separation; and thus becoming able to move freely, their Mass might become fluid, and continue such, as long as the small parts of the Fire should abide betwixt them: and that those departing, and leaving the former Vacuities, their wonted attractions might return, and consequently the Cohesion of the parts. And, as to the Allegation made by *Simplicius*, it may, in my opinion, be thus resolved; That although such Vacuities should be very small, and consequently each of them easie to be overcome, yet neverthelesss their innumerable multitude innumerably (if it be proper so to speak) multiplieth the Resistances: and we have an evident proof what, and how great is the Force that resulteth from the conjunction of an immense number of very weak Moments, in seeing a Weight of many thousands of pounds, held by

Most small Vacuities disseminated and interposed between the small Corpuscles of Solids the probable cause of the consistence or connection of those Corpuscles to one another.

Innumerable Atomes of Water insinuating into Cables draw and raise an immense weight

by mighty Cables, to yield, and suffer it self at last to be overcome by the assault of the innumerable Atomes of Water; which, either carryed by the South wind, or else by being distended into very thin Mists that move to and fro in the Air, insinuate themselves between string and string of the Hemp of the hardest twisted Cables; nor can the immense force of the pendent Weight prohibit their entrance; so that penetrating the strict passages between the Pores, they swell the Ropes, and by consequence shorten them, whereupon that huge Mass is forcibly raised.

Sagr. There's no doubt but that so long as a Resistance is not infinite, it may by a multitude of most minute Forces be overcome; insomuch that a competent number even of Ants would be able to carry to shore a whole ship lading of Corn: for Sense giveth us quotidian examples, that an Ant carrieth a single grain with ease; and its cleer, that in the Ship there are not infinite grains, but that they are comprised in a certain number; and if you take another number four or six times bigger than that, and take also another of Ants equal to it, and set them to work, they shall carry the Corn, and the Ship also. It is true indeed, that it will be needful that the number be great, as also in my judgment that of the *Vacuities*, which hold together the small parts of the Mettal.

Any finite Resistance is superable by any the least Force, multiplied.

Salv. But though they were required to be infinite, do you think it impossible?

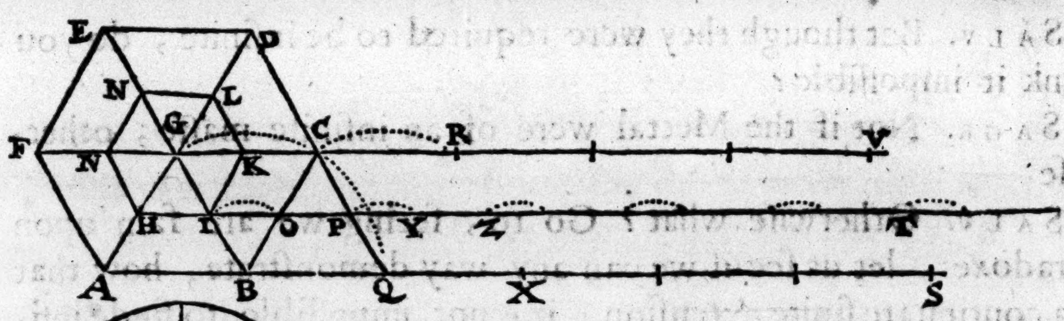
Sagr. Not if the Mettal were of an infinite masse; otherwise ——

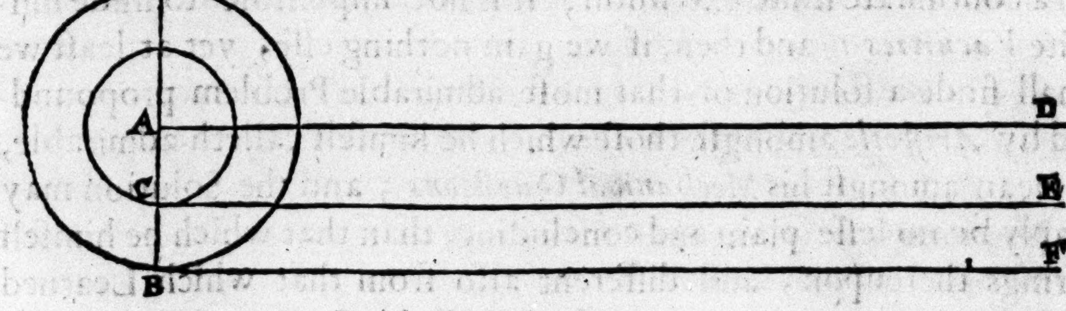
Salv. Otherwise what? Go to, seeing we are faln upon Paradoxes, let us see if we can any way demonstrate, how that in a continuate finite extension, it is not impossible to finde infinite *Vacuities*: and then, if we gain nothing else, yet at least we shall finde a solution of that most admirable Problem propounded by *Aristotle* amongst those which he himself calleth admirable, I mean amongst his *Mechanical Questions*; and the Solution may haply be no lesse plain and concluding, than that which he himself brings thereupon, and different also from that which Learned *Monsig. di Guevara* very acutely discusseth. But it is first requisite to declare a Proposition not toucht by others, on which the solution of the question dependeth, which afterwards, if I deceive not my self, will draw along with it other new and admirable Notions; for understanding whereof the more exactly, we will give it you in a Scheme: We suppose, therefore an equilateral, and equiangled Poligon of any number of Sides at pleasure, described about this Center G; and in this example let it be a Hexagon ABCDEF; like to which, and concentrick with the same must be distributed another lesser, which we mark HIKLMN;

Aristotles *admirable Problem of two Concentrick Circles that turn round, and its true resolution.*

Monsig. Guevara *honourably mentioned.*

and let one Side of the greater A B be prolonged indeterminately towards S, and of the lesse the correspondent Side H I is to be produced in like manner towards the same part, representing the Line H T, parallel to A S; and let another passe by the Center equidistant from the former, namely G V. This done, we suppose the greater Poligon to turn about upon the Line A S, carrying with it the other lesser Poligon, It is manifest, that the point B, the term of the Side A B, standing still, whilst the Revolution begins, the angle A riseth, and the point C descendeth, describing the arch C Q; so that the Side B C is applyed to the line B Q, equal to it self: but in such conversion the angle I of the lesser Poligon riseth above the Line I T. for that I B is oblique upon A S: nor will the point I fall upon the parallel I T, before the point C come to Q: and by that time I shall be descended unto O after it had described the Arch I O, without the Line H T: and at the same time the Side I K shall have pass'd to O P. But the Center G shall have gone all this time out of the Line G V, on which it shal not fall, until it shall first have described the Arch G C. Having made this first step, the greater Poligon shall be transposed to rest with the Side B C upon the Line B Q; the Side I K of the lesser upon the Line O P, having skipt all the Line I O without touching





it; and the Center G shall be removed to C, making its whole course without the Parallel G V: And in fine all the Figure shall be remitted into a Position like the first; so that the Revolution being continued, and coming to the second step, the Side of the greater Poligon D C shall remove to Q X; K L of the lesser (having first skipt the Arch P Y) shall fall upon Y Z, and the Center proceeding evermore without G V shall fall on it in R, after the great skip C R. And in the last place, having finished an entire Conversion, the greater Poligon will have impressed upon A S, six

Lines

Lines equal to its Perimeter without any interpositions or skips: the lesser Poligon likewise shall have traced six Lines equal to its Perimeter, but discontinued by the interposition of five Arches, under which are the Chords, parts of the parallel H T not toucht by the Poligon: And lastly, the Center G never hath toucht the Parallel G V except in six points. From hence you may comprehend, how that the Space passed by the lesser Poligon, is almost equal to that passed by the greater, that is the Line H T is almost equal to A S, then which it is lesser only the quantity of one of these Arches, taking the Line H T, together with all its Arches. Now, this which I have declared and explained to you in the example of these Hexagons, I would have you understand to hold true in all other Poligons, of what number of Sides soever they be, so that they be like Concentrick, and Conjoyned; and that at the Conversion of the greater, the other, how much soever lesser, be supposed to revolve therewith: that is, you must understand, I say, that the Lines by them passed are very near equal, computing into the Space past by the lesser, the Intervals under the little Arches not toucht by any part of the Perimeter of the said lesser Poligon. Let therefore the greater Poligon, of a thousand Sides, pass round, and measure out a continued Line equal to its Perimeter; and in the same time the less passeth a Line almost as long, but compounded of a thousand Particles equal to its thousand Sides, but discontinued with the interposition of a thousand void Spaces: for such may we call them, in relation to the thousand little Lines toucht by the Sides of the Poligon. And what hath been spoken hitherto admits of no doubt or scruple. But tell me, in case that about a Center, as suppose the point A, (in the former Scheme) we should describe two Circles concentrick, and united together; and that from the points C and B of their Semi-Diameters, there be drawn the Tangents C E, and B F, and by the Center A the Parallel A D; supposing the greater Circle to be turned upon the Line B F, (drawn equal to its Circumference, as likewise the other two C E, and A D;) when it hath compleated one Revolution, what shall the lesser Circle, and Center have done? The Center shall doubtless have run over, and touched the whole Line A D, and the less Circumference shall with its touches have measured all C E, doing the same as did the Poligons above; and different only in this, that the Line H T was not touched in all its Parts by the Perimeter of the lesser Poligon, but there were as many parts left untoucht with the interposition of salts, or skipped spaces; as were these parts touched by the Sides: but here in the Circles, the Circumference of the lesser Circle, never separates from the Line C E, so as to leave any of its parts untoucht; nor is the parts touching of the Circumference, less than the part toucht of the

Right-line. Now how is it possible that the lesser Circle should without skips run a Line so much bigger than its Circumference?

SAGR. I was considering whether one might not say, that like as the Center of the Circle trailed alone upon A D toucht, it all being yet but one sole Point; so likewise might the Points of the lesser Circumference, drawn by the revolution of the greater, go gliding along some small part of the Line C E.

SALV. This cannot be for two reasons; first, because there is no reason why some of the touches like to C should go gliding along some part of the Line C E, more than others: and though there should; such touches being (because they are points) infinite, the glidings along upon C E would be infinite; and so being, they would make an infinite Line, but the Line C E is finite. The other reason is, that the greater Circle, in its Revolution continually changing contact, the lesser Circle must of necessity do the like; there being no other Point but B, by which a Right Line can be drawn to the Center A, and passing through C; so that the greater Circumference changing Contact, the less doth change it also; nor doth any Point of the less touch more than one Point of its Right-Line C E: besides, that also in the conversion of the Poligons, no Point of the Perimeter of the less falls on more than one Point of the Line, which was by the said Perimeter traced, as may be easily understood, considering the Line I K is parallel to B C, whereupon, till just that B C fall on B R, I K continueth elevated above I P, and toucheth it not before B C is on the very Point of uniting with B Q, and then all in the same instant I K uniteth with O P, and afterwards immediately riseth above it again.

SAGR. The business is really very intricate, nor can I think on any Solution of it, therefore do you explain it to us as far as you judge needful.

SALV. I should, for the evincing hereof, have recourse to the consideration of the fore-described Poligons, the effect of which is intelligible and already comprehended, and would say, that like as in the Poligons of an hundred thousand Sides, the Line passed and measured by the Perimeter of the greater, that is by its hundred thousand Sides continually distended, is not considerably bigger than that measured by the hundred thousand Sides of the less, but with the interposition of an hundred thousand void spaces intervening; so I would say in the Circles (which are Poligons of innumerable Sides) that the Line measured by the infinite Sides of the great Circle, lying continued one with another, to be equalled in length by the Line traced by the infinite Sides of the less, but by these including the interposition of the like number of intervening Spaces: and like as the Sides are not quantitative, but yet infinite

in

in number, ſo the interpoſing Vacuities are not quantitative, but infinite in number; that is, thoſe are infinite Points all filled, and theſe are infinite points,part filled, and part empty. And here I would have you note,that reſolving,and dividing a Line into quantitative parts,and conſequently of a finite number,it is not poſſible to diſpoſe them into a greater extention than that which they poſſeſt whilſt they were continued,and connected, without the interpoſition of a like number of void Spaces; but imagining it to be reſolved into parts not quantitative,namely, into its infinite indiviſibles, we may conceive it produced to immenſity without the interpoſition of quantitative void ſpaces, but yet of infinite indiviſible Vacuities. And this which is ſpoken of ſimple lines,ſhould alſo be underſtood of Superficies,and Solid Bodies,conſidering that they are compoſed of infinite Atomes not non-quantitative;if we would divide them into certain quantitative parts, there's no queſtion,but that we cannot diſpoſe them into Spaces more ample than the Solid before occupied, unleſs with the interpoſition of a certain number of quantitative void Spaces;void,I ſay, at leaſt of the matter of the Solid: but if we ſhould propoſe the higheſt,and ultimate reſolution made into the firſt, non-quantitative, but infinite firſt compounding parts,we may be able to conceive ſuch compounding parts extended unto an immenſe Space without the interpoſition of quantitative void Spaces;but only of infinite non-quantitative Vacuities: and in this manner a man may draw out, *v.gr.* a little Ball of Gold into a very vaſt expanſion without admitting any quantitative void Spaces; yet nevertheleſs we may admit the Gold to be compounded of infinite inducible ones.

SIMP. Me thinks that in this point you go the way of thoſe diſſeminated Vacuities of a certain *Ancient Philoſopher* ——

SALV. But you add not: [*who denied Divine Providence*:) as on ſuch another occaſion, ſufficiently beſides his purpoſe, a certain Antagoniſt of our *Accademick* did ſubjoyn.

SIMP. I ſee very well, and not without indignation,the malice of ſuch contradictors; but I ſhall forbear theſe Cenſures, not only upon the ſcore of Good-Manners, but becauſe I know how diſagreeing ſuch Tenets are to the well-tempered, and well-diſpoſed mind of a perſon,ſo Religious and Pious,yea, Orthodox and Holy,as you, Sir. But returning to my purpoſe; I find many ſcruples to ariſe in my mind about your laſt Diſcourſe, which I know not how to reſolve. And this preſents its ſelf for one, that if the Circumferences of two Circles are equall to the two Right Lines C E,and B F, this taken continually, and that, with the interpoſition of infinite void Points; how can A D,deſcribed by the Center, which is but one ſole Point, be ſaid to be equal to the ſame,it containing infinite of them? Again, that ſame compoſing the Line of

Points,

Points, the divisible of indivisibles, the quantitative of non-quantitative, is a rock very hard, in my judgment, to pass over: And the very admitting of Vacuity, so thorowly confuted by *Aristotle*, no less puzleth me than those difficulties themselves.

SALV. There be, indeed, these and other difficulties; but remember, that we are amongst Infinites, and Indivisibles: those in comprehensible by our finite understanding for their Grandure; and these for their minuteness: nevertheless we see that Humane Discourse will not be beat off from ruminating upon them, in which regard, I also assuming some liberty, will produce some of my conceits, if not necessarily concluding, yet for novelty sake, which is ever the messenger of some wonder: but perhaps the carrying you so far out of your way begun, may seem to you impertinent, and consequently little pleasing.

SAGR. Pray you let us enjoy the benefit, and priviledge, of free speaking which is allowed to the living, and amongst friends; especially, in things arbitrary, and not necessary; different from Discourse with dead Books, which start us a thousand doubts, and resolve not one of them. Make us therefore partakers of those Considerations, which the course of our Conferences suggest unto you; for we want no time, seeing we are disengaged from urgent businesses, to continue and discusse the other things mentioned; and particularly, the doubts, hinted by *Simplicius*, must by no means escape us.

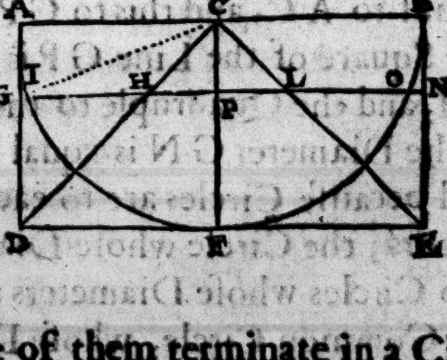
SALV. It shall be so, since it pleaseth you: and beginning at the first, which was, how it's possible to imagine that a single Point is equal to a Line; in regard I can do no more for the present, I will attempt to satisfie, or, at least, qualifie one improbability with another like it, or greater; as some times a Wonder is swallowed up in a Miracle. And this shall be by shewing you two equal Superficies, and at the same time two Bodies, likewise equal, and placed upon those Superficies as their Bases; and that go (both these and those) continually and equally diminishing in the self-same time, and that in their remainders rest alwaies equal between themselves, and (lastly) that, as well Superficies, as Solids, determine their perpetual precedent equalities, one of the Solids with one of the Superficies in a very long Line; and the other Solid with the other Superficies in a single Point: that is, the latter in one Point alone, the other in infinite.

The equal Superficies of two Solids continually substracting from them both equal parts, are reduced, the one into the Circumference of a Circle, and the other into a Point.

SAGR. An admirable proposal, really, yet let us hear you explain and demonstrate it.

SALV. It is necessary to give you it in Figure, because the proof is purely Geometrical. Therefore suppose the Semicircle A F B, and its Center to be C, and about it describe the Rectangle A D E B, and from the Center unto the Points D and E let there be drawn the Lines C D, and C E; Then drawing the Semi-Diameter

meter C F, perpendicular to one of the two Lines A B, or D E and immoveable; we suppose all this Figure to turn round about that Perpendicular: It is manifest, that there will be described by the Parallelogram A D E B, a Cylinder; by the Semi-circle A F B, an Hemi-Sphære; and by the Triangle C D E a Cone. This presupposed, I would have you imagine the Hemisphære to be taken away, leaving behind the Cone, and that which shall remain of the Cylinder; which for the Figure, which it shall retain like to a Dish, we will hereafter call a Dish: touching which, and the Cone, we will first demonstrate that they are equal; and next a Plain being drawn parallel to the Circle, which is the foot or Base of the Dish, whose Diameter is the Line D E, and its Center F; we will demonstrate, that should the said Plain pass, *v. gr.* by the Line G H, cutting the Dish in the points G I, and O N; and the Cone in the points H and L; it would cut the part of the Cone C H L, equal alwaies to the part of the Dish, whose Profile is represented to us by the Triangles G A I, and B O N: and moreover we will prove the Base also of the same Cone, (that is the Circle, whose Diameter is H L) to be equal to that circular Superficies, which is Base of the part of the Dish; which is, as we may say, a Rimme as broad as G I; (note here by the way what Mathematical Definitions are: they be an imposition of names, or, we may say, abreviations of speech, ordain'd and introduced to prevent the trouble and pains, which you and I meet with, at present, in that we have not agreed together to call *v. gr.* this Superficies a circular Rimme, and that very sharp Solid of the Dish a round Razor:) now howsoever you please to call them, it sufficeth you to know, that the Plain produced to any distance at pleasure, so that it be parallel to the Base, *viz.* to the Circle whose Diameter D E cuts alwaies the two Solids, namely, the part of the Cone C H L, and the upper part of the Dish equal to one another: and likewise the two Superficies, Basis of the said Solids, *viz.* the said Rimme, and the Circle H L, equal also to one another. Whence followeth the forementioned Wonder; namely, that if we should suppose the cutting-plain to be successively raised towards the Line A B, the parts of the Solid cut are alwaies equall, as also the Superficies, that are their Bases, are evermore equal; and, in fine, raising the said Plain higher and higher, the two Solids (ever equal) as also their Bases, (Superficies ever equal) shall one couple of them terminate in a Circumference of a Circle, and the other couple in one sole point;



for

for such are the upper Verge or Rim of the Dish, and the Vertex of the Cone. Now whilst that in the diminution of the two Solids, they till the very last maintain their equality to one another, it is, in my thoughts, proper to say, that the highest and ultimate terms of such Diminutions are equal, and not one infinitely bigger than the other. It seemeth therefore, that the Circumference of an immense Circle may be said to be equal to one single point; and this that befalls in Solids, holdeth likewise in the Superficies their Bases; that they also in the common Diminution conserving alwaies equality, in fine, determine at the instant of their ultimate Diminution the one, (that is, that of the Dish) in their Circumference of a Circle, the other (to wit, that of the Cone) in one sole point. And why may not these be called equal, if they be the last remainders, and footsteps left by equal Magnitudes? And note again, that were such Vessels capable of the immense Cœlestial Hemispheres: both their upper Rims, and the points of the contained Cones (keeping evermore equally to one another) would finally determine, those, in Circumferences equal to those of the greatest Circles of the Cœlestial Orbes, and these in simple points. Whence, according to that which such Speculations perswade us to, all Circumferences of Circles, how unequal soever, may be said to be equal to one another, and each of them equal to one sole point.

SAGR. The Speculation is, in my esteem, so quaint and curious, that, for my part, though I could, yet would I not oppose it, for I take it for a piece of Sacriledge to deface so fine a Structure, by spurning at it with any pedantick contradiction; yet for our entire satisfaction, give us the proof (which you say is Geometrical) of the equality alwaies retained between those Solids, and those their Bases, which I think must needs be very subtil, the philosophical Contemplation being so nice, which depends on the said Conclusion.

SALV. The Demonstration is but short, and easie. Let us keep to the former Figure, in which the Angle I P C being a Right Angle, the Square of the Semi-Diameter I C is equal to the two Squares of the Sides I P, and P C. But the Semi-Diameter I C, is equal to A C, and this to G P, and C P is equal to P H; therefore the Square of the Line G P is equal to the two Squares of I P, and P H, and the Quadruple to the Quadruples; that is, the Quadrate of the Diameter G N is equal to the two Quadrates I O, and H L: and because Circles are to each other, as the Squares of their Diameters; the Circle whose Diameter is G N, shall be equall to the two Circles whose Diameters are I O, and H L; and taking away the Common Circle, whose Diameter is I O; the residue of the Circle G N shall be equal to the Circle, whose Diameter is H L.

And

And this is as to the first part. Now as for the other part, we will, for the present, omit its Demonstration, as well because that if you would see it, you shall find it in the twelfth Proposition of the Second Book *De centro Gravitatis Solidorum*, published by Signour *Lucas Valerius*, the new *Archimedes* of our Age; who upon another occasion hath made use of it; as because in our case it sufficeth to have seen, how the Superficies, already explained, are evermore equal; and that alwaies diminishing equally, they in the end determine, one in a single point, and the other in the Circumference of a Circle, be it never-somuch bigger, for in this lyeth our Wonder.

Lucas Valerius, *the other* Archimedes *of our Age, hath written admirably*, De Centro Gravitatis Solidorum.

SAGR. The Demonstration is as ingenious, as the reflection grounded upon it is admirable. Now let us hear somewhat about the other Doubt suggested by *Simplicius*, if you have any particulars worth note to hint thereupon, but I should incline to think it impossible to be, in regard it is a Controversie that hath been so canvassed.

SALV. You shall have some of my particular thoughts thereon; first repeating what but even now I told you, namely, that Infinity alone, as also Indivisibility, are things incomprehensible to us; now think how they will be conjoyned together: and yet if you would compound the Line of indivisible points, you must make them infinite; and thus it will be requisite to apprehend in the same instant both Infinite, and Indivisible. The things that at several times have come into my mind, on this occasion, are many; part whereof, and the more considerable, it may be, I cannot upon such a sudden remember; but it may happen, that in the sequal of the Discourse, coming to put questions and doubts to you, and particularly to *Simplicius*, they may, on the other side, re-mind me of that, which without such excitement would have lain dormant in my Fancy: and therefore, with my wonted freedom, permit me that I produce any wild conjectures, for such may we fitly call them in comparison of supernatural Doctrines, the only true and certain determiners of our Controversies, and unerring guides in our obscure, and dubious paths, or rather Laberinths.

Amongst the first Instances that are wont to be produced against those that compound *Continuum* of Indivisibles, this is usually one; That an Indivisible, added to another Indivisible, produceth not a thing divisible; for if that were so, it would follow, that even the Indivisibles were divisible: for if two Indivisibles, as for example, two Points conjoyned, should make a Quantity that should be a divisible Line, much more such should one be that is compounded of three, five, seven, or others, that are odd numbers; the which Lines, being to be cut in two equal parts, render divisible that Indivisible which was placed in the middle. In this

Continuum *compounded of indivisibles.*

and other Objections of this kind you may satisfie the proposer of them, telling him, that neither two Indivisibles, nor ten, nor an hundred, no, nor a thousand can compound a Magnitude divisible, and quantitative, but being infinite they may.

SIMP. Here already riseth a doubt, which I think unresolvable; and it is, that we being certain to find Lines one bigger than another, although both contain infinite Points, we must of necessity confess, that we have found in the same Species a thing bigger than infinite; because the Infinity of the Points of the greater Line, shall exceed the Infinity of the Points of the lesser. Now this assigning of an Infinite bigger than an Infinite is, in my opinion, a conceit that can never by any means be apprehended.

SALV. These are some of those difficulties, which result from the Discourses that our finite Judgments make about Infinites, giving them those attributes which we give to things finite and terminate; which I think is inconvenient; for I judge that these terms of Majority, Minority, and Equality sute not with Infinites, of which we cannot say that one is greater, or less, or equal to another: for proof of which there cometh to my mind a Discourse, which, the better to explain, I will propound by way of Interrogatories to *Simplicius* that started the question.

I suppose that you very well understand which are Square Numbers, and which not Square.

SIMP. I know very well, that the Square Number is that which proceeds from the multiplication of another Number into it self; and so four, and nine, are Square Numbers, that arising from two, and this from three multiplied into themselves.

SALV. Very well; And you know also, that as the Products are called Squares: the Produsors, that is, those that are multiplied, are called Sides, or Roots; and the others, which proceed not from Numbers multiplied into themselves, are not Squares. So that if I should say, all Numbers comprehending the Square, and the not Square Numbers, are more than the Square alone, I should speak a most unquestionable truth: Is it not so?

SIMP. It cannot be denied.

SALV. Farther questioning, if I ask you how many are the Numbers Square, you can answer me truly, that they be as many, as are their propper Roots; since every Square hath its Root, and every Root its Square, nor hath any Square more than one sole Root, or any Root more than one sole Square.

SIMP. True.

An Infinite Number, as it contains infinite Square and Cube Roots, so it containeth infinite Square and Cube Numbers.

SALV. But if I shall demand how many Roots there be, you cannot deny but that they be as many as all Numbers, since there is no Number that is not the Root of some Square: And this being granted, it is requisite to affirm, that Square Numbers are as many

many as their Roots, and Roots are all Numbers: and yet in the beginning we said, that all Numbers are far more than all Squares, the greater part not being Squares: and yet neverthelesſs the number of the Squares goeth diminiſhing alwaies with greater proportion, by how much the greater number it riſeth to; for in an hundred there are ten Squares, which is as much as to ſay, the tenth part are Squares: in ten thouſand only the hundredth part are Squares: in a Million only the thouſandth, and yet in an Infinite Number, if we are able to comprehend it, we may ſay the Squares are as many, as all Numbers put together.

SAGR. What is to be reſolved then on this occaſion?

SALV. I ſee no other deciſion that it may admit, but to ſay, that all Numbers are infinite, Squares are infinite, their Roots are infinite; and that neither is the multitude of Squares leſs than all Numbers, nor this greater than that: and in concluſion, that the Attributes of Equality, Majority, and Minority, have no place in Infinites, but only in terminate quantities. And therefore when *Simplicius* propoundeth to me many unequal Lines, and demandeth of me, how it can be, that in the greater there are no more Points than in the leſs: I anſwer him, That there are neither more, nor leſs, nor juſt ſo many; but in each of them infinite. Or if I had anſwered him, that the Points in one, are as many as there are Square Numbers; in another bigger, as many as all Numbers; in a leſs, as many as the Cubick Numbers, might not I have given ſatisfaction, by aſſigning more to one, than to another, and yet to every one infinite? And thus much as to the firſt difficulty.

SAGR. Hold, I pray you, and give me leave to add unto what hath been ſpoken hitherto, a thought which I juſt now light on, and it is this, that granting what hath been ſaid, me-thinks, that not only it's improper to ſay, one Infinite is greater than another Infinite, but alſo, that it's greater than a Finite; for if an Infinite Number were greater, *v. gr.* than a Million; it would thereupon follow, that paſſing from the Million to others, and ſo to others continually greater, one ſhould paſs on towards Infinity; which is not ſo: but on the contrary, to how much the greater Numbers we go, ſo much the more we depart from Infinite Number; becauſe in Numbers, the greater you take, ſo much the rarer and rarer alwaies are Square Numbers contained in them; but in Infinite Number the Squares can be no leſs than all Numbers, as but juſt now was concluded: therefore the going towards Numbers alwaies greater, and greater, is a departing farther from Infinite Number.

SALV. And ſo by your ingenious Diſcourſe we may conclude, that the Attributes of Greater, Leſſer, or Equal, have no place, not only amongſt Infinites; but alſo betwixt Infinites, and Finites.

I pass now to another Consideration; and it is, that in regard that the Line, and every continued quantity are divideable continually into divisibles, I see not how we can avoid granting that the composition is of infinite Indivisibles: because a division and subdivision that may be prosecuted perpetually supposeth that the parts are infinite; for otherwise the subdivision would be terminable: and the parts being Infinite, it followeth of consequence that they be non-quantitative; for infinite quantitative parts make an infinite extension: and thus we have a *Continuum* compounded of infinite Indivisibles.

SIMP. But if we may continually prosecute the division in quantitative parts, what need have we, for such respect, to introduce the non-quantitative?

SALV. The very possibility of perpetually prosecuting the division in quantitative parts induceth the necessity of the composition of infinite non-quantitative. Therefore, coming closer to you, I demand you to tell me resolutely, whether the quantitative parts in *Continuum* be in your judgment finite or infinite?

SIMP. I reply, that they are both Infinite, and Finite; Infinite in Power, and Finite in Act. Infinite in Power, that is, before the Division; but Finite in Act, that is, after they are divided: for the parts are not actually understood to be in the whole, till it is divided, or at least marked; otherwise we say that they are in Power.

SALV. So that a Line *v. gr.* twenty foot long, is not said to contain twenty Lines of one foot a piece, actually, but only after it is divided into twenty equal parts; but is till then said to contain them only in power. Now be it as you please; and tell me whether, when the actual Division of such parts is made, that first whole encreaseth or diminisheth, or else continueth of the same bigness?

SIMP. It neither encreaseth, nor diminisheth.

SALV. So I think also. Therefore the quantitative parts in *Continuum* quantity, be they in Act, or be they in Power, make not its quantity bigger or lesser: but it is very plain that these quantitative parts, actually contained in their whole, if they be infinite, make it an infinite Magnitude; therefore quantitative parts, though infinite only in power, cannot be contained, but only in an infinite Magnitude: therefore in a finite Magnitude infinite quantitative parts can be contained neither in Act, nor Power.

SAGR. How then can it be true, that the *Continuum* may be incessantly divided into parts still capable of new divisions?

SALV. It seems that that distinction of Power, and Act, makes that feasible one way, which another way would be impossible. But I will see to adjust these matters by making another account:

And

And to the Question, which was put, Whether the quantitative parts in a terminated *Continuum* be finite or infinite; I will answer directly contrary to that which *Simplicius* replied, namely, that they be neither finite, nor infinite.

Simp. I should never have found such an answer, not imagining that there was any mean term between finite and infinite; so that the division or distinction which makes a thing to be either Finite, or Infinite, is imperfect and deficient.

Salv. In my opinion it is; and speaking of * Discrete Quantities, me thinks that there is a third mean term between Finite and Infinite, which is that which answereth to every assigned Number: So that being demanded in our present case, Whether the quantitative parts in *Continuum* be Finite, or Infinite, the most congruous reply is to say, that they are neither Finite, nor Infinite, but so many, as that they *Answer* to any number assigned: the which to do, it is necessary that they be not comprehended in a limited Number, for then they would not answer to a greater: nor, again, is it necessary, that they be infinite, for no assigned Number is infinite. And thus at the pleasure of the Demander, a Line being propounded, we may be able to assign in it an hundred quantitative parts, or a thousand, or an hundred thousand, according to the number which he best likes; so that it be not divided into infinite. I grant therefore to the Philosophers, that *Continuum* containeth as many quantitative parts as they please, and grant them that it containeth the same either in Act, or in Power, which they best like: but this I add again, that in like manner, as in a Line of ten yards, there are contained ten Lines of one yard a piece, and thirty Lines of a foot a piece, and three hundred and sixty Lines of an inch a piece, so it contains infinite Points; denominate them in Act, or in Power, as you will: and I remit my self in this matter to your opinion and judgment, *Simplicius*.

Quantitative parts in Discrete Quantity are neither finite nor infinite, but answerable to every given Number.

Simp. I cannot but commend your Discourse: but am greatly afraid, that this parity of the Points, being contained in the like manner as the quantitative parts, will not agree with absolute exactness; nor shall it be so easie a matter for you to divide the given Line into infinite Points, as for those Philosophers to divide it into ten yards, or thirty feet, nay, I hold it wholly impossible to effect such a division: so that this will be one of those Powers that are never reduced to Act.

Salv. The trouble, pains, and long time without which a thing is not feasible, render it not impossible; for I think also, that you cannot so easily effect a division to be made of a Line into a thousand parts, and much less being to divide it into 937, or some other great Prime Number. But if I dispatch this, which you, it may be, judge an impossible division, in as short a time, as another would

would require to divide it into forty, you will be content more willingly to admit of it in our future Discourse?

SIMP. I am pleased with your way of arguing, as you now do mix it with some pleasantness: and to your question I reply, that the facility would seem more than sufficient, if the resolving it into Points were but as easie, as to divide it into a thousand parts.

SALV. Here I will tell you a thing, which haply will make you wonder in this matter of going about, or being able to resolve the Line into its Infinites, keeping that order which others observe in dividing it into forty, sixty, or an hundred parts; namely, by dividing it first into two, then into four: in which order he that should think to find its infinite Points would grosly delude himself; for by that progression, though continued to eternity, he should never arrive to the division of all its quantitative parts: yea, he is in that way so far from being able to arrive at the intended term of Indivisibility, that he rather goeth farther from it; and whilst he thinks by continuing the division, and multiplying the multitudes of the parts, to approach to Infinite, I am of opinion, that he more and more removes from it: and my reason is this; In the Discourse, we had even now, we concluded, that, in an infinite Number, there was, of necessity, as many Square, or Cube Numbers, as there were Numbers; since that those and these were as many as their Roots, and Roots comprehend all Numbers: Next we did see, that the greater the Numbers were that were taken, the seldomer are their Squares to be found in them, and seldomer yet their Cubes: Therefore it is manifest, that the greater the Number is to which you pass, the farther you remove from Infinite Number: from whence it followeth, that turning backwards, (seeing that such a progression more removes us from the desired term) if any number may be said to be infinite it is the Unite: and, indeed, there are in it those conditions, and necessary qualities of the Infinite Number, I mean, of containing in it as many Squares as Cubes, and as Numbers.

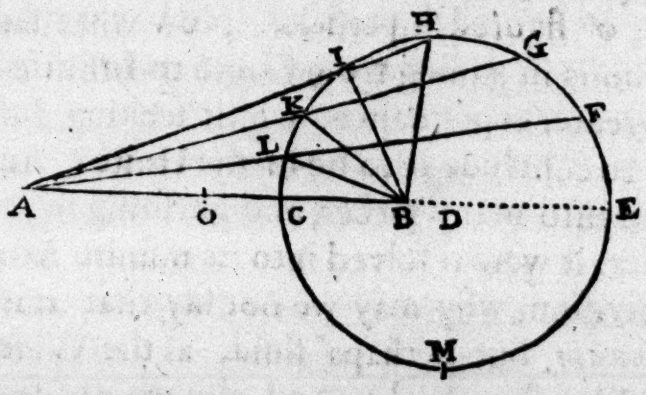
The Unite of all Numbers may most properly be said to be Infinite.

SIMP. I do not apprehend very well, how this business should be understood.

SALV. The thing hath no difficulty at all in it, for the Unite is a Square, a Cube, a Squared Square, and all other Powers; nor is there any particular whatsoever essential to the Square, or to the Cube, which doth not agree with the Unite; as *v. gr.* one property of two Square-numbers is to have between them a Number mean-proportional; take any Square number for one of the terms, and the Unite for the other, and you shall likewise ever find between them a Number Mean-proportional. Let the two Square Numbers be 9 and 4, you see that between 9 and 1 the Mean-proportional is 3, and between 4 and 1 the Mean-proportional

is

is 2, and between the two Squares 9 and 4, 6 is the Mean. The property of Cubes is to have necessarily between them two Numbers Mean-proportional. Suppose 8, and 27, the Means between them are 12 and 18; and between the Unite and 8 the Means are 2 and 4; betwixt the Unite and 27 there are 3, and 9. We therefore conclude, *That there is no other Infinite Number but the Unite.* And these be some of those Wonders, that surmount the comprehension of our Imagination, and that advertize us how exceedingly they err, who discourse about Infinites with those very Attributes, that are used about Finites; the Natures of which have no congruity with each other. In which affair I will not conceal from you an admirable accident, that I met with some time since, explaining the vast difference, yea, repugnance and contrariety of Nature, that a terminate quantity would incur by changing or passing into Infinite. We assign this Right Line A B, of any length at pleasure, and any point in the same, as C being taken, dividing it into two unequal parts: I say, that many couples Lines, (holding the same proportion between themselves as have the parts A C, and B C,) departing from the terms A and B to meet with one another; the points of their Intersection shall all fall in the Circumference of one and the same Circle: as for example, A L and B L departing [or *being drawn*] from the Points A and B, and having between themselves the same proportion, as have the parts A C and B C, and concurring in the point L: and the same proportion being between two others A K, and B K, concurring in K, also others as A I, and B I; A H, and B H; A G, and B G; A F, and B F; A E, and B E: I say, that the points of their Intersection L, K, I, H, G, F, E, do all fall in the Circumference of one and the same Semi-circle: so that we should imagine the point C to mve continually after such a sort, that the Lines produced from it to the fixed terms A and B retain alwaies the same proportion that is between the first



parts A C and G B, that point C shall decribe the Circumference of a Circle, as we shall shew you presently. And the Circle in such sort described shall be alwaies greater and greater successively, according as the point C is taken nearer to the middle point which is O; and the Circle shall be lesser which shall be described from a point nearer to the extremity B, insomuch, that from the infinite

infinite Points which may be taken in the Line O B, there may be described Circles (moving them in such sort as above is prescribed) of any Magnitude; lesser than the Pupil of the eye of a Flea, and bigger than the Equinoctial of the *Primum Mobile.* Now, if raising any of the Points comprehended betwixt the terms O and B, from every one we may describe Circles, and vast ones from the Points nearer to O; then if we raise the Point O it self, and continue to move it in such sort as aforesaid, that is, that the Lines drawn from it to the terms A and B keep the same proportion as have the first Lines A O, and O B, what Line shall be described? There would be described the Circumference of a Circle, but of a Circle bigger than the biggest of all Circles, therefore of a Circle that is infinite: but it doth also describe a Right Line, and perpendicular upon A B, erected from the Point O, and produced *in infinitum* without ever turning to reunite its last term with the first, as the others did; for the limited motion of the Point C, after it had designed the upper Semi-circle C H E, continued to describe the Power E M C, reuniting its extream terms in the point C: But the Point O being moved to design (as all the other Points of the Line A B, for the Points taken in the other part O A shall design their Circles, and those Points nearest to O the greatest) its Circle; to make it the biggest of all, and consequently infinite, it can never return any more to its first term, and in a word designeth an Infinite Right-Line for the Circumference of its Infinite Circle. Consider now, what difference there is between a finite Circle, and an infinite; seeing that this in such manner changeth its being that it wholly loseth both its being, and power of being; for we have already well comprehended, that there cannot be assigned an infinite Circle; by which we may consequently know that there can be no infinite Sphære, or other Body, or figured Superficies. Now what shall we say to this Metamorphosis in passing from Finite to Infinite? And why should we find greater repugnance, whilst seeking Infinity in Numbers, we come to conclude it to be in the Unite? And whilst that breaking a Solid into many pieces, and pursuing to reduce it into very small powder, it were resolved into its infinite Atomes, admitting no farther division, why may we not say that it is returned into one sole *Continuum*, but perhaps fluid, as the Water, or Quicksilver, or other Metall melted? And do we not see Stones liquified into Glass, and Glass it self with much Fire to become more fluid than Water?

The difference betwixt a finite and infinite Circle.

Unity participates of Infinity.

SAGR. Should we therefore think Fluids to be so called, because they are resolved into their first, infinite, indivisible compounding parts?

SALV. I know not how to find a better answer to resolve certain

tain sensible appearances, amongst which this is one: When I take a hard Body, be it either Stone, or Metal, and with a Hammer, or very fine File, endeavour to divide it, as much as is possible, into its most minute and impalpable powder; it is very clear, that its least Atomes, albeit for their smalness they are imperceptible, one by one, to our sight and touch; yet are they quantitative, figured, and numerable: and it happens in them, that being accumulated together, they continue in heap; and being laid hollow, or with a pit in the midst, the hollowness or pit remains, the parts heaped about it not returning to fill it up; and being stirr'd, or shaken, they suddenly settle so soon as their external mover leaves them, And the like effects are seen in all the Aggregates of small Bodies, bigger, and bigger, and of any kind of Figure, although Spherical; as we see in heaps of Pease, Wheat, Bird shot, and other matters. But if we try to find the like accidents in Water, you will meet with none of them; but, being raised, it instantly returns to a level, if it be not by a vessel, or some other external stay upheld; being made hollow, it presently diffuseth to fill up the Cavity; and being long moved, it continually undulates, and spreads its waves very far. From this, I think, we may very rationally infer, that the minute parts of Water, into which it seemeth to be resolved, (since it hath less consistence than any the finest powder, yea, hath no consistence at all,) are vastly differing from Atomes quantitative and divisible; nor know I how to find any other difference therein than that of being indivisible. Methinks, also, that its most exquisite transparency, affords us sufficient grounds to conjecture thereof; for if we take the most diaphanous Christal that is, and begin to break, and pound it to powder, when it is in powder it loseth its transparency, and so much the more, the smaller it is pounded; but yet Water which is ground to the highest degree, hath also the highest degree of Diaphaneity. Gold and Silver, reduced by *Aqua-fortis* into a smaller Powder than any File can make, yet they continue powder, and become not fluid; nor do they liquifie till the Indivisibles of the Fire, or of the Sun-beams dissolve them, as, I believe, into their first and highest infinite and indivisible compounding parts.

Fluid Bodies are such, for that they are resolved into their first Indivisible Atomes.

SAGR. This which you have hinted of the Light I have many times observed with admiration: I have seen, I say, a burning-Glass, of a foot Diameter, liquifie or melt lead in an instant; whence I came to be of opinion, that if the Glasses were very big, and very polite, and of Parabolical Figure, they would no less melt every other Metal in a very short time; seeing that that, not very big, nor very clear, and of a Spherical Concave, with such force melted Lead, and burnt every combustible matter: effects, that make the wonders, reported of the Burning-glasses of *Archimedes*, credible to me.

Archimedes his Burning -- Glasses admirable.

SALV. Touching the Effects of the Glasses, invented by *Archimedes*, all the Miracles, that several Writers record of them, are to me rendred credible by the reading of *Archimedes* his own Books, which I have with infinite amazement perused and studied: and if any doubts had been left me; that which last of all Father *Buonaventura Cavalieri* hath published, touching *Lo Specchio Ustorio*, (or the Burning-glass) and which I have read with admiration, is sufficient to resolve them all.

Buonaventura Cavalieri, the Jesuate, a famous Mathematician, and his Book entitled, Lo Specchio Ustorio.

SAGR. I have also seen that Tract, and perused it with much delight and wonder; and because I formerly had knowledge of the Author, I was confirmed in the opinion which I had conceived of him, that he was like to prove one of the principal Mathematicians of our Age. But returning to the admirable effects of the Sun-Beams in melting of Metals, are we to believe that such, and so violent an operation is without Motion, or else that it is with Motion, but extream swift?

Burnings are performed with a most swift Motion.

SALV. We see other burnings, and meltings to be performed with Motion, and with a most swift Motion. Observe the operations of Lightnings, of Powder in Mines, and in Petards, and, in sum, how by quickning the flame of Coles, mixt with gross and impure vapours, by Bellows, encreaseth its force in the melting of Metals: so that I cannot see how the Action of Light, albeit most pure, can be without Motion, and that also very swift.

SAGR. But what and how great ought we to judge this Velocity of the Light? Is it haply *Instantaneous*, and done in a moment, or, as the rest of Motions, performed in Time? May we not by Experiment be assured what it is?

SIMP. Quotidian experience shews the expansion of Light to be *Instantaneous*, in that beholding a Cannon, let off at a great distance, the flash of the fire, without interposition of time, is transmitted to our eye, but so is not the Report to our ear untill a considerable time after.

SAGR. True, but, I pray you, what doth this obvious experiment evince; but only this, that the Report is longer in arriving at our Ear, than the Flash at our Eye; but it assures me not, that the transmission of the Light is therefore *Instantaneous* rather than in Time, but only most swift. Nor doth such an observation conclude more than that other, of such who say, that as soon as the Sun cometh to the Horizon, its Light arriveth at our eye: for who shall assure me, that its beams arrive not at the said term, afore they reach our sight?

SALV. The inconcludency of these, and other observations of the like Nature, made me once think of some other way, whereby we may without errour be ascertained whether the illumination, that

that is, whether the expansion of the Light were really *Instantaneous*; seeing that the very swift Motion of Sound, assureth us, that that of Light cannot but be extream swift. And the experiment I hit upon, was this; I would have two persons take each of them a Light, which, by holding it in a Lanthorn, or other coverture, they may cover, and discover at pleasure by interposing their hand to the sight of each other; and, that placing themselvs against one another, some few paces distance, they may practice the speedy discovery, and occultation of their Lights from the sight of each other: So that when one seeth the others Light, he immediatly disclose his: which correspondence, after some Responses mutually made, will become so exactly Instantaneous, that, without sensible variation, at the discovery of the one, the other shall at the same time appear to the sight of him that disclos'd the first. Having adjusted this practice at this small distance, let us place the two persons with two such Lights at two or three miles distance; and by night renewing the same experiment; Let them intensely observe if the Responses of the disclosures, and occultations do follow the same tenour which they did near hand: for if they keep the same proportion, it may be with certainty enough concluded, that the expansion of Light is Instantaneous; but if it should require time in a distance of three miles, which importeth six for the going of one, and return of the other, the stay would be sufficiently observable. And if this Experiment be made at greater distances, namely, at eight or ten miles, we may make use of the *Telescope*, the Observators accommodating each of them one at the places, where by night the Lights are to be observed; which though not very big, and so not visible, at that great distance, to the eye at large; (though easie to be disclosed, and hid) by help of the *Telescopes* before admitted, and fixed they may commodiously be discerned.

The Velocity of Light, how to find by Experiment whether it be Instantaneous or not.

SAGR. The Invention seems to me no less certain than ingenuous; but tell us what upon experimenting it you concluded.

SALV. Really, I have not tryed it, save only at a small distance, namely, less than a Mile: whereby I could come to no certainty whether the apparence of the opposite Light was truly Instantaneous; But if not Instantaneous, yet it was of exceeding great Velocity, and I may say Momentary: and for the present, I would resemble it to that Motion which we see a flash of Lightning make in the Clouds ten or more Miles off: of which Light we distinguish the beginning, and, I may say, the source and rise of it, in a particular place in those Clouds; but yet its wide expansion immediatly succeeds amongst those adjacent: which to me seems an argument that it is some small time in doing; because had the illumination been made all at once, and not by degrees, it seems to

me that we could not have distinguished its original, or rather the Center of its flake, and extream Dilatations. But into what Oceans do we by degrees engage our selves? Amongst *Vacuities, Infinites, Indivisibles,* and *Instantaneous Motions*; so that we shall not be able by a thousand Discourses to recover the Shore?

SAGR. They are things, indeed, very disproportionate to our understanding. Behold Infinite, sought amongst Numbers, seemeth to determine in the Unite: From Indivisibles ariseth things that are continually divisible: Vacuity seems only to reside indivisibly mixt with Repletion: and, in brief, these things so change the nature of those understood by us, that even the Circumference of a Circle becometh an Infinite Right-Line; which, if I well remember, is that Proposition which you, *Salviatus*, are to manifest by Geometrical Demonstration. Therefore, if you think fit, it would be well, without any more digressions, to make it out to us.

SALV. I am ready to serve you in demonstrating the ensuing Problem for your fuller information.

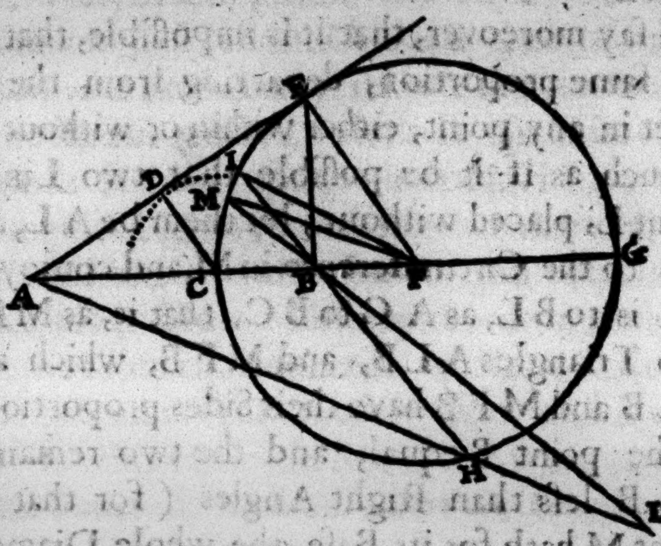
PROPOSITION.

A Right-Line being given, divided, according to any proportion, into unequal parts, to describe a Circle, to the Circumference of which, at any point of the same, two Right-Lines being produced from the terms of the given Right Line, they may retain the same proportion that the parts of the said Line given have to one another, so that those be Homologous which depart from the same terms.

LET the given Right-Line be A B, unequally divided according to any proportion in the point C; it is required to describe a Circle at any point of whose Circumference two Right Lines, produced from the terms A and B, concurring, have the same proportion to each other, that A B, hath to A C, so that those be Homologous which depart from the same term. Upon the Center C, at the distance of the lesser part C B, let a Circle be supposed to be described, to the Circumference of which from the point A the Right-line A D is made a Tangent, and indeterminately prolonged towards E; and let the Contact be in D, and draw a Line from C to D, which shall be perpendicular to A E; and let B E be perpendicular to B A, which produced, shall inter-

sect

ſect A E, the Angle A being acute: Let the interſection be in E; from whence let fall a Perpendicular to A E, which produced, will meet with A B infinitely prolonged in F. I ſay, firſt, that the Right-lines F E, and F C are equal: ſo that drawing the Line E C, we ſhall, in the two Triangles D E C, B E C, have the two Sides of the one, D E, and C E, equal to the two Sides of the other B E, and E C; the two Sides, D E, and E B, being Tangents to the Circle D B, and the Baſes D C, and C B, are likewiſe equal: wherefore the two Angles D E C, and B E C, ſhall be equal. And becauſe the Angle B C E wanteth of being a Right-Angle, as much as the Angle B E C; and the Angle C E F, to make it a Right-Angle, wants the Angle C E D, thoſe Supplements being equal, the Angles F C E, and F E C ſhall be equal, and ſo conſequently the Sides F E, and F C; wherefore making the point F a Center, and at the diſtance F E, deſcribing a Circle, it ſhall paſs by the point C. Deſcribe it, and let it be C E G. I ſay, that this is the Circle required, by any point of the Circumference of which, any two Lines that ſhall interſect, departing from the terms A and B, ſhall be in proportion to each other, as are the two parts A C, and B C, which before did concur in the point C. This is manifeſt in the two that concur or interſect in the point E, that is A E, and B E; the Angle E of the Triangle A E B being divided in the midſt by C E; ſo that as A C is to C B, ſo is A E to B E. The ſame we prove in the two A G, and B G, determined in the point G. Therefore being (by the Similitude of the Triangles A F E, and E F B) that as A F is to E F, ſo is E F to F B; that is, as A F is to F C, ſo is C F to F B: So by Diviſion; as A C is to C F, (that is, to F G) ſo is C B to B F; and the whole A B is to the whole B G, as the part C B to the part B F: and by Compoſition; as A G is to G B, ſo is C F to F B; that is, as E F to F B, that is, as A E to E B, and A C to C B: Which was to be demonſtrated. Again, let any other Point be taken in the Circumference, as H; in which the two Lines A H and B H concur. I ſay, in like manner as before, that as A C is to C B, ſo is A H to B H. Continue H B untill it interſect the Circumference in I, and draw



a Line joyning I to F. And because it hath been proved already that as A B is to B G, so is C B to B F, the Rectangle A B F shall be equall to the Rectangle C B G, that is I B H: and therefore, as A B is to B H, so is I B to B F, and the Angles at B are equal: Therefore A H is to H B, as I F, that is E F, to F B, and as A E to E B.

I say moreover, that it is impossible, that the Lines, which have this same proportion, departing from the terms A and B, should meet in any point, either within or without the said Circle: Forasmuch as if it be possible that two Lines should concur in the point L, placed without; let them be A L, and B L; and continue L B to the Circumference in M, and conjoyn M to F. If therefore A L is to B L, as A C to B C, that is, as M F to F B, we shall have two Triangles A L B, and M F B, which about the two Angles A L B and M F B have their Sides proportional, their upper Angles in the point B equal, and the two remaining Angles F M B and L A B less than Right Angles (for that the Right-angle at the point M hath for its Base the whole Diameter C G, and not the sole part B F, and the other at the point A is acute by reason the Line A L Homologous to A C, is greater than B L Homologous to B C) Therefore the Triangles A B L, and M B F are like; and therefore as A B is to B L, so is M B to B F; Wherefore the Rectangle A B F shall be equall to the Rectangle M B L. But the Rectangle A B F hath been demonstrated to be equal to that of C B G: Therefore the Rectangle M B L is equal to the Rectangle C B G, which is impossible: Therefore the Concourse of the Lines cannot fall without the Circle. And in like manner it may be demonstrated that it cannot fall within; Therefore all the Concourses fall in the Circumference it self.

But it is time that we return to give satisfaction to the Intreaty of *Simplicius*, shewing him that the resolving the Line into its infinite Points is not only not impossible, but that it hath in it no more difficulty than to distinguish its quantitative parts; presupposing one thing (notwithstanding) which I think, *Simplicius*, you will not deny me, and that is this; that you will not require me to sever the Points one from another, and shew you them one by one distinctly upon this paper: for I my selfe should be content, if without enjoyning to pull the four or six parts of a Line from one another, you should but shew me its divisions marked, or at most inclined to Angles, framing them into a Square, or a Hexagon; therefore I perswade my self, that for the present you will grant them then sufficiently, and actually distinguished.

SIMP. I shall indeed.

How infinite points are assigned in a finite Line.

SALV. Now if the inclining of a Line to Angles, framing therewith sometimes a Square sometimes an Octagon, sometimes a

a Poligon of Forty, of an *Hundred*, of a Thousand Angles be a mutation sufficient to reduce into Act those four, eight, forty, hundred, or thousand parts, which were, as you say, Potentially in the said Line at first: if I make thereof a Poligon of infinite Sides, namely, when I bend it into the Circumference of a Circle, may not I, with the like leave, say, that I have reduced those infinite parts into Act, which you before, whilst it was straight, said were Potentially contained in it? Nor may such a Resolution be denied to be made into its Infinite Points, as well as that of its four parts in forming thereof a Square, or into its thousand parts in forming thereof a Mill-angular Figure; by reason that there wants not in it any of the Conditions found in the Poligon of a thousand, or of an hundred thousand Sides. This applied or layed to a Right-Line covereth it, touching it with one of its Sides, that is, with one of its hundred thousandth parts; the Circle, which is a Poligon of infinite Sides, toucheth the said Right-line with one of its Sides, that is one single Point divers from all its Colaterals, and therefore divided, and distinct from them, no less than a Side of the Poligon from its Conterminals. And as the Poligon turned round upon a Plane describes, with the consequent tacts of its Sides, a Right-line equal to its Perimeter: so the Circle, rowled upon such a Plane, describes or stamps upon it, by its infinite successive Contacts, a Right-line, equall to its own Circumference. I know not at present, *Simplicius*, whether or no the Peripateticks, (to whom I grant, as a thing most certain, that *Continuum* may be divided into parts alwaies divisible, so that continuing the division and subdivision there can be no end thereof) will be content to yield to me, that none of those divisions are the ultimate, as indeed they be not, since that there alwaies remains another; but that only to be the last, which resolves it into infinite Indivisibles; to which I yield we can never attain, dividing and subdividing it successively into a greater, and greater multitude of parts: but making use of the way which I propound to distinguish and resolve all the infinite parts at one only draught, (an Artifice which ought not to be denied me) I could perswade my self they would satisfie themselves, and admit this composition of *Continuum* to consist of Atomes absolutely indivisible: And especially, this one path being more current than any other to extricate us out of very intricate Laberinths; such as are, (besides that already touched of the Coherence of the parts of Solids) the conceiving the business of Rarefaction and Condensation, without running into the inconvenience of being forced to admit forth of void Spaces or Vacuities; and for this a Penetration of Bodies: inconveniences, which both, in my opinion, may easily be avoided, by granting the foresaid Composition of Indivisibles.

Continuum *compounded of Indivisibles.*

SIMP.

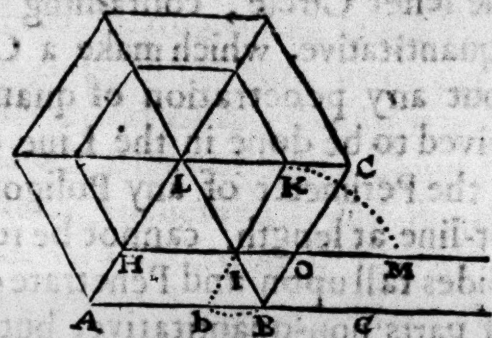
SIMP. I know not what the Peripateticks would say, in regard that the Considerations you have proposed would be, for the most part, new unto them, and as such, it is requisite that they be examined: and it may be, that they would find you answers, and powerful Solutions, to unty these knots, which I, by reason of the want of time and ingenuity proportionate, cannot for the present resolve. Therefore, suspending this particular for this time, I would gladly understand how the introduction of these Indivisibles facilitateth the knowledge of Condensation, and Rarefaction, avoiding at the same time a *Vacuum*, and the Penetration of Bodies.

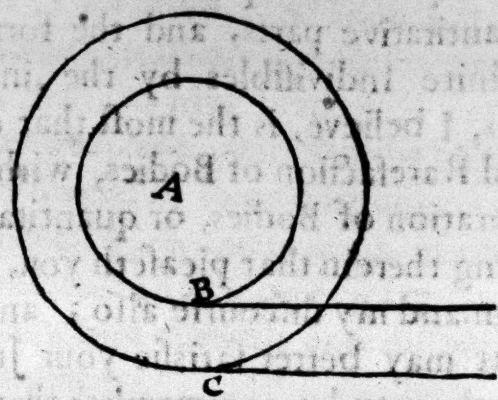
SAGR. I also much long to understand the same, it being to my Capacity so obscure: with this *proviso*, that I be not couzened of hearing (as *Simplicius* said but even now) the Reasons of *Aristotle* in confutation of a *Vacuum*, and consequently the Solutions which you bring, as ought to be done, whilst that you admit what he denieth.

SALV. I will do both the one and the other. And as to the first it's necessary, that like as in favour of Rarefaction, we make use of the Line described by the lesser Circle bigger than its own Circumference, whilst it was moved at the Revolution of the greater; so, for the understanding of Condensation, we shall shew, how that, at the conversion made by the lesser Circle, the greater describeth a Right-line less than its Circumference; for the clearer explication of which, let us set before us the consideration of that which befalls in the Poligons. In a description like to that other, suppose two Hexagons about the common Center L, which let be A B C, and H I K, with the Parallel-lines H O M, and A B C, upon which they are to make their Revolutions; and the Angle I, of the lesser Poligon, resting at a stay, turn the said Poligon till such time as I K fall upon the Parallel, in which motion the point K shall describe the Arch K M, and the Side K I, shall unite with the part I M; while this is in doing, you must observe what the Side C B of the greater Poligon will do. And because the Revolution is made upon the Point I, the Line I B with its term B shall describe, turning backward the Arch B b, below the Parallel c A, so that when the Side K I shall fall upon the Line M I, the Line B C shall fall upon the Line b c, advancing forwards only so much as is the Line B c, and retiring back the part subtended by the Arch B b, which falls upon the Line B A, and intending to continue after the same manner the Revolution of the lesser Poligon, this will describe, and pass upon its Parallel, a Line equal to its Perimeter; but the greater shall pass a Line less than its Perimeter, the quantity of so many of the lines *B b* as it hath Sides, wanting one; and that same line shall be very near equal to that described by the

the lesser Poligon, exceeding it only the quantity of b B. Here then, without the least repugnance the cause is seen, why the greater Poligon passeth or moveth not (being carried by the less) with its Sides a greater Line than that passed by the less; that is, because that one part of each of them falleth upon its next coterminal and precedent.

But if we should consider the two Circles about the Center A, resting upon their Parallels, the lesser touching his in the point B, and the greater his in the point C; here, in beginning to make the Revolution of the less, it shall not occur as before, that the point B rest for some time immoveable, so that the Line B C giving back, carry with it the point C, as it befell in the Poligons, which resting fixed in the point I till that the Side K I falling upon the Line I M, the Line I B carried back B, the term of the Side C B, as far as b, by which means the Side B C fell on b c, super-posing or resting the part B b upon the Line B A, and advancing forwards only the part B c, equal to I M, that is to one Side of the lesser Poligon: by which superpositions, which are the excesses of the greater Sides above the less, the advancements which remain equal to the Sides of the lesser Poligon come to compose in the whole Revolution the Right-line equal to that traced, and measured by the lesser Poligon. But now, I say, that if we would apply this same discourse to the effect of the Circles, it will be requisite to confess, that whereas the Sides of whatsoever Poligon are comprehended by some Number, the Sides of the Circle are infinite; those are quantitative and divisible, these non-quantitative and Indivisible: the terms of the Sides of a Poligon in the Revolution stand still for some time, that is, each such part of the time of an entire Conversion, as it is of the whole Perimeter: in the Circles likewise the stay of the terms of its infinite Sides are momentary, for a Moment is such part of a limited Time, as a Point is of a Line, which containeth infinite of them; the regressions made by the Sides of the greater Poligon, are not of the whole Side, but only of its excess above the Side of the





A Circle is a Poligon of infinite indivisible quantitative Sides.

An Instant or Moment of quantitative Time, is the same as a Point of a quantitative Line.

lesser, getting forwards as much space as the said lesser Side: in Circles, the Point, or Side C in the instantaneous rest of B recedeth as much as is its excess above the Side B, advancing forward as much as the quantity of the same B: And in short, the infinite indivisible Sides of the greater Circle with their infinite indivisible Regressions, made in the infinite instantaneous staies of the infinite terms of the infinite Sides of the lesser Circle, and with their infinite Progresses, equal to the infinite Sides of the said lesser Circle, they compose and measure a Line equall to that described by the lesser Circle, containing in it self infinite superpositious non-quantitative, which make a Constipation and Condensation without any penetration of quantitative parts: which cannot be contrived to be done in the Line divided into quantitative parts, as is the Perimeter of any Poligon, which being distended in a Right-line at length, cannot be reduced to a lesser length, unless the Sides fall upon and Penetrate one the other. This Constipation of parts non-quantitative, but infinite without Penetration of quantitative parts, and the former Distraction above declared of infinite Indivisibles by the interposition of indivisible Vacuities, I believe, is the most that can be said for the Condensation and Rarefaction of Bodies, without being driven to introduce Penetration of Bodies, or quantitative Void Spaces. If there be any thing therein that pleaseth you, make use of it, if not, account it vain, and my discourse also; and seek out some other explanation that may better satisfie your Judgment. Only these two words by the way, let us remember that we are amongst Infinites, and Indivisibles.

Rarefaction is the distraction of infinite indivisibles by the interposition of infinite indivisible Vacuities.

Condensation, according to the operation of the Author, proceeds from the Constipation of quantitative and indivisible parts.

SAGR. That the Conceit is ingenious, and to my eares wholly new, and strange, I freely confess, but whether or no Nature proceed in this order, I know not how to resolve; Truth is, that till such time as I hear something that may better satisfie me, that I may not stand silent, I will adhere to this. But haply *Simplicius* may have somwhat, which I have not yet met with, to explicate the explication, which is produced by Philosophers in so abstruce a matter; for, indeed, what I have hitherto read about Condensation, is to me so dense, and that of Rarefaction so subtill, that my weak sight neither penetrates the one, nor comprehends the other.

SIMP. I am full of confusion, and find great Rubbs in the one path, and in the other, and more particularly in this new one: for according to this Rule, an Ounce of Gold might be rarefied and drawn forth into a Mass bigger than the whole Earth, and the whole Earth condensed and reduced into a less Mass than a Nut; which I neither believe, nor think that you your self do believe: and the Considerations and Demonstrations by you hitherto delivered,

livered, as they are things Mathematical, abstract and separate from Sensible Matter, I believe, that when they come to be applyed to Matters Physical and Natural, they will not exactly comply with these Rules.

SALV. It is not in my power, nor, as I believe, do you desire, that I should make that visible which is invisible; but as to such things as may be comprehended by our Senses, in regard that you have instanced in Gold, do we not see an immense extension to be made of its parts? I know not whether you may have seen the Method that Wyer-drawers observe in disgrossing Gold Wyer: which in reality is not Gold, save only in the Superficies, for the internal substance is Silver; and the way of disgrossing it is this. They take a Cylinder, or, if you will, Ingot of Silver, about half a yard long, and about three or four * Inches thick, and this they gild or over-lay with Leaves of beaten Gold, which, you know, is so thin that the Wind will blow it to and again, and of these Leaves they lay on eight or ten, and no more. So soon as it is gilded, they begin to draw it forth with extraordinary force, making it to pass thorow the hole of the Drawing Iron, and then reiterate this forceable disgrosment again and again thorow holes successively narrower, so that, after several of these disgrosments, they bring it to the smalness of the hair of a womans head, if not smaller, and yet it still continueth gilded in its Superficies or outside: Now I leave you to consider to what a fineness and distension the substance of the Gold is brought.

Gold in the gilding of Silver is drawn forth and disgrossed immensly.

* Or Thumbbreadths.

SIMP. I do not see how it can be inferred from this Experiment, that there may be a disgrosment of the matter of the Gold sufficient to effect those wonders which you speak of: First, For that the first gilding was with ten Leaves of Gold, which make a considerable thickness: Secondly, howbeit in the extension and disgrosment that Silver encreaseth in length, it yet withall diminisheth so much in thickness, that compensating the one dimension with the other, the Superficies doth not so enlarge, as that for overlaying the Silver with Gold, the said Gold need to be reduced to a greater thinness than that of its first Leaves.

SALV. You much deceive your self, *Simplicius*, for the encrease of the Superficies is Subduple to the extension in length, as I could Geometrically demonstrate to you.

SAGR. I beseech you, both in the behalf of my self, and of *Simplicius*, to favour us with that Demonstration, if so be you think that we can comprehend it.

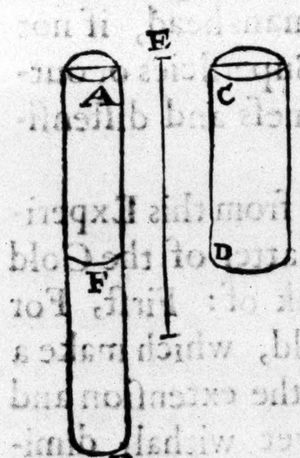
SALV. I will see whether I can, thus upon the sudden, recall it to mind. It is already manifest, that that same first gross Cylinder of Silver, and the Wyer distended to so great a length are two equal Cylinders, in regard that they are the same Silver; so that

if I ſhall ſhew you what proportion the Superficies of equall Cylinders have to one another, we ſhall obtain our deſire. I ſay, therefore, that

PROPOSITION.

The Superficies of Equal Cylinders, their Baſes being ſubſtracted, are to one another in ſubduple proportion of their lengths.

TAke two equall Cylinders, the heights of which let be A B, and C D: and let the Line E be a Mean-proportional between them. I ſay, the Superficies of the Cylinder A B, the Baſes ſubſtracted, hath the ſame proportion to the Superficies of the Cylinder C D, the Baſes in like manner ſubſtracted, as the Line A B hath to the Line E, which is ſubduple of the proportion of A B to C D. Cut the part of the Cylinder A B in F, and let the height A F be equal to C D: And becauſe the Baſes of equal Cylinders anſwer Reciprocally to their heights, the Circle, Baſe of the Cylinder C D, to the Circle, Baſe of the Cylinder A B, ſhall be as the height B A to D C: And becauſe Circles are to one another as the Squares of their Diameters, the ſaid Squares ſhall have the ſame proportion, that B A hath to C D: But as B A, is to C D, ſo is the Square B A to the Square of E: Therefore thoſe four Squares are Proportionals: And therefore their Sides ſhall be Proportionals. And as the Line A B is to E, ſo is the Diameter of the Circle C to the Diameter of the Circle A: But as are the Diameters, ſo are the Circumferences; and as are the Circumferences, ſo likewiſe are the Superficies of Cylinders equal in Height. Therefore as the Line A B is to E, ſo is the Superficies of the Cylinder C D to the Superficies of the Cylinder A F. Becauſe therefore the height A F to the height A B, is as the Superficies A F to the Superficies A B: And as is the height A B to the Line E, ſo is the Superficies C D to the Superficies A F: Therefore by Perturbation of Proportion as the height A F is to E, ſo is the Superficies C D to the Superficies A B: And, by Converſion, as the Superficies of the Cylinder A B is to the Superficies of the Cylinder C D, ſo is the Line E to the Line A F; that is, to the Line C D: or as A B to E: Which is in ſubduple proportion of A B to C D: Which is that which was to be proved.



Now

Now if we apply this, that hath been demonstrated, to our purpose; presupposing that that same Cylinder of Silver, that was gilded whilst it was no more than half a yard long, and four or five Inches thick, being disgrossed to the fineness of an hair, is prolonged unto the extension of twenty thousand yards (for its length would be much greater) we shall find its Superficies augmented to two hundred times its former greatness: and consequently, those Leaves of Gold, which were laid on ten in number, being distended on a Superficies two hundred times bigger, assure us that the Gold which covereth the Superficies of the so many yards of Wyer is left of no greater thickness than the twentieth part of a Leaf of ordinary Beaten-Gold. Consider, now, how great its thinness is, and whether it is possible to imagine it done without an immense distention of its parts: and whether this seem to you an Experiment, that tendeth likewise towards a composition of infinite Indivisibles in Physical Matters: Howbeit there want not other more strong and necessary proofs of the same.

SAGR. The Demonstration seemeth to me so ingenuous, that although it should not be of force enough to prove that first intent for which it was produced, (and yet, in my opinion, it plainly makes it out) yet nevertheless that short space of time was well spent which hath been employed in hearing of it.

SALV. In regard I see, that you are so well pleased with these Geometrical Demonstrations, which bring with them certain profit, I will give you the fellow to this, which satisfieth to a very curious Question. In the former we have that which hapneth in Cylinders that are equall, but of different heights or lengths: it will be convenient, that you also hear that which occurreth in Cylinders equal in Superficies, but unequal in heights; my meaning alwaies is, in those Superficies only that encompass them about, that is, not comprehending the two Bases superiour and inferiour. I say, therefore, that

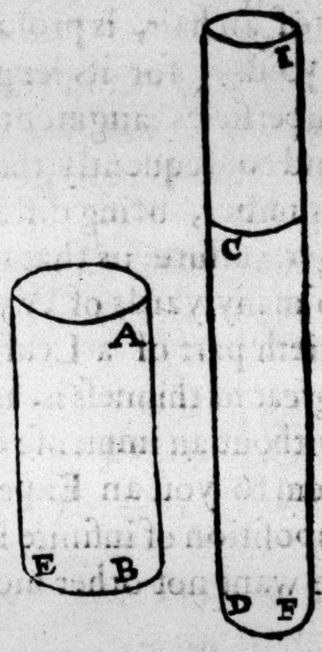
PROPOSITION.

Upon Cylinders, the Superficies of which the Bases being substracted are equal, have the same proportion to one another as their heights Reciprocally taken.

LET the Superficies of the two Cylinders A E and C F be equall; but the height of this C D greater than the height of the other A B. I say, that the Cylinder A E hath the same proportion to the Cylinder C F, that the height C D hath to A B. Because therefore the Superficies C F is equall to the

Superficies

superficies A E, the Cylinder C F shall be lesse than A E: For if they were equal, its Superficies, by the last Proposition would be greater than the Superficies A E, and much the more, if the said Cylinder C F were greater than A E. Let the Cylinder I D be supposed equal to A E: Therefore, by the precedent Proposition, the Superficies of the Cylinder I D shall be to the Superficies A E, as the height I F to the Mean-proportional betwixt I F & A B. But the Superficies A E being by Supposition equal to C F and I D, having the same proportion to C F that the height I F hath to C D: Therefore C D is the Mean-Proportional between I F and A B. Moreover, the Cylinder I D being equal to the Cylinder A E, they shall both have the same proportion to the Cylinder C F: But I D is to C F, as the height I F is to C D: Therefore the Cylinder A E shall have the same proportion to the Cylinder C F, that the line I F hath to C D; that is, that C D hath to A B: Which was to be demonstrated.



Of Corn-sacks with a Board at the Bottom, made of the same Stuffe, but different in height, which are the more capacious.

* Or Sacking.

From hence is collected the Cause of an Accident, which the Vulgar do not hearken to without admiration; and it is, how it is possible that the same piece of *Cloth, being longer one way than another, if a Sack be made thereof to hold Corn, as the usual manner is, with a Board at the bottom, will hold more, making use of the lesser breadth of the Cloth, for the height of the Sack, and with the other encompassing the Board at the bottom, than if it be made up the other way: As if for Example, the Cloth were one way six foot, and the other way twelve, it will hold more, when with the length of twelve one encompasseth the Board at the bottom, the Sack being six foot high, than if it encompassed a bottom of six foot, having twelve for its height. Now, by what hath been demonstrated, there is added to the Knowledge in general that it holds more that way than this, the Specifick, and particular Knowledge of how much it holdeth more: which is, That it will hold more in proportion as it is lower, and lesser, as it is higher. And thus in the measures afore taken, the Cloth being twice as long as broad, when it is sewed the length-ways it will hold but half so much, as it will do the other way. And likewise having a Mat to make a *Frale or Basket twenty five foot long, and suppose seven broad; made up the long-way it will hold but onely seven of those measures, whereof the other way it will hold five and twenty.

* Bugnola, any Vessel made of Rushes or Wicker.

SAGR.

SAGR. And thus to our particular content we continually discover new Notions of great Curiosity, and not unaccompanyed with Utility. But in the particular glanced at but even now, I really believe, that amongst such as are altogether void of the knowledge of Geometry, there would not be found one in twenty, but at the first dash would not be mistaken, and wonder that those Bodies that are contained within equal Superficies, should not likewise be in every respect equal; like as they run into the same errour, speaking of the Superficies, when for determining, as it frequently falls out, of the amplenesse of several Cities, they think they have obtained their desire so soon as they know the space of their Circuits, not knowing that one Circuit may be equal to another, and yet the place conteined by this much larger than the place of that: which befalleth not onely in irregular Superficies, but in the regular; amongst which those of more Sides are alwayes more capacious than those of fewer; so that in fine, the Circle, as being a Poligon of infinite Sides, is more capacious than all other Poligons of equal Perimeter; of which I remember, that I with particular delight saw the Demonstration on a time when I studied the Sphere of *Sacrobosco*, with a very learned Commentary upon the same.

SALV. It is most certain; and I having likewise light upon that very place, it gave me occasion to investigate, how it may with one sole Demonstration be concluded, that the Circle is greater than all the rest of regular Isoperemitral Figures, and of others, those of more Sides bigger than those of fewer.

SAGR. And I that take great pleasure in certain select and nowise-trivial Demonstrations, entreat you with all importunity to make me a partaker therein.

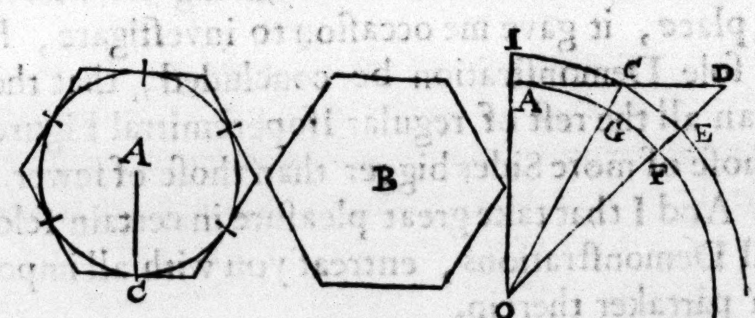
SALV. I shall dispatch the same in few words, demonstrating the following Theorem, namely;

PRO-

PROPOSITION.

The Circle is a Mean-Proportional betwixt any two Regular Homogeneal Poligons, one of which is circumscribed about it, and the other Isoperimetral to it: Moreover, it being lesse than all the circumscribed, it is, on the contrary, bigger than all the Isoperimetral. And, again of the circumscribed, those that have more angles are lesser than those that have fewer; and on the other side of the Isoperimetral, those of more angles are bigger.

OF the two like Poligons A and B, let A be circumscribed about the Circle A, and let the other B, be Isoperimetral to the said Circle: I say, that the Circle is the Mean-proportional betwixt them. For that (having drawn the Semidiameter A C) the Circle being equal to that Right-angled Triangle, of whose Sides including the Right angle, the one is equal



to the Semidiameter A C, and the other to the Circumference: And likewise the Poligon A being equal to the right angled Triangle, that about the right angle hath one of its Sides equal to the said right line A C, and the other to the Perimeter of the said Poligon: It is manifest, that the circumscribed Poligon hath the same proportion to the Circle, that its Perimeter hath to the Circumference of the said Circle; that is, to the Perimeter of the Poligon B, which is supposed equal to the said Circumference: But the Poligon A hath a proportion to the Poligon B, double to that of its Perimeter, to the Perimeter of B (they being like Figures:) Therefore the Circle A is the Mean-proportional between the two Poligons A and B. And the Poligon A being bigger than the Circle A, it is manifest that the said Circle A is bigger than the Poligon B, its Isoperimetral, and consequently the greatest of all Regular Poligons that are Isoperimetral

to

to it. As to the other particular, that is to prove, that of the Poligons circumscribed about the same Circle, that of fewer Sides is bigger than that of more Sides; but that, on the contrary, of the Isoperimetral Poligons, that of more Sides is bigger than that of fewer Sides, we will thus demonstrate. In the Circle whose Center is O, and Semidiameter O A, let there be a Tangent A D, and in it let it be supposed, for example, that A D is the half of the Side of the Pentagon circumscribed, and A C the half of the Side of the Heptagon, and draw the right lines O G C, and O F D; and on the Center O, at the distance O C, draw the Arch E C I: And because the Triangle D O C is greater than the Sector E O C, and the Sector C O I greater than the Triangle C O A; the Triangle D O C shall have greater proportion to the Triangle C O A, than the Sector E O C, to the Secant C O I, that is, than the Secant F O G to the Secant G O A. And, by Composition, Permutation of Proportion, the Triangle D O A shall have greater proportion to the Secant F O A, than the Triangle C O A to the Secant G O A: And ten Triangles D O A shall have greater proportion to ten Secants F O A, than fourteen Triangles C O A to fourteen Sectors G O A: That is the circumscribed Pentagon shall have greater proportion to the Circle, than hath the Heptagon: And therefore the Pentagon shall be greater than the Heptagon. Let us now suppose an Heptagon and a Pentagon Isoperimetral to the same Circle. I say, that the Heptagon is bigger than the Pentagon. For that the said Circle being the Mean proportional between the Pentagon circumscribed and the Pentagon its Isoperimetral, and likewise the Mean between the Circumscribed and Isoperimetral Heptagon: It having been proved that the Circumscribed Pentagon is greater then the Circumscribed Heptagon, the said Pentagon shall have greater proportion to the Circle, than the Heptagon: that is, the Circle shall have greater proportion to its Isoperimetral Pentagon, than to its Isoperimetral Heptagon: Therefore the Pentagon is lesser than the Isoperimetral Heptagon. Which was to be demonstrated.

SAGR. A most ingenious Demonstration, and very acute. But whither are we run to ingulph our selves in Geometry, when as we were about to consider the Difficulties proposed by *Simplicius*, which indeed are very considerable, and in particular, that of Condensation, is in my opinion, very abstruce.

SALV. If Condensation and Rarefaction are opposite Motions, where there is seen an immense Rarefaction, one cannot deny an extraordinary Condensation: but immense Rarefactions, and, which encreaseth the wonder, almost Momentary, we see every day: for what a boundlesse Rarefaction is that of a little quan-

Rarefaction immense is that of a little Gunpowder into a vast mass of fire.

tity of Gunpowder resolved into a vast masse of Fire? And what, beyond this, the (I could almost say) indeterminate Expansion of its Light? And if that Fire and this Light should reunite together, which yet is no impossibility, in regard, that at the first they lay in that little room, what a Condensation would this be? If you study for them, you will find hundreds of such Rarefactions, which are much more readily observed, than Condensations: for Dense matters are more tractable, and subject to our Senses. For we can easily order Wood at pleasure, and we see it resolved into Fire, and into Light, but we do not in the same manner see the Fire and the Light Condense to the making of Wood: We see Fruits, Flowers, and many other solid matters resolved in a great measure into Odors, but we do not after the same manner see the odoriferous Atomes concurre to the constitution of the Oderate Solids; but where Sensible Observation is wanting, we are to supply it with Reason, which will suffice to make us apprehensive, no lesse of the Motion to the Rarefaction and resolution of Solids, than, to the Condensation of rare and most tenuous Substances. Moreover, we question how to effect the Condensation and Rarefaction of the Bodies which may be rarefied and condensed, studying in what manner it may be done without introducing of a *Vacuum*, and Penetration of Bodies; which doth not hinder, but that in Nature there may be matters which admit no such accidents, and consequently do not allow roome for those things which you phrase inconvenient and impossible. And lastly, *Simplicius*, I have on the the score of satisfying you, and those Philosophers that hold with you, taken some pains in considering how Condensation and Rarefaction may be understood to be performed without admitting Penetration of Bodies, and introducing the Void Spaces called Vacuities, Effects which you deny and abhorre: for if you would but grant them, I would no longer so resolutely contradict you. Therefore either admit these Inconveniences, or accept of my Speculations, or else finde out others more conducing to the purpose.

SAGR. As to the denying of Penetration, I am wholly of opinion with the Peripatetick Philosophers; as to that of a *Vacuum*, I would see the Demonstration of *Aristotle* thorowly examined, wherewith he opposeth the same, and what you, *Salviatus*, will answer to it. *Simplicius* shall do me the favour punctually to recite the proof of the Philosopher; and you, *Salviatus*, to answer it.

SIMP. *Aristotle*, as neer as I can remember, breaks out against certain of the Ancients, who introduced Vacuity, as necessary to Motion, saying, that this without that could not be effected;

to

to this *Aristotle* making opposition, demonstrateth, that on the contrary, the effecting of Motion (as we see) destroyeth the Position of *Vacuum*; and his method therein is this. He maketh two Suppositions, one is touching Moveables different in Gravity moved in the same *Medium*; the other is concerning the same Moveable moved in several *Medium's*. As to the first, he supposeth that Moveables different in Gravity, move in the same *Medium* with unequal Velocities, which bear to each other the same proportion as their Gravities: so that, for example, a Moveable ten times heavier than another, moveth ten times more swiftly. In the other Position he assumes, that the Velocity of the same Moveable in different *Medium's* are in Reciprocal to that of the thicknesse or Density of the said *Medium's*: so that, supposing *v. gr.* that the Crassitude of the Water was ten times as great as that of the Air, he will have the Velocity in the Air to be ten times more than the Velocity in the Water. And from this second Assumption he draweth his Demonstration in this manner: Because the tenuity of *Vacuum* infinitely surpasseth the corpulence, though never so subtil, of any whatever Replete *Medium*, every Moveable that in the Replete *Medium* moveth a certain space in a certain time, in a *Vacuum* would passe the same in an instant: But to make a Motion in an instant is impossible: Therefore to introduce Vacuity in the accompt of Motion is impossible.

SALV. The Argument one may see to be *ad hominem*, that is, against those who would make a *Vacuum* necessary to Motion; but if I shall admit of the Argument as concludent, granting withal, that in Vacuity there would be no Motion; yet the Position of Vacuity taken absolutely, and not in relation to Motion, is not thereby overthrown. But to tell you what those Ancients, peradventure, might answer, that so we may the better discover how far the Demonstration of *Aristotle* holds good, methinks that one might oppose his Assumptions, denying them both. And as to the first: I greatly doubt that *Aristotle* never experimented how true it is, that two stones, one ten times heavier than the other, let fall in the same instant from an height, *v. gr.* of an hundred yards, were so different in their Velocity, that upon the arrival of the greater to the ground, the other was found not to have descended so much as ten yards.

Aristotle's *Argument against a* Vacuum *is* ad hominem.

SIMP. Why, it may be seen by his own words, that he confesseth he had made the Experiment, for he saith, [*We see the more grave*] now that *Seeing* implieth that he had tried the Experiment.

SAGR. But I, *Simplicius*, that have made proof thereof, do assure you, that a Cannon bullet that weigheth one hundred, two

hundred, and more pounds, will not one Palme anticipate the arrival of a Musket-bullet to the ground, that weigheth but half a pound, falling likewise from an height of two hundred yards.

SALV. But without any other Experiments, we may by short and necessary Demonstrations cleerly prove, that it is not true that a Moveable more grave moveth more swiftly than another lesse grave, confining our meaning still to Moveables of the same Matter; and, in short, to those of which *Aristotle* speaketh. For tell me, *Simplicius* whether you admit, that to every cadent grave Body there belongeth by nature one determinate Velocity; so as that it cannot be encreased or diminished in it without using violence to it, or imposing some impediment upon it?

SIMP. It cannot be doubted, but that the same Moveable in the same *Medium* hath one established and by-nature-determinate Velocity, which cannot be increased, unlesse with new *Impetus* conferred on it, or diminished, save onely by some impediment that retards it.

SALV. If therefore we had two Moveables, the natural Velocities of which were unequal, it is manifest, that if we joyned the slower with the swifter, this would be in part retarded by the slower, and that in part accelerated by the other more swift. Do not you concur with me in this opinion?

SIMP. I think that it ought undoubtedly so to succeed.

SALV. But if this be so, and, it be likewise true that a great Stone moveth with (suppose) eight degrees of Velocity, and a lesser with fewer, then joyning them both together, the compound of them will move with a Velocity lesse than eight Degrees: But the two Stones joyned together make one Stone greater than that before, which moved with eight degrees of Velocity: Therefore this greater Stone moveth lesse swiftly than the lesser, which is contrary to your Supposition. You see therefore, that from the supposing that the more grave Moveable moveth more swiftly than the lesse grave, I prove unto you that the more grave moveth lesse swiftly.

SIMP. I find my self at a losse, for the truth is, that the lesser Stone being joyned to the greater, weight is added unto it, and weight being added to it, I cannot see why there should not Velocity be added to it, or at least why it should be diminished in it.

SALV. Here you run into another errour, *Simplicius*, for it is not true, that that same lesser Stone encreaseth the weight of the greater.

SIMP. Oh wonderful! this quite surpasseth my apprehension.

SALV. Not at all, if you will but stay till I have discovered to you the Equivokes, of which you are in doubt: Therefore

you

you must know that it is necessary to distinguish betwixt grave Bodies set on Moving, and the same constituted in Rest; a Stone put into the Ballance not onely acquireth greater weight, by laying another Stone upon it, but also the addition of a Flake of Hemp will make it weigh more by those six or ten ounces that the Hemp shall weigh; but if you should freely let fall the Stone tied to the Hemp from an high place, do you think that in the Motion the Hemp weigheth down the Stone, so as to accelerate its Motion; or else do you believe that it will retard it, sustaining it in part? We indeed feel our shoulders laden, so long as we will oppose the Motion that the weight would make which lyeth upon our backs; but if we should descend with the same Velocity wherewith that same grave Body would naturally descend, in what manner will you that it presse or bear upon us? Do not you see that this would be a wounding one with a Lance that runneth before you, with as much or more speed than you pursue him. You may conclude therefore that in the free and natural fall, the lesser Stone doth not bear upon the greater, and consequently doth not encrease their weight, as it doth in Rest.

SIMP. But what if the greater was put upon the lesser?

SALV. It would encrease their weight, in case its Motion were more swift; but it hath been already concluded, that in case the lesser should be more slow it would in part retard the Velocity of the greater, so that their Compound would move lesse swiftly, being greater than the other, which is contrary to your Assumption: Let us conclude therefore, that great Moveables, and likewise little, being of the same Specifical Gravity, move with like Velocity.

SIMP. Your discourse really is full of ingenuity, yet methinks it is hard to conceive that a drop of Bird-shot, should move as swiftly as a Canon-bullet.

SALV. You may say a grain of Sand as fast as a Mill-stone. I would not have you, *Simplicius*, to do as some others are wont to do, and diverting the discourse from the principal design, fasten upon some one saying of mine that may want an hairs-breadth of the truth, and under this hair hide a defect of another man as big as the Cable of a Ship. *Aristotle* saith, a Ball of Iron of an hundred pounds weight falling, from an height of an hundred yards, commeth to the ground before that one of one pound is descended one sole yard: I say, that they arrive at the earth both in the same time: You find, that the bigger anticipates the lesser two Inches, that is to say, that when the great one falls to the ground, the other is distant from it two inches: you go about to hide under these two inches the ninety nine yards of *Aristotle*, and speaking onely to my small errour, passe over in silence the other great one.

Ari-

Aristotle affirmeth, that Moveables of different Gravities in the same *Medium* move (as far as concerneth Gravity) with Velocities proportionate to their Weights; and exemplifieth it by Moveables, wherein one may discover the pure and absolute effect of Weight, omitting the other Considerations, as well of Figures, as of the minute Motions, which things receive great alteration from the *Medium*, which altereth the simple effect of the sole Gravity; wherefore we see Gold, that is heavier than any other matter, being reduced into a very thin Leaf, to go flying to and again through the Air; the like do Stones beaten to very small Powder. But if you would maintain the Universal Proposition, it is requisite that you shew the proportion of the Velocities to be observed in all grave Bodies, and that a Stone of twenty pounds moveth ten times swifter than one of two: which, I tell you, is false, and that falling from an height of fifty or an hundred yards, they come to the ground in the same instant.

Simp. Perhaps in very great heights of Thousands of yards that would happen, which is not seen to occur in these lesser heights.

Salv. If this was the Meaning of *Aristotle*, you have involved him in another Errour, which will be found a Lie; for there being no such perpendicular altitudes found on the Earth, its a clear case, that *Aristotle* was not able to have made an Experiment thereof; and yet would perswade us that he had, whilst he saith, that the said effect is *seen*.

Simp. *Aristotle* indeed makes no use of this Principle, but of that other, which I believe is not obnoxious to these doubts.

Salv. Why that also is no lesse false than this; and I admire that you do not of your self perceive the fallacy, and discern, that should it be true, that the same Moveable in *Medium's* of different Subtilty and Rarity, and, in a word, of different Cession, such, for example, as are Water and Air, move with a greater Velocity in the Air than in the Water, according to the proportion of the Airs Rarity to the Rarity of the Water, it would follow that every Moveable that descendeth in the Air would descend also in the Water: Which is so false, that very many Bodies descend in the Air, that in the Water do not onely not descend, but also rise upwards.

Simp. I do not understand the necessity of your Consequence: and I will say farther, that *Aristotle* speaketh of those Grave-bodies that descend in the one *Medium* and in the other, and not of those that descend in the Air and ascend in the Water.

Salv. You produce for the Philsopher such Pleas as he, without all doubt, would never alledge, for that they aggravate the first mistake. Therefore tell me, if the Crassitude of the Water,

or

or whatever it be that retardeth the Motion, hath any proportion to the Crassitude of the Air that lesse retards it; and if it have, do you assign it us, at pleasure.

SIMP. It hath such a proportion, and we will suppose it to be decuple; and that therefore the Velocity of a Grave Body, that descends in both the Elements, shall be ten times slower in the Water than in the Air.

SALV. I will take one of those Grave-Bodies that descend in the Air, but not in the Water; as for instance, a Ball of Wood, and desire that you will assign it what Velocity you please, whilst it descends through the Air.

SIMP. Suppose we, that it move with twenty degrees of Velocity.

SALV. Very well: And it is manifest, that that Velocity to some other lesser, may have the same proportion, that the Crassitude of the Water hath to that of the Air; and that this shall be the Velocity of the two only degrees: so that exactly to an hair, and in direct conformity to the Assumption of *Aristotle*, it should be concluded, That the Ball of Wood, which in the Air, ten times more yielding, moveth descending with twenty degrees of Velocity, in the Water should descend with two, and not return from the bottom to flote a-top, as it doth: unless you will say, that the ascending of the Wood to the top is the same in the Water, as its sinking to the bottom with two degrees of Velocity; which I do not believe. But seeing that the Ball of Wood descends not to the bottom, I rather think that you will grant me, that some other Ball, of other matter different from Wood, might be found that descends in the Water with two degrees of Velocity.

SIMP. Questionlesse there might; but it must be of a matter considerably more grave than Wood.

SALV. This is that which I desired to know. But this second Ball, which in the Water descendeth with two degrees of Velocity, with what Velocity will it descend in the Air? It is requisite (if you will maintain *Aristotles* Rule) that you answer that it will move with twenty degrees: But you your self have assigned twenty degrees of Velocity to the Ball of Wood: Therefore this, and the other that is much more grave, will move thorow the Air with equall Velocity. Now how doth the Philosopher reconcile this Conclusion with that other of his, that the Moveables of different Gravity, move in the same *Medium* with different Velocities, and so different as are their Gravities? But, without any deep studies, how comes it to pass that you have not observed very frequent, and very palpable Accidents, and not considered two Bodies, that in the Water will move one an hundred times more swiftly than the other, but that again in the Air that swifter one will not out-go the other

other one sole Centesm? As for example, an Egge of Marble will descend in the Water an hundred times faster than one of an Hen, when as in the Air, at the height of twenty Yards it will not anticipate it four Inches: and, in a word, such a certain Grave Body will sink to the bottom in three hours in ten fathom VVater, that in the Air will pass the same space in one or two pulses, and such another (as for instance a Ball of Lead) will pass that number of fathoms with ease in less than double the time. And here I see plainly, *Simplicius*, that you find, that herein there is no place left for any distinction, or reply. Conclude we therefore, that that same Argument concludeth nothing against *Vacuum*; and if it should, it would only overthrow Spaces considerably great, which neither I, nor, as I take it, those *Ancients* did suppose to be naturally allowed, though, perhaps, with violence they may be effected, as, me thinks, one may collect from several Experiments, which it would be two tedious to go about at present to produce.

SAGR. Seeing that *Simplicius* is silent, I will take leave to say something. In regard you have with sufficient plainnesse demonstrated, that it is not true, That Moveables unequally grave move in the same *Medium* with Velocities proportionate to their Gravities, but with equal: desiring to be understood to speak of Bodies of the same Matter, or of the same Specifick Gravity, but not (as I conceive) of Gravities different *in Spetie*, (for I do not think that you intend to prove unto us, that a Ball of Cork moveth with like Velocity to one of Lead;) and having moreover very manifestly demonstrated, that it is not true, That the same Moveable in *Mediums* of different Resistances retain in their Velocities and Tardities the same proportion as have their Resistances: to me it would be a very pleasing thing to hear, what those be which are observed as well in the one case as in the other.

SALV. The Questions are ingenuous, and I have many times thought of them: I will relate unto you the Contemplations made upon them, and what at length I did from thence infer. After I had assured my self that it was not true, That the same Moveable in *Medium's* of different Resistance observeth in its Velocity the proportion of the Cession of those *Media*; nor yet, again, That in the same *Medium* Moveables of different Gravity retain in their Velocities the proportion of those Gravities (speaking alwaies of Gravities different *in specie*) I began to put both these Accidents together, observing that which befell the Moveables different in Gravity put into *Mediums* of different Resistance, and I perceived that the inequality of the Velocities were found to be alwaies greater in the more resisting *Medium's*, than in the more yielding; and that with such a diversity, that of two Moveables that, descending thorow the Air, differ very little in Velocity of Motion,

one

one will, in the Water, move ten times faster than the other; yea: that such, as in the Air do swiftly descend, in the Water not only will not descend, but will be wholly deprived of Motion, and, which is yet more, will move upwards: for one shall sometimes find some kind of Wood, or some knot, or root of the same, that in the Water will lye still, when as in the Air it will swiftly descend.

SAGR. I have many times set my self with an extream patience to see if I could reduce a Ball of Wax, (which of it self doth not go to the bottom) by adding to it grains of sand, to such a degree of Gravity like to the Water, as to make it stand still in the midst of that Element; but I could never, by all the care I used, succeed in my attempt; so that I cannot tell, whether any Solid matter may be found so naturally alike in Gravity to Water, as that being put into any place of the same, it can rest or lye still.

SALV. In this, as well as in a thousand other actions, many Animals are more ingenuous than we. And, in this case, Fishes would have been able to have given you some light, being in this affair so skilful, that at their pleasure they * equilibrate themselves, not only with one kind of Water, but with such, as, either of their own nature, or by means of some supervenient muddiness, or for their saltness (which maketh a great alteration) are very different; equilibrate themselves, I say, so exactly, that without stirring in the least they lye still in every place: and this, in my opinion, they do, by making use of the Instrument given them by Nature to that end, *scilicet*, of that Bladder which they have in their Bodies, which by a very narrow neck answereth to their mouth; and by that they either, when they would stand still, send forth part of the Air that is contained in the said Bladders, or, swimming to the top they draw in more, making themselves by that art one while more, another while less heavy than the Water, and at their pleasures equilibrating themselves to the same.

Fishes equilibrate themselves admirably in the Water.

* Or poise.

SAGR. I deceived some of my Friends with another device; for I had made my boast unto them, that I would reduce that Ball of Wax to an exact *equilibrium* with the Water, and having put some salt Water in the bottom of the Vessel, and a-top of that some fresh, I shewed them the Ball, which in the midst of the Water stood still, and being thrust to the bottom, or to the top, staid neither in this nor that scituation, but returned to the midst.

SALV. This same Experiment is not void of utility; for Physicians, in particular, treating of sundry qualities of Waters, and amongst other things, principally of the more or less Gravity or Levity of this or that: by such a Ball, in such manner poised and adjusted that it may rest ambiguous, if I may so say, between

A Ball of Wax prepared to make the Experiment of the different Gravities of Waters.

ascending and descending in a Water, upon the least difference of weight between two Waters, if that Ball shall descend in the one, in the other, that is more grave, it shall ascend. And the Experiment is so exact, that the addition of but only two grains of Salt, put into six pounds of Water, shall make that Ball to ascend from the bottom to the surface, which was but a little before descended thither. And moreover, I will tell you this in confirmation of the exactness of this Experiment, and withall for a clear proof of the Non-resistance of Water to division, that not only the ingravitating it with the mixture of some matter heavier than it, maketh that so notable difference, but the warming or cooling of it a little produceth the same effect, and with so subtil an operation, that the infusing four drops of other Water, a little warmer, or a little colder, than the six pounds, shall cause the Ball to rise or sink in the same; to sink in it upon the infusion of the warm, and to rise at the infusion of the cold. Now see how much those Philosophers are deceived, who would introduce in Water viscosity, or other conjunction of parts which make it to resist Division or Penetration.

Water hath no Resistance to Division.

SAGR. I have seen many Convincing Discourses touching this Argument in a * Treatise of our *Accademick*; yet neverthe less there is resting in me a strong scruple, which I know not how to remove: For if nothing of Tenacity, or Coherence resides amongst the parts of Water, how can it bear it self up in reasonable big and high Tumours; in particular, upon the leaves of Cole-worts without dispersing or levelling?

* The Tract cited in this place is that which we dispose first in Order, in the first part of this Tome,

SALV. Although it be true, that he who is Master of a true Conclusion, may resolve all Objections that can be brought against it, yet will not I arrogate to my self the power so to do; nor ought my insufficiency becloud the splendour of Truth. First, therefore, I confess that I know not how it cometh to pass, that those Globes of Water sustain themselves at such an height and bigness, albeit I certainly know that it doth not proceed from any internal Tenacity that is between its parts; so that it remaineth necessary, that the Cause of that Effect do reside without. That it is not Internal, besides those Experiments already shewn you, I can prove by another most convincing one. If the parts of that Water, which conserveth it self in a Globe or Tumour whilst it is encompassed by the Air, had an internal Cause for so doing, they would much better sustain themselves being environed by a *Medium*, in which they had less propension to descend, than they have in the Ambient Air: But every Fluid Body more grave than the Air would be such a *Medium*; as, for instance, Wine: And therefore, infusing Wine about that Globe of Water, it might raise it self on every side, and yet the parts of the Water, conglutinated by

Water formed into great drops upon the Leaves of Colworts, how they consist.

by the internal Viscosity, never dissolve: But it doth not happen so; nay, no sooner doth the circumfused liquor approach thereto, but, without staying till it rise much about it, the little globes of Water will dissolve and become flat, resting under the Wine, if it was red. The Cause therefore of this Effect is External, and perhaps in the Ambient Air: and, indeed, one may observe a great dissention between the Air and Water; which I have observed in another Experiment; and this it is: If I fill a * Ball of Christal, that hath a mouth as narrow as the hollow of a straw, with water, and when it is thus full, turn it with its mouth downwards, yet will not the Water, although very heavy, and prone to descend thorow the Air, nor the Air, as much disposed on the other hand, as being very light, to ascend thorow the Waters, yet will they not (I say) agree that that should descend, issuing out at the mouth, and this ascend, entering in at the same: but they both continue averse and contumacious. Again, on the contrary, if I present to that mouth a vessel of red Wine, which is almost insensibly less grave than Water, we shall see it in an instant gently to ascend by red streams thorow the Water, and the Water with like Tardity to descend through the Wine, without ever mixing with each other, till that in the end, the Ball will be full of Wine, and the Water Will all sink unto the bottom of the Vessel underneath. Now what are we to say, or what are we to infer, but a disagreement between the Water and Air, occult to me, but perhaps ——

* Or bottle.

Simp. I can scarce refrain my laughter to see the great Antipathy that *Salviatus* hath to Antipathy, so that he will not so much as name it, and yet it is so accommodate to resolve the doubt.

Salv. Now let this, for the sake of *Simplicius* be the solution of our scruple; and leaving the Digression, let us return to our purpose. Seeing that the difference of Velocity in Moveables of divers Gravities is found to be more and more, as the *Mediums* are more and more Resisting: And withall, that in a *Medium* of Quicksilver, Gold doth not only go to the bottom more swiftly than Lead, but it alone descends in it, and all other Metals and Stones move upwards therein, and flote thereon; whereas between Balls of Gold, Lead, Brass, Porphiry, or other grave matters, the inequality of motion in the Air shall be almost wholly insensible, for it is certain, that a Ball of Gold in the end of the descent of an hundred yards shall not out-strip one of Brass four Inches: seeing this, I say, I have thought, that if we wholly took away the Resistance of the *Medium*, all Matters would descend with equall Velocity.

Resistance of the Medium *removed, all Matters, though of different Gravities would move with like Velocity.*

Simp. This is a bold speech, *Salviatus*, I shall never believe that in *Vacuity* it self, if so be one should allow Motion in it, a lock of Wooll would move as swiftly as a piece of Lead.

SALV. Fair and softly, *Simplicius*, your scruple is not so abstruce, nor I so incautelous, that you should need to think that I was not advised of it, and that consequently I have not found a reply to it. Therefore, for my explanation, and your information, hearken to what I shall say. We are upon the examination of what would befall Moveables exceeding different in weight in a *Medium*, in case it should have no Resistance, so that all the difference of Velocity that is found between the said Moveables ought to be referred to the sole inequality of Weight. And because only a Space altogether void of Air, and of every other, though tenuous and yielding Body, would be apt sensibly to shew us what we seek, since we want such a Space, let us successively observe that which happeneth in the more subtill and lesse resisting *Mediums*, in comparison of that which we see to happen in others lesse subtill and more resisting: for if we should really find the Moveables different in Gravity to differ lesse and lesse in Velocity, according as the *Mediums* are found more and more yielding; and that, finally, although extreamly unequal in weight, in a *Medium* more tenuous than any other, though not void, the difference of Velocity discovers it self to be very small, and almost unobservable, I conceive that we may, and that upon very probable conjecture, believe, that in a *Vacuum* their Velocities would be exactly equal. Therefore let us consider that which hapneth in the Air; wherein to have a Figure of an uniform Superficies, and very light Matter, I will that we take a blown Bladder, in which the included Air will weigh little or nothing in a *Medium* of the Air it self, because it can make but very small Compression therein, so that the Gravity is only that little of the said film, which would not be the thousandth part of the weight of a lump of Lead of the bigness of the said Bladder when blown. These, *Simplicius*, being let fall from the height of four or six yards, how great a space, do you judge, that the Lead would anticipate the Bladder in its descent? Assure your self that would not move thrice, no nor twice as fast, although even now you would have had it to have been a thousand times more swift.

SIMP. It is possible that at the beginning of the Motion, that is, in the first five or six yards this might happen that you say; but in the progresse, and in a long continuation I believe, that the Lead would leave it behind, not only six, but also eight and ten parts of twelve.

SALV. And I also believe the same: and make no question, but that in very great distances the Lead will have passed an hundred miles of *way*, ere the Bladder will have passed so much as one. But this, *Simplicius*, which you propound, as an effect contrary to my Assertion, is that which most especially confirmeth it. It is (I once

once more tell you) my intent to declare, That the difference of Gravity is in no wise the cause of the divers velocities of Moveables of different Gravity, but that the same dependeth on exteriour accidents,& in particular,on the Resistance of the *Medium*, so that, this being removed, all Moveables move with the same degrees of Velocity. And this I chiefly deduce from that which but now you your self did admit, and which is very true, namely, that of two Moveables,very different in weight,the Velocities more and more differ, according as the * Spaces are greater and greater that they passe : an Effect which would not follow, if it did depend on the different Gravities : for they being alwaies the same, the proportion betwixt the Spaces would likewise alwaies continue the same, which proportion we see still successively to encrease in the continuance of the Motion ; for that the heaviest Moveable in the descent of one yard will not anticipate the lightest the tenth part of that Space or VVay, but in the fall of twelve yards will out-go it a third part, in that of an hundred will outstrip it $\frac{90}{100}$.

* Or Waies.

SIMP. Very well : But following you step by step, if the difference of weight in Moveables of different Gravities cannot cause the difference of proportion in their Velocities, for that the Gravities do not alter ; neither then can the *Medium*, which is supposed alwaies to continue the same, cause any alteration in the proportion of the Velocities.

SALV. You wittily bring an instance against my Position, that it is very necessary to remove. I say therefore, that a Grave Body hath, by Nature, an intrinsick Principle of moving towards the Common Center of heavy things, that is to that of our Terrestrial Globe, with a Motion continually accelerated, and accelerated alwaies equally, *scilicet*, that in equal times there are made equal * additions of new Moments, and degrees of Velocities : and this ought to be understood to hold true at all times when all accidental and external impediments are removed ; amongst which there is one that we cannot obviate, that is the Impediment of the *Medium*, which is Repleat, when as it should be opened and latterally moved by the falling Moveable, to which transverse Motion the *Medium*, though fluid, yielding and tranquile, opposeth it self with a Resistance one while lesser, and another while greater and greater, according as it is more slowly or hastily to open to give passage to the Moveable , which, because, as I have said, it goeth of its own nature continually accelerating, it cometh of consequence to encounter continually greater Resistance in the *Medium*, and therefore Retardment, and diminution in the acquist of new degrees of Velocity ; so that in the end, the Velocity arriveth to that swiftnesse, and the Resistance of the *Medium*, to that strength , that ballancing each other, they take away all further

The Velocity of Grave Bodies descending Naturally to the Center do go continually encreasing till that by the encrease of the Resistance of the Medium *it becometh uniform.*

* Or aquists.

Acceleration,

Acceleration, and reduce the Moveable to an Equable and Uniform Motion, in which it afterwards continually abides. There is therefore in the *Medium* augmentation of Resistance, not because it changeth its Essence, but because the Velocity altereth wherewith it ought to open, and laterally move, to give passage to the falling Body, which goeth continually accelerating. Now the observing, that the Resistance of the Air to the small Moment or *Impetus* of the Bladder is very great, and to the great weight of the Lead is very small, makes me hold for certain, that if one should wholly remove it, by adding to the Bladder great assistance, and but very little to the Lead, their Velocities would equalize each other. Taking this Principle therefore for granted, That in the *Medium* wherein, either by reason of Vacuity, or otherwise, there were no Resistance that might abate the Velocity of the Motion, so that of all Moveables the Velocities were alike, we might congruously enough assign the proportions of the Velocities of like and unlike Moveables, in the same and in different, Repleat, and therefore Resisting *Medium's*. And this we might effect by studying how much the Gravity of the *Medium* abateth from the Gravity of the Moveable, which Gravity is the Instrument wherewith the Moveable makes its VVay, repelling the parts of the *Medium* on each Side: an operation that doth not occur in void *Mediums*; and therefore there is no difference to be expected from the diverse Gravity: and because it is manifest, that the *Medium* abateth from the Gravity of the Body by it contained, as much as is the weight of such another mass of its own Matter, if the Velocities of the Moveables that in a non-resisting *Medium* would be (as hath been supposed) equal, should diminish in that proportion, we should have what we desired. As for example; supposing that Lead be ten thousand times more grave than Air, but Ebony a thousand times only; of the Velocities of these two Matters, which absolutely taken, that is, all Resistance being removed, would be equal, the Air substracts from the ten thousand degrees of the Lead one, and from the thousand degrees of the Ebony likewise abateth one, or, if you will, of its ten thousand, ten. If therefore the Lead and the Ebony shall descend thorow the Air from any height, which, the retardment of the Air removed, they would have passed in the same time, the Air will abate from the ten thousand degrees of the Leads Velocity one, but from the ten thousand degrees of Ebony's Velocity it will abate ten: which is as much as to say, that dividing that Altitude, from which those Moveables departed into ten thousand parts, the Lead will arrive at the Earth, the Ebony being left behind, ten, nay, nine of those same ten thousand parts. And what else is this, but that a Ball of Lead, falling from a Tower two hundred yards high, to find how

To find the Proportions of the Velocities of different Moveables in the same, and in different Mediums.

much

much it will anticipate one of Ebony of lesse than four Inches? The Ebony weigheth a thousand times more than the Air, but that Bladder so blown, weigheth only four times so much; the Air therefore from the intrinsick and natural Velocity of the Ebony subducteth one degree of a thousand, but from that, which also in the Bladder would absolutely have been the same, the Air subducts one part of four: so that by that time the Ball of Ebony falling from the Tower, shall come to the ground, the Bladder shall have passed but three quarters of that height. Lead is twelve times heavier than Water, but Ivory only twice as heavy; the Water therefore, from their absolute Velocities which would be equal, shall abate in the Lead the twelfth part, but in the Ivory the half: when therefore, in the Water, the Lead shall have descended eleven fathom, the Ivory shall have descended six. And, arguing by this Rule, I believe, that we shall find the Experiment much more exactly agree with this same Computation, than with that of *Aristotle*. By the like method we might find the Velocities of the same Moveable in different fluid *Mediums*, not comparing the different Resistances of the *Mediums*, but considering the excesses of the Gravity of the Moveable over and above the Gravities of the *Mediums*: *v. gr.* * Tin is a thousand times heavier than Air, and ten times heavier than Water; therefore dividing the absolute Velocity of the Tin into a thousand degrees, it shall move in the Air, which deducteth from it the thousandth part, with nine hundred ninety nine, but in the Water with nine hundred only; being that the Water abateth the tenth part of its Gravity, and the Air the thousandth part. Take a Solid somewhat heavier than Water, as for instance, the Wood called Oake, a Ball of which weighing, as we will suppose, a thousand drams, a like quantity of Water will weigh nine hundred and fifty, but so much Air will weigh but two drams,: it is manifest, that supposing that its absolute Velocity were of a thousand degrees, in Air there would remain nine hundred ninety eight, but in the Water only fifty; because that the Water of the thousand degrees of Gravity taketh away nine hundred and fifty, and leaves fifty only; that Solid therefore would move well-near twenty times as fast in the Air as VVater; like as the excesse of its Gravity above that of the VVater is the twentieth part of its own. And here I desire that we may consider, that no matters, having a power to move downwards in the VVater, but such as are more grave in Species than it; and consequently many hundreds of times, more grave than the Air, in seeking what the proportions of their Velocities are in the Air and VVater, we may, without any considerable errour, make account that the Air doth not deduct any thing of moment from the absolute Gravity, and consequently, from the absolute Velocity of such mat-

* Or Pewter.

matters: so that having easily found the excesse of their Gravity above the Gravity of the Water, we may say that their Velocity in the Air, to their Velocity in the Water hath the same proportion, that their total Gravity hath to the excesse of this above the Gravity of the VVater. For example, a Ball of Ivory weigheth twenty ounces, a like quantity of VVater weigheth seventeen ounces: therefore the Velocity of the Ivory in Air, to its Velocity in VVater is very neer as twenty to three.

SAGR. I have made a great acquist in a businesse of it self curious, and in which, but without any benefit, I have many times wearied my thoughts: nor would there any thing be wanting for the putting these Speculations in practice, save onely the way how one should come to know of what Gravity the Air is in comparison to the Water, and consequently to other heavy matters.

SIMP. But in case one should finde, that the Air instead of Gravity had Levity, what ought one to say of the foregoing discourses, otherwise very ingenuous?

SALV. It would be necessary to confesse that they were truly Aerial, Light, and Vain. But will you question whether the Air be heavy, having the expresse *Text* of *Aristotle* that affirmeth it, saying, That all the Elements have Gravity, even the Air it self; a signe of which (subjoyns he) we have in that a * Bladder blown, weigheth heavier than unswell'd.

* Or *Boracho*; a bottle made of a Goat skin, used to hold wine and other Liquids.

SIMP. That a *Boracho*, or Bladder blown, weigheth more, might proceed, as I could suppose, not from the Gravity that is in the Air, but in the many grosse Vapours intermixed with it in these our lower Regions; by means whereof I might say, that the Gravity of the Bladder, or *Boracho* encreaseth.

SALV. I would not have you say it, and much lesse that you should make *Aristotle* speak it, for he treating of the Elements, and desiring to perswade me that the Element of Air is grave, making me to see it by an Experement: if in comming to the proof he should say: Take a Bladder, and fill it with grosse Vapours; and observe that its weight will encrease; I would tell him that it would weigh yet more if one should fill it with bran; but would afterwards adde; that those Experiments prove, that bran, and grosse Vapours are grave: but as to the Element of Air, I should be left in the same doubt as before. The Experiment of *Aristotle* therefore is good, and the Proposition true. But I will not say so much, for a certain other reason taken expresly out of a Philosopher whose name I do not remember, but am sure that I have read it, who argueth the Air to be more grave than light, because it more easily carrieth grave Bodies downwards, than the light upwards.

SAGR. Good i-faith. By this reason then, the Air shall be much

much heavier than the Water, since, that all Bodies are carried more easily downwards thorow the Air than thorow the Water, and all light Bodies more easily upwards in this than in that: nay, infinite matters ascend in the Water, that in the Air descend. But be the Gravity of the Bladder, *Simplicius*, either by reason of the grosse Vapours, or pure Air, this nothing concerns our purpose, for we seek that which happeneth to Moveables that move in this our Vaporous Region. Therefore, returning to that which more concerneth me, I would for a full and absolute information in the present businesse, not onely be assured that the Air is grave, as I hold for certain, but I would, if it be possible, know what its Gravity is. Therefore, *Salviatus*, if you have wherewith to satisfie me in this also, I entreat you to favour me with the same.

The Air hath Positive Gravity.

SALV. That there resideth in the Air positive Gravity, and not, as some have thought, Levity, which haply is in no Matter to be found, the Experiment of the Blown-Bladder, alledged by *Aristotle*, affordeth us a sufficiently-convincing Argument; for if the quality of absolute and positive Levity were in the Air, then the Air being multiplied and compressed, the Levity would encrease, and consequently the propension of going upwards: but Experience shews the contrary. As to the other demand, that is, of the Method how to investigate its Gravity, I have tried to do it in this manner: I have taken a pretty bigge Glasse * Bottle, with its neck bended, and a Finger-stall of Leather fast about it, having in the top of the said Finger-stall inserted and fastened a Valve of Leather, by which with a Siringe I have made passe into the Bottle by force a great quantity of Air, of which, because it admits of great Condensation, it may take in two or three other Bottles-ful over and above that which is naturally contained therein. Then I have in an exact Ballance very precisely weighed that Bottle with the Air compressed within it, adjusting the weight with small Sands. Afterwards, the Valve being opened, and the Air let out, that was violently conteined in the Vessel, I have put it again into the Scales, and finding it notably aleviated, I have by degrees taken so much Sand from the other Scale, keeping it by it self, that the Ballance hath at last stood *in Equilibrio* with the remaining counter-poise, that is with the Bottle. And here there is no question, but that the weight of the reserved Sand is that of the Air that was forceably driven into the Bottle, and which is at last gone out thence. But this Experiment hitherto assureth me of no more but this, that the Air violently deteined in the Vessel, weigheth as much as the reserved Sand, but how much the Air resolutely and determinately weigheth in respect of the Water, or other grave matter, I do not as yet know, nor can

How that Gravity may be computed.

* *Un Fiasco*, those long-neckt glasse bottles in which we have our *Florence* Wine brought to us.

I tell, unlesse I measure the quantity of the Air compressed: and for the discovering of this a Rule is necessary, which I have found may be performed two manner of wayes, one of which is to take such another Bottle or Flask as the former, and in like manner bended, with a Finger-stall of Leather, the end of which may closely imbrace the Volve of the other, and let it be very fast tied about it. It's requisite, that this second Bottle be bored in the bottom, so that as by that hole we may thrust in a VVier, wherewith we may, at pleasure, open the said Volve, to let out the superfluous Air of the other Vessel, after it hath been weighed: but this second Bottle ought to be full of VVater. All being prepared in the manner aforesaid, and with the VVier opening the Volve, the Air issuing out with impetuosity, and passing into the Vessel of VVater, shall drive it out by the hole at the Bottom: and it is manifest, that the quantity of VVater which shall be thrust out, is equal to the Masse and quantity of Air that shall have issued from th'other Vessel: that VVater therefore being kept, and returning to weigh the Vessel lightned of the Air compressed (which I suppose to have been weighed likewise first with the said forced Air) and the superfluous sand being laid by, as I directed before; it is manifest, that this is the just weight of so much Air in masse, as is the masse of the expulsed and reserved Water; which we are to weigh, and see how many times its weight shall contain the weight of the reserved sand: and we may without errour affirme, that the VVater is so many times heavier than Air; which shall not be ten times, as it seemeth *Aristotle* held, but very neer four hundred, as the said Experiment sheweth.

The other way is more expeditious, and it may be done with one Vessel onely, that is with the first accomodated after the manner before directed, into which I will not that any other Air be put, more than that which naturally is found therein; but I will, that we inject VVater without suffering any Air to come out, which being forced to yield to the supervenient VVater must of necessity be compressed: having gotten in, therefore, as much Water as is possible, (but yet without great violence one cannot get in three quarters of what the Bottle will hold) put it into the Scales, and very carefully weigh it: which done, holding the Vessel with the neck upwards, open the Volve, letting out the Air, of which there will precisely issue forth so much as there is Water in the Bottle. The Air being gone out, put the Vessel again into the Scales, which by the departure of the Air will be found lightened, and abating from the opposite Scale the superfluous weight, it shall give us the weight of as much Air as there is VVater in the Bottle.

SIMP. The Contrivances you found out cannot but be confessed

fessed to be witty and very ingenuous, but whilst, me thinks, they fully satisfie my understanding, they another way occasion in me much Confusion, for it being undoubtedly true that the Elements in their proper Region are neither heavy nor light, I cannot comprehend, how and which way that portion of Air, which seemeth to have weighed *v. gr.* four drams of sand, should afterwards have that same Gravity in the Air, in which the sand is contained that weigheth against it: and therefore me thinks that the Experiment ought not to be practiced in the Element of Air, but in a *Medium* in which the Air it self might exercise its quality of Gravitation, if it really be owner thereof.

SALV. Certainly the Objection of *Simplicius* is very acute, and therefore its necessary, either that it be unanswerable, or that the Solution be no lesse acute. That that Air, which compressed, appeared to vveigh as much as that sand, left at liberty in its Element is no longer to weigh any thing as the Sand doth, is a thing manifest: and therefore for making of such an Experiment, its requisite to choose a place and *Medium* wherein the Air as well as the Sand might weigh: for, as hath several times been said, the *Medium* substracts from the Weight of every Matter that is immerged therein, so much, as such another quantity of the said *Medium*, as is that of the masse immersed, weigheth: so that the Air depriveth the Air of all its Gravity. The operation, therefore, to the end it were made exactly, ought to be tried in a *Vacuum*, wherein every grave Body would exercise its Moment without any diminution. In case therefore, *Simplicius*, that we should weigh a portion of Air in a *Vacuum*, would you then be convinced and assured of the businesse?

The Air compressed and violently pent up, weigheth in a Vacuum; *and how its weight is to be estimated.*

SIMP. Verily I should: but this is to desire, or enjoyn that which is impossible.

SALV. And therefore the obligation must needs be great that you owe to me, when ever I shall for your sake effect an impossibility: but I will not sell you that which I have already given you: for we, in the foregoing Experiment, weigh the Air in a *Vacuum*, and not in the Air, or in any other Replete *Medium*. That from the Mass, *Simplicius*, that in the fluid *Medium* is immerged certain Gravity is substracted by the said *Medium*, this commeth to pass by reason that it resisteth its being opened, driven back, and in a word commoved; a sign of which is its pronenesse to return instantly to fill the Space up again, that the immersed mass occupied in it, as soon as ever it departeth thence; for if it suffered not by that immersion, it would not operate against the same. Now tell me, when you have in the Air the Bottle before filled with the same Air naturally contained therein, what division, repulse, or, in short, what mutation doth the external ambient Air receive from the se-

cond Air that was newly infused with force into the Vessel? Doth it enlarge the Bottle, whereupon the Ambient ought the more to retire it self to make room for it? Certainly no: And therefore we may say, that the second Air is not immersed in the Ambient, not occupying any Space therein; but is as if it was in a *Vacuum*, nay more, is really constituted in it, and is placed in Vacuities that were not repleted by the former un-condensed Air. And, really, I know not how to discern any difference between the two Constitutions of Inclosed and *Ambient*, whilst in this the *Ambient* doth no-ways press the Inclosed, and in that the Inclosed doth not repulse the *Ambient*: and such is the placing of any matter in a *Vacuum*, and the second Air compressed in the Flask. The weight therefore that is found in that same condensed Air, is the same that it would have, were it freely distended in a *Vacuum*. Tis true indeed, that the weight of the Sand that weigheth against it, as having been in the open Air, would in a *Vacuum* have been a little more than just so heavy; and therefore it is necessary to say, that the weighed Air is in reality somewhat lesse heavy than the Sand that counterpoiseth it, that is, so much, by how much the like quantity of Air would weigh in a *Vacuum*.

SIMP. I had thought that there was something to have been wished for in the Experiments before produced; but now I am thorowly satisfied.

The difference, though very great, of the Gravity of Moveables hath no part in differencing their Velocities.

SALV. The things by me hitherto alledged, and in particular, this, That the difference of Gravity, although exceeding great, hath no part in diversifying the Velocities of Moveables, so that, notwithstanding any thing depending on that, they would all move with equal Celerity, is so new, and at the first apprehension so remote from probability, that, were there not a way to delucidate it, and make it as clear as the Sun, it would be better to passe it over in silence, than to divulge it: therefore seeing that I have let it escape from me, its fit that I omit neither Experiment nor Reason that may corroborate it.

SAGR. Not onely this, but many other also of your Assertions are so remote from the Opinions and Doctrines commonly received, that sending them abroad, you would stir up a great number of Antagonists: in regard, that the innate Disposition of Men doth not see with good eyes, when others in their Studies discover Truths or Fallacies, that were not discovered by themselves: and with the title of Innovators of Doctrines, little pleasing to the ears of many, they study to cut those knots which they cannot untie, and with sub-terranean Mines to blow up those Structures, which have been with the ordinary Tools by patient Architects erected: but with us here, who are far from any such thoughts, your Experiments and Arguments are suf-

sufficient to give full satisfaction: yet neverthelesse, if so be you have other more palpable Experiments, and more convincing Reasons we would very gladly hear them.

SALV. The Experiment made with two Moveables, as different in weight as may be, by letting them descend from a place on high, thereby to see whether their Velocity be equal, meets with some difficulty: for if the height shall be great, the *Medium*, which is to be opened and laterally repelled by the *Impetus* of the cadent Body, shall be of much greater prejudice to the small Moment of the light Moveable, than to the violence of the heavy one; whereupon in a long way the light one will be left behind; and in a little altitude it might be doubted whether there were really any difference, or if there were, whether it would be sensible. Therefore I have oft been thinking to reiterate the descent so many times from small heights, and to accumulate together so many of those minute differences of time, as might intercede between the arrival or fall of the heavy Body to the ground, and the arrival of the light one, which so conjoyned, would make a time not onely observable, but observable with much facility. Moreover, that I might help my self with Motions as slow as possible may be, in which the Resistance of the *Medium* operates lesse in altering the effect that dependeth on simple Gravity, I have had thoughts to cause the Moveable to descend upon a declining Plane; not much raised above the Plane of the Horizon; for upon this, no lesse than in perpendicularity, we may discover that which is done by Grave Bodies different in weight: and proceeding farther, I have desired to free my self from any whatsoever impediment, that might arise from the Contact of the said Moveables upon the said declining Plane: and lastly, I have taken two Balls, one of Lead, and one of Cork, that above an hundred times more grave than this, and have fastened them to two small threads, each equally four or five yards long, tyed on high: and having removed aswel the one as the other Ball from the state of Perpendicularity, I have let them both go in the same Moment, and they descending by the Circumferences of Circles described by the equal Strings their Semidiameters, and having passed beyond the Perpendicular, they afterwards by the same way returned back, and reiterating these Vibrations, and returns of themselves neer an hundred times, they have shewn very sensibly, that the grave *Pendulum* moveth so exactly under the time of the light one, that it doth not in an hundred, no nor in a thousand Vibrations, anticipate the time of one small moment, but that they keep an equal passe in their Recursions. They also shew the Operation of the *Medium*, which conferring some impediment on the Motion, doth much more diminish the Vibrati-

ons

ons of the Cork, than that of the Lead: not that it maketh them more or lesse frequent, nay, when the Arches passed by the Cork were not of above five or six degrees, and those of the Lead fifty, they did pass them under the same times.

SIMP. If this be so, how is it then that the Velocity of the Lead is not greater than that of the Cork? that passing a journey of sixty degrees, in the time that this passeth hardly six?

SALV. But what would you say, *Simplicius*, in case they should both dispatch their Recursions in the same time, when the Cork being removed thirty degrees from the Perpendicular, should pass an arch of sixty, and the Lead removed from the same middle point onely two degrees, should run an arch of four? would not then the Cork be so much more swift than the Lead? and yet Experience shews that so it happeneth: therefore observe, The *Pendulum* of Lead being carried *v. gr.* fifty degrees from the Perpendicular, and thence let go, swingeth, and passing beyond the Perpendicular, neer fifty more degrees, describeth an arch of well neer an hundred degrees; and returning of its self back again, it describeth another arch, not much lesse than the former, and continuing its Vibrations, after a great number of them, it finally returneth to Rest: Each of those Vibrations are made under equal times aswel those of ninety degrees, as those of fifty, twenty, ten, or four; so that by consequence, the Velocity of the Moveable doth successively languish and abate, in regard, that under equal times it doth successively passe arches continually lesser and lesser. The like, yea the self same effect is performed by the Cork, hanging by a string of the like length, save that in a lesse number of Vibracions it returneth to Rest, as being less apt, by means of its Levity, to overcome the obstacle of the Air: and yet neverthelesse all the Vibrations, both great and small, are made under times equal to one another, and equal also to the times of the times of the Vibrations of the Lead. Whereupon it is true, that if whilst the Lead passeth an arch of fifty degrees, the Cork passeth one but of ten, the Cork is then more slow than the Lead: but it will also happen on the other side, that the Cork passeth the arch of fifty degrees, when the Lead passeth but that of ten or six; and so in several times the Lead shall be swifter onewhile, and the Cork another while: but if the same Moveables shall also under the same equal times, pass arches that are equal, one may then very safely say, that their Velocities are equal.

SIMP. This discourse seems to me concluding, and not concluding, and I finde in my thoughts such a Confusion, arising from the one while swift, another while slow, another while extreme slow motion of both the one and other Moveable; as that it

it permits me not to discern clearly, whether it be true, That their Velocities are alwaies equal.

SAGR. Give me leave, I pray you, *Salviatus*, to interpose two words. And tell me, *Simplicius*, whether you admit, that it may be said with absolute verity that the Velocities of the Cork and of the Lead are equal, in case, that both of them, departing at the same moment from Rest, and moving by the same declivities, they should alwaies passe equal Spaces in equal times?

SIMP. This admits of no doubt, nor can it be contradicted.

SAGR. It hapneth now in the Pendulums that each of them passeth now sixty degrees, now fifty, now thirty, now ten, now eight, four, and two; and when each of them passeth the Arch of sixty degrees they passe it in the same time; in the Arch of fifty the same time is spent by both the one and the other Moveable; so in the Arch of thirty, of ten, and of the rest: and therefore it is concluded, that the Velocity of the Lead in the Arch of sixty degrees, is equal to the Velocity of the Cork in the same Arch of sixty degrees: and that the Velocities in the Arch of fifty, are likewise equal to one the other, and so in the rest. But it is not said, that the Velocity that is exercised in the Arch of sixty is equal to the Velocity that is exercised in the Arch of fifty, nor this to that of the Arch of thirty. But the Velocities are alwaies lesser, in the lesser Arches. And this is collected from our sensibly seeing the same Moveable consume as much time in passing the great Arch of sixty degrees, as in passing the lesser of fifty, or the least of ten: and, in a word, in their being all passed alwaies under equal times. It is true therefore, that both the Lead and the Cork successively retard the Motion, according to the Diminution of the Arches, but yet do not alter their harmony in keeping the equality of Velocity in all the same Arches by them passed. I desired to say thus much, more to try whether I have rightly apprehended the Conceit of *Salviatus*, than out of any necessity that I thought *Simplicius* to stand in of a more plain Explanation than that of *Salviatus*, which is, as in all other things, extreamly clear, and such, that, it being frequent with him to resolve Questions, in appearance not only obscure, but repugnant to Nature, and to the Truth, with Reasons, or Observations, or Experiments very trite and familiar to every one, it hath (as I have understood from divers) given occasion to one of the most esteemed Professors of our Age to put the lesse esteem upon his Novelties, holding them to have as much of Sordidnesse, for that they depend on over low and popular Fundamentals: as if the most admirable and most-to-be-prized Property of the Demonstrative Sciences, were not to spring and arise from Principles known, understood, and granted by every one. But let us, for all that, continue to banquet our selves with this diet

that

that is so light of digestion; and supposing that *Simplicius* is fully satisfied in understanding and admitting, That the intern Gravity of different Moveables hath no share in differencing their Velocities, so that all of them, for ought that dependeth on that, would move with the same Velocities; tell us, *Salviatus*, in what you place the sensible and apparent inequalities of Motion; and answer to that Instance that *Simplicius* produceth, and which I likewise confirm, I mean, of seeing a Cannon Bullet move more swiftly than a drop of Bird-shot, for the difference of Velocity shall be but small, in respect of that which I object against you of Moveables of the same matter, of which some of the greater will descend in a *Medium*, in lesse than one beat of the Pulse, that space, that others which are lesser will not passe in an hour, nor in four, nor in twenty; such are pebbles and minute gravel-stones, especially, that small sand which muddieth the Water; in which *Medium* they will not descend in many hours so much as two fathoms, which Stones, and those of no great bignesse, do passe in one beat of the Pulse.

SALV. That which the *Medium* operates, in retarding Moveables, the more according as they are compared to one another, less grave *in specie*, hath been already declared, shewing that it proceeds from the substraction of weight. But how one and the same *Medium* can with so great difference diminish the Velocity in Moveables that differ only in Magnitude, although they are of the same Matter, and of the same Figure, requireth for its explication a more subtil discourse, than that which sufficeth for understanding how the more dilated Figure of the Moveable, or the Motion of the *Medium* that is made contrary to the Moveable, retardeth the Velocity of the said Moveable. I reduce the cause of the said Problem to the Scabrosity, and Porosity, that is commonly, and, for the most part, necessarily found in the Superficies of Solid Bodies, the which Scabrosities, in their Motion, go repulsing and commoving the Air, or other Ambient *Medium*: of which we have an evident testimony, in that we hear the Bodies, though made as round as is possible for them to be, to hum whilst they passe very swiftly thorow the Air; and they are not only heard to hum, but to whir and whistle, if there be but in them some more than ordinary cavity or prominency. We see also, that in turning round every rotund Solid maketh a little wind: And what need more? Do we not hear a notable whirring, and in a very sharp Accent, made by a Top, while it turneth round on the ground with great Celerity? The shrilness of which whizzing groweth flatter according as the Velocity of the *Vertigo* doth by degrees more and more slacken: a necessary Argument likewise of the commotion and percussion of the Air by those (though very small) Scabrosities

The greater or less Scabrosity and Porosity of the Superficies of Moveables, a probable cause of their greater or lesser Retardation.

ties of their Superficies. It is not to be doubted, but that these in the descent of Moveables, grating upon, and repulsing the fluid Ambient, procure retardment in the Velocity, and so much the greater, by how much the Superficies shall be greater, as is that of lesser Solids compared to bigger.

SIMP. Stay, I pray you, for here I begin to be at a losse: for though I understand and admit, that the Confrication of the *Medium* with the Superficies of the Moveable retardeth the Motion, and that it more retardeth it where, *ceteris paribus*, the Superficies is greater, yet do I not comprehend upon what ground you call the Superficies of lesser Solids greater: & farthermore if, as you affirm, the greater Superficies ought to cause greater retardment, the greater Solids ought to be the slower, which is not so: but this Objection may easily be removed, by saying, that although the greater hath a greater Superficies, it hath also a greater Gravity, upon which the impediment of the greater Superficies hath not so much more prevalent influence, than the impediment of the lesser Superficies hath upon the lesser Gravity, as that the Velocity of the greater Solid should become the lesser. And therefore I see no reason why one should alter the equality of the Velocities, whilst, that looking how much the Moving Gravity diminisheth, the faculty of the Retarding Superficies doth diminish at the same rate.

SALV. I will resolve all that which you object in one word. Therefore, *Simplicius*, you will without controversie admit, that when, of two equal Moveables of the same Matter, and alike in Figure (which undoubtedly would move with equal swiftnesse) as well the Gravity, as the Superficies of one of them diminisheth, (yet still retaining the similitude of Figure) the Velocity likewise, for the same reason, would not be diminished in that which was lessened.

SIMP. Really, I think, that it ought so to follow as you say, granting the present Doctrine with a *salvo* still to our Doctrine, which teacheth, that the greater or lesser Gravity hath no operation in accelerating or retarding Motion.

SALV. And this I confirm; and grant you likewise your Position, from whence, in my opinion, may be inferred, That in case the Gravity diminisheth more than the Superficies, there may be introduced in the Moveable, in that manner diminished, some retardment of Motion, and that greater and greater, by how much in proportion, the diminution of the Weight was greater than the diminution of the Superficies.

SIMP. I make not the least question of it.

SALV. Now know, *Simplicius*, that in Solids one cannot diminish the Superficies so much as the Weight keeping the similitude of Figure. For it being manifest, that in diminishing of grave Solids,

Solids cannot be diminished at the same rate in Superficies as in Weight, retaining the similitude of the Figures.

Solids, the Weight lesseneth as much as the Bulk, when ever the Bulk happens to be diminished more than the Superficies, (care being had to retain the similitude of Figure) the Gravity likewise would come to be more diminished than the Superficies. But *Geometry* teacheth us, that there is much greater proportion between the Bulk and the Bulk in like Solids, than between their Superficies. Which for your better understanding, I shall explain in some particular case. Therefore fancy to your self, for example, a Dye, one of the Sides of which is *v. gr.* two Inches long, so that one of its Surfaces shall be four Square Inches, and all six, that is, all its Superficies twenty four Square Inches. Then suppose the same Dye at three sawings cut into eight small Dice, the Side of every one of which will be one Inch, and one of its Surfaces an Inch Square, and its whole Superficies six Square Inches, of which the whole Dye contained twenty four in its Superficial content. Now, you see, that the Superficial content of the little Dye is the fourth part of the Superficial content of the great one, (for six is the fourth part of twenty four) but the Solid content of the said Dye is only the eighth part: therefore the Bulk, and consequently the Weight, doth much more diminish than the Superficies. And if you subdivide the little Dye into eight others, we shall have for the whole Superficial content of one of these, one and an half Square Inches, which is the sixteenth part of the Superficies of the first Dye; but its Bulk, or Mass, is only the sixty fourth part of that. You see therefore, how that in only these two divisions the Bulks decrease four times faster than their Superficies: and if we should prosecute the Subdivision, untill that we had reduced the first Solid into a small powder, we should find the Gravity of the minute Atomes to be lessened an hundred and an hundred times more than their Superficies. And this which I have exemplified in Cubes, hapneth in all like Solids, the Bulks of which are in Sesquialter proportion of their Superficies. You see, therefore, in how much greater proportion the Impediment of the Contact of the Superficies of the Moveable with the *Medium* encreaseth in small Moveables, than in greater: and if we should add, that the Scabrosities in the very small Superficies of the minute Atomes are not happily lesser than those of the Superficies of greater Solids, that are diligently polished, observe how fluid, and void of all Resistance being opened, the *Medium* is required to be, when it is to give passage to so feeble a Virtue. And therefore take notice, *Simplicius*, that I did not equivocate, when even now I said, That the Superficies of lesser Solids is greater, in comparison of that of bigger.

SIMP. I am wholly satisfied: and I verily believe, that if I were to begin my Studies again, I should follow the Counsel of *Plato*, and

and enter my self first in the Mathematicks, which I see to proceed very scrupulously, and refuse to admit any thing for certain, save that which they necessarily demonstrate.

SAGR. I have taken great delight in this Discourse; but, before we passe any further, I would be glad to be satisfied in one particular, which newly came into my thoughts, when but just now you said, that Like Solids are in Sesquialter proportion to their Superficies, for I have seen, and understood, the Proposition with its Demonstration, in which it is proved, That the Superficies of Like Solids are in duplicate proportion of their Sides; and another that proveth the same Solids to be in triple proportion of the same Sides; but the proportion of Solids to their Superficies, I do not remember that I ever so much as heard it mentioned.

Solids are to each other in Sesquialter proportion to their Superficies.

SALV. You your self have answered and declared the doubt. For that which is triple of a thing of which another is double, doth it not come to be Sesquialter of this double? Yes doubtlesse. Now, if Superficies are in double proportion of the Lines, of which the Solids are in triple proportion, may not we say, That the Solids are in Sesquialter proportion of their Superficies?

SAGR. I understand you very well. And although other particulars, pertaining to the matter of which we have treated, do remain for me to ask, yet if we should thus run from one Digression to another, it will be late before we should come to the Questions principally intended, which concern the diversities of the Accidents of the Resistances of Solids against Fraction; and therefore, if you so please, we may return to the first Theme, which we proposed in the beginning.

SALV. You say very well; but the so many, and so different things that have been examined, have stoln so much of our time, that there is but little of it left in this day to spend in our other principal Argument, which is full of Geometrical Demonstrations that are to be considered with attention: so that I should think it were better to adjourn our meeting till to morrow, as well for this which I have told you, as also because I might bring with me some Papers, on which I have, in order, set down the Theorems and Problems, in which are proposed and demonstrated the different Passions of this Subject, which, it may be, would not otherwise with requisite Method come into my mind.

SAGR. I very gladly comply with your advice, and so much the more willingly, in regard that, for a Conclusion of this daies Conference, I shall have time to hear you resolve some doubts that I find in my mind concerning the Point last handled. Of which one is, Whether we are to hold, that the Impediment of the *Medium* may be sufficient to assign bounds to the Acceleration of Bodies of very grave Matter, that are of great Bulk, and of a Spherical Figure:

and I instance in the Spherical Figure, that I might take that which is contained under the least Superficies, and therefore lesse subject to Retardment. Another shall be, touching the Vibrations of Pendulums, and this hath many heads: One shall be, Whether all, both Great, Mean, and Little, are made really and precisely under equal Times: And another, What is the proportion of the Times of Moveables, suspended at unequal strings, of the Times of their Vibrations I mean.

SALV. The Questions are ingenious, and, like as it is incident to all Truths, I suppose, that, which ever of them we handle, it will draw after it so many other Truths, and curious Consequences, that I cannot tell whether the remainder of this day may suffice for the discussing of them all.

SAGR. If they shall be but as delightful as the precedent, it would be more grateful for me to employ as many daies, not to say, hours, as it is unto night, and I believe that *Simplicius* will not be cloy'd with such Argumentations as these.

SIMP. No certainly: and especially, when the Questions treated of are Physical, touching which we read not the Opinions or Discourses of other Philosophers.

Any Body, of any Figure, Greatness, and Gravity, is checked by the Renitence of the Medium, *though never so tenuous, in such sort, that the Motion continuing, it is reduced to equability.*

SALV. I come therefore to the first, affirming without any hæsitation, that there is not a Sphere so big, nor of Matter so grave, but that the Renitence of the *Medium*, though very tenuous, checks its Acceleration, and in the continuation of the Motion reduceth it to Equability, of which we may draw a very clear Argument from Experience it self. For if any falling Moveable were able in its continuation of Motion to attain any degree of Velocity, no Velocity that should be conferred upon it, could be so great but that it would depose it, and free it self of it by help of the Impediment of the *Medium*. And thus, a Cannon-bullet, that had descended through the Air, *v. gr.* four yards, and had, for example, acquired ten degrees of Velocity, and that with these should enter into the Water, in case the Impediment of the Water were not able to prohibit such a certain *Impetus* in the Ball, it would encrease it, or at least would continue it unto the bottom; which is not observed to ensue: nay, the Water, although it were but a few fathoms in depth, would impede and debilitate it in such a manner, that it will make but a small impression in the bottom of the River or Lake. It is therefore manifest, that that Velocity, of which the Water had ability to deprive it in a very short way, would never be permitted to be acquired by it, though in a depth of a thousand Fathoms. And why should it be permitted to gain it in a thousand, to be taken from it again in four? What need we more? Do we not see the immense *Impetus* of the Ball, shot from the Cannon it self, to be in such a manner flatted by the interposition

ſition of a few Fathom of Water, that without any harm to the Ship, it but very hardly reacheth to make a dent in it? The Air alſo, though very yielding, doth neverthleſſe repreſſe the Velocity of the falling Moveable, although it be very heavy, as we may by ſuch like Experiments collect; for if from the top of a very high Tower we ſhould diſcharge a Muſquet downwards, this will make a leſſer impreſſion on the ground, than if we ſhould diſcharge the Muſquet at the height of four or ſix yards above the Plane: an evident ſign, that the *Impetus*, wherewith the Bullet iſſueth from the Gun, diſcharged on the top of the Tower, doth gradually diminiſh in deſcending thorow the Air: therefore the deſcending from any whatſoever great height will not ſuffice to make it acquire that *Impetus*, of which the Reſiſtance of the Air deprived it, when it had in any manner been conferred upon it. The battery likewiſe that the force of a Bullet, ſhot from a Culverin, ſhall make in a Wall at the diſtance of twenty Paces, would not, I believe, be ſo great, if the Bullet was ſhot perpendicularly from any immenſe Altitude. I believe, therefore, that there is a Bound or term belonging to the Acceleration of every Natural Moveable that departs from Reſt, and that the Impediment of the *Medium* in the end reduceth it to * Equality, in which it afterwards alwaies continueth.

* Or Equability.

SAGR. The Experiments are really, in my opinion, much to the purpoſe: nor doth any thing remain, unleſſe the Adverſary ſhould fortifie himſelf, by denying, that they will hold true in great and ponderous Maſſes, and that a Cannon-bullet coming from the Concave of the Moon, or from the upper Region of the Air, would make a greater percuſſion than coming from the Cannon.

SALV. There is no queſtion, but that many things may be objected, and that they may not be all ſalved by Experiments; neverthleſſe in this contradiction, me thinks, there is ſomething that may fall under conſideration; *ſcilicet*, that it is very probable, that the Grave Body, falling from an Altitude, acquireth ſo much *Impetus*, at its arrival to the ground, as would ſuffice to return it to that height, as is plainly ſeen in a *Pendulum* reaſonable weighty, that being removed fifty or ſixty degrees from the Perpendicular, gaineth that Velocity and Virtue which exactly ſufficeth to force it to the like Recurſion, that little abated, which is taken from it by the Impediment of the Air. To conſtitute, therefore, the Cannon-bullet in ſuch an Altitude as may ſuffice for the acquiſt of an *Impetus*, as great as that which the Fire giveth it in its iſſuing from the Piece, it would ſuffice to ſhoot it upwards perpendicularly with the ſaid Cannon, and then obſerving, whether in its fall it maketh an impreſſion equal to that of the percuſſion made near at hand in its iſſuing forth; but, indeed, I believe, that it would not be any whit

A Grave Body, falling from an Altitude, acquireth ſo much Impetus *at its arrival to the ground, as in all probability, would ſuffice to recarry it to the ſame height from whence it fell.*

whit near so forcible. And therefore I hold that the Velocity, which the Bullet hath near to its going out of the Piece, would be one of those that the Impediment of the Air would never suffer it to acquire, whilst it should with a natural Motion descend, leaving the state of Rest, from any great height. I come now to the other Questions belonging to *Pendulums*, matters which to many would seem very frivolous, and more especially to those Philosophers that are continually busied in the more profound Questions of Natural Philosophy: yet, notwithstanding, will not I contemn them, being encouraged by the Example of *Aristotle* himself, in whom I admire this above all things; that he hath not, as one may say, omitted any matter that any waies merited consideration, which he hath not spoken of: and now upon the Questions you propounded, I think I can tell you a certain conceit of mine upon some Problems concerning Musick, a noble Subject, of which so many famous men, and *Aristotle* himself, have written; and touching it, he considereth many curious Problems: so that if I likewise shall from so familiar and sensible Experiments, draw Reasons of admirable accidents on the Argument of Sounds, I may hope that my discourses will be accepted by you.

SAGR Not only accepted, but by me, in particular, most passionately desired, in regard that I taking a great delight in all Musical Instruments, and being reasonably well instructed concerning Consonances, have alwaies been ignorant and perplexed with endeavouring to know, whence it cometh that one should more please and delight me than another; and that some not only procure me no delight, but highly displease me: the trite Problem also of the two Chords set to an Unison, one of which moveth and actually soundeth at the touching of the other, I also am unresolved in: nor am I very clearly informed concerning the Forms of Consonances, and other particularities.

SALV. We will see, if from these our *Pendulums* one may gather any satisfaction in all these Doubts. And as to the first Question, that is, Whether the same *Pendulum* doth really and punctually perform all its Vibrations, great, lesser, and least, under Times precisely equal; I refer my self to that which I have heretofore learnt from our *Academian*, who plainly demonstrateth, that the Moveable that should descend along the Chords, that are Subtenses to any Arch, would necessarily passe them all in equal Times, as well the Subtense under an hundred and eighty degrees, (that is, the whole Diameter) as the Subtenses of an hundred, sixty, ten, two, or half a degree, or of four minutes: still supposing that they all determine in the lowest Point touching the Horizontal Plane. Next as to the descendents by the Arches of the same Chords elivated above the Horizon, and that are not greater than a Quadrant,

Moveables descending along the Chords, that are Subtenses to any Arch of a Circle, passe as well the greater as the lesser Chords in equal Times.

drant, that is, than ninety degrees, Experience likewise shews, that they passe all in Times equal, but yet shorter than the Times of the passages by the Chords: an effect which hath so much of wonder in it, by how much at the first apprehension one would think the contrary ought to follow: For the terms of the beginning, and the end of the Motion being common, and the Right-Line being the shortest, that can be comprehended between the said Terms, it seemeth reasonable, that the Motion made by it should be finished in the shortest Time, which yet is not so: but the shortest Time, and consequently, the swiftest Motion, is that made by the Arch of which the said Right-Line is Chord. In the next place, as to the Times of the Vibrations of Moveables, suspended by strings of different lengths, those Times are in Subduple proportion to the lengths of the strings, or, if you will, the lengths are in duplicate proportion to the Times, that is, are as the Squares of the Times: so that if, for example, the Time of a Vibration of one *Pendulum* is double to the Time of a Vibration of another, it followeth, that the length of the string of that is quadruple to the length of the string of this. And in the Time of one Vibration of that, another shall then make three Vibrations, when the string of that shall be nine times as long as the other. From whence doth follow, that the length of the strings have to each other the same proportion, that the Squares of the Numbers of the Vibrations that are made in the same Times have.

Moveables and Pendula descending along the Arches of the same Chords, elivated as far as 90 deg. pass the said Arches in Times equal, but that are shorter than the transitions along the Chords.

The Times of the Vibrations of Moveables, hanging at a longer or shorter thread, are to one another in proportion subduple the lengths of the strings, at which they hang.

SAGR. Then, if I have rightly understood you, I may easily know the length of a string, hanging at any never-so-great height, although the sublime term of the suspension were invisible to me, and I only saw the other lower extream. For if I shall fasten a weight of sufficient Gravity to the said string here below, and set it on vibrating to and again, and a friend telling some of its Recursions, and I at the same time tell the Recursions of another Moveable, suspended at a string that is precisely a yard long, by the Numbers of the Vibrations of these *Pendula*, made in the same Time, I will find the length of the string. As for example, suppose that in the time that my friend hath counted twenty Recursions of the long string, I had told two hundred and forty of my string, that is one yard long: squaring the two numbers twenty and two hundred and forty, which are 400, and 57600, I will say, that the long string containeth 57600 of those Measures, of which my string containeth 400, and because the string is one sole yard, I will divide 57600 by 400, and the quotient will be 144, and I will affirm that string to be 144 yards long.

To find the Length of any Rope, or string, at which a Moveable hangeth, by the frequency of its Vibrations

SALV. Nor will you be mistaken one Inch; and especially, if you take a great Number of Vibrations.

SAGR. You give me frequent occasion to admire the Riches, and

and withal the extraordinary bounty of Nature, whil'st by things so common, and, I might in a certain sence say, vile, you go collecting of Notions very curious, new, and oftentimes, remote from all imagination. I have an hundred times considered the Vibrations, in particular, of the Lamps in some Churches, hanging by very long ropes, when they have been unawares stirred by any one: but the most that I inferred from that same Observation, was the improbability of the Opinion of those who hold, that such-like Motions are maintained and continued by the *Medium*, that is by the Air: for it should seem to me, that the Air had a great judgment, and withal but little businesse to spend so many hours time in vibrating an hanging Weight with so much Regularity: but that I should have learnt, that that same Moveable, suspended at a string of an hundred yards long, being removed from Perpendicularity one while ninety degrees, and another while one degree onely, or half a degree, should spend as much time in passing this little, as in passing that great Arch, certainly would never have come into my head, for I still think, that it bordereth upon Impossibility. Now I am in expectation to hear that these petty Notions will asign me such Reasons of those Musical Problems, as may, in part at least, give me satisfaction.

Every Pendulum *hath the Time of its Vibration so limited, that it is not possible to make it move under any other Period.*

SALV. Above all things, you are to know, that every *Pendulum* hath the Time of its Vibrations so limited, and prefixed, that it is impossible to make it move under any other Period, than that onely one, which is natural unto it. Let any one take the string in hand, to which the Weight is fastened, and trie all the wayes he can to encrease or decrease the frequency of its Vibrations, and he shall finde it labour in vain: but we may, on the contrary, on a *Pendulum*, though grave and at rest, by onely blowing upon it, conferre a Motion, and a Motion considerably great, by reiterating the blasts, but under the Time that is properly belonging to its Vibrations: for if at the first blast we should have removed it from Perpendicularity half an Inch, adding a second, after that it, being returned towards us, is ready to begin the second Vibration, we should conferre new Motion on it, and so successively with other blasts, but given in Time, and not when the *Pendulum* is comming towards us (for so we should impede; and not help the Motion) and so continuing with many Impulses, we should confer upon it such an *Impetus*, that a greater force by much than that of a blast of our breath, will be required to stay it.

SAGR. I have, from my childhood, observed, that one man alone, by means of these Impulses, given in Time, hath been able to towl a very great Bell, and when it was to cease, I have seen four or six men more lay hold on the Bell-rope, and they have all been

been raised from the ground: so many together being unable to arrest that *Impetus*, which one alone, with regular Pulls, had conferred upon the Bell.

The Chord of a Musical Instrument touched, moveth, and maketh the Chords set to an Unison, Fifth and Eighth, with it to sound; and why.

SALV. An example, that declareth my meaning with no lesse propriety than this that I have premised, doth sute to render the reason of the admirable Problem of the Chord of the Lute or Viol, which moveth, and maketh not onely that really to sound, which is tuned to the Unison, but that also which is set to an Eighth and a Fifth. The Chord being toucht, its Vibrations begin, and continue all the Time that its Sound is heard to endure: these Vibrations make the Air neer adjacent to vibrate and tremble, whose tremblings and quaverings distend themselves a great way, and strike upon all the Chords of the Instrument, and also of others neer unto it: the Chord that is set to an Unison, with that which is toucht, being disposed to make its Vibrations * in the same Time, beginneth at the first impulse to move a little, and a second, a third, a twentieth, and many more, overtaking it, all in just and Periodick Times, it receiveth at last, the same Tremulation, with that first touched, and one may clearly see it go, dilating its Vibrations exactly according to the Pace of its Mover. This Undulation that distendeth it self thorow the Air, moveth, and makes to vibrate, not onely the Chords, but likewise any other Body disposed to trembling, and to vibrate in the very Time of the trembling Chord: so that if we fix in the Sides of the Instrument several small pieces of Bristles, or of other flexible matters, you shall see upon the sounding of the Viol, now one, now another of those Corpuscles tremble, according as that Chord is toucht, whose Vibrations return in the same Time: the others will not move at the striking of this Chord, nor will that Bristle tremble at the striking of another Chord. If with the Bow one smartly strike the Base-Chord of a Viol, and set a drinking Glasse, thin and smooth, neer unto it, if the Tone of the Chord be an Unison to the Tone of the Glasse, the Glasse shall dance, and sensibly re-sound. Again, the ample dilating of the Tremor or Undulation of the *Medium* about the Body resounding, is apparently seen in making the Glasse to sound, by putting a little Water in it, and then chafing the brim or edge of it with the tip of the finger: for the included Water is observed to undulate in a most regular order: and the same effect will be yet more clearly seen, by setting the foot of the Glasse in the bottom of a reasonable large Vessel, in which there is Water as high almost as to the brim of the Glasse, for making it to sound, as before, with the Confrication of the finger, we shall see the trembling of the Water to diffuse it self most regularly, and with great Velocity, to a great distance round about the Glasse; and it hath many

Sundry Problems touching Musical Proportions, and their Solutions.

* Or under.

times been my fortune, in making a reasonable big Glasse, almost full of Water, to sound as aforesaid, to see the Waves in the Water, at first formed with an exact equality; and it hapning sometimes, that the Tone of the Glasse riseth an Eighth higher, at the same instant, I have seen every one of the said Waves to divide themselves in two: an accident that very clearly proveth the forme of the Octave to be the double.

SAGR. The same hath also befaln me more than once, to my delight, and also benefit: for I stood a long time perplexed about these Forms of Consonants, not conceiving, that the Reason, commonly given thereof by the Authours that have hitherto written learnedly of Musick, were sufficiently convincing, they tell us, that the Diapason, that is the Eighth, is contained by the double, the Diapente, which we call the Fifth, by the Sesquialter: for a Chord being distended on the * Monochord, striking it all; and afterwards striking but the half of it, by placing a Bridge in the middle, one heareth an Eighth; and if the Bridge be placed at a third of the whole Chord, touching the whole, and then the two thirds, it soundeth a Fifth; whereupon they infer, that the Eighth is contained between two and one, and the Fifth between three and two. This Reason, I say, seemed to me not necessarily concluding for the assigning justly the double and the Sesquialter, for the natural Forms of the Diapason and the Diapente. And that which moved me so to think, was this. There are three ways, by which we may sharpen the Tone of a Chord: one is, by making it shorter, the other is by distending; or making it more tense; and the third is by making it thinner. If, retaining the same Tention and thicknesse, we would hear an Eighth, it is necessary to shorten it to one half, which is done by striking it all, and then half. But if, retaining the same length and thicknesse, we would have it rise to an Eighth, by screwing it higher, it will not suffice to stretch it double as much, but we shall need the quadruple, so that, if before it was stretched by a Weight of one pound, it will be needful to fasten four pound to it to sharpen it to an Eighth. And lastly, if, keeping the same length and Tention, we would have a Chord, that by being smaller, rendereth an Eighth, it will be necessary, that it retain onely a fourth part of the thicknesse of the other more Grave. And this which I speak of the Eighth, that is, that its form taken from the Tention, or from the thicknesse of the Chord, is in duplicate proportion to that which it receiveth from the length, is to be understoood of all other Musical Intervals: for that which the length giveth us in a Sesquialter proportion, *i. e.* by striking it all, and then the two thirds, if you would have it proceed from the Tention, or from the disgrossing, you must double the Sesquialter

* An Instrument of but one string; called by *Marsennus la Trompette Marine.*

alter proportion, taking the double Sesquiquartan: and if the Grave Chord were stretched by four pound weight, fasten to the Acute not six, but nine: and, as to the thicknesse, make the Grave Chord thicker than the Acute, according to the proportion of nine to four, to have the Fifth. These being most exact Experiments, I thought, that I saw no reason, why these Sage Philosophers should establish the form of the Eighth to be rather the double, than quadruple; and the Form of the Fifth to be rather the Sesquialter, than the double Sesquiquartan. But because the numbring of the Vibrations of a Chord, which in giving a sound, are extreme frequent, is altogether impossible, I should always have been in doubt, whether or no it were true, that the more Acute Chord of the Eighth, made in the same time, double the number of the Vibrations of the more Grave, if the Waves, which may be continued as long as you please, by making the Glass to sound and vibrate, had not sensibly shewn me, that in the self same moment that (sometimes) the Sound is heard to rise to an Eighth, there are seen to arise other Waves more minute, which with infinite smoothness cut in the middle each of those first.

SALV. An excellent Observation for distinguishing, one by one, the Undulations arising from the Tremulation of the resounding Body: which are those that diffusing themselves thorow the Air, make the titillation upon the Drum of our Ear, that in our Soul becommeth a Sound: But whereas beholding and observing them in the Water, endure no longer than the confrication of the finger lasteth, and also in that time they are not permanent, but are continually made and dissolved, would it not be an ingenious undertaking, if one could make, with much exquisitenesse, such, as would continue a long time; I mean Moneths and Years, so as to give a man opportunity, measure, and with ease to number them?

SAGR. I assure you I should highly value such an Invention.

SALV. The discovery was accidental, and the Observation and applicative improvement of it onely were mine, and I hold it to be a Circumstance of noble Contemplation, although a businesse in its self sufficiently homely. Scraping a Brasse Plate with an Iron Chizzel to fetch out some Spots, in moving the Chizzel to and again upon it pretty quick, I heard it (once or twice amongst many gratings) to Sibilate and send forth a whistling noise, very shrill and audible: and looking upon the Plate, I saw a long row of small streaks, parallel to one another, and distant from one another by most equal Intervals: returning to my scraping again, I perceived by several trials, that in those scrapings, and those onely that whistled, the Chizzel left the streaks upon the

Plate: but when the Scraping passed without any Sibilation, there was not so much as the least sign of any such streaks. Repeating the Experiment several times afterwards, scraping now with greater, now with lesse velocity, the Sibilation hapned to be of a Tone sometimes acuter, sometimes graver; and I observed the marks made in the more acute sounds to be closer together, and those of the more grave farther asunder: and sometimes also, according as the self same scrape was made towards the end, with greater velocity than at the beginning, the sound was heard to grow sharper, and the streaks were observed to stand thicker, but ever with extream neatnesse, and marked with exact equidistance: and farther-more, in the Sibilating scrapes; I felt the Chizzel to shake or tremulate in my hand, and a certain chilnesse to run along my arm; and in short, I saw the same effected upon the Toole, which we use to observe in whispering, and afterwards speaking aloud, for sending forth the breath without forming a sound, we do not perceive any moving in the throat and mouth, in comparison of that which we discern to be in the Wind-pipe and Throat of every one, in sending forth the voice; and especially in grave and loud Tones. I have likewise sometimes amongst the Chords of the Viols, observed two that were Unisons to the Sibilations made by scraping after the manner I told you, and that were most different in Tone, from which two they precisely were distant a perfect Fifth, and then measuring the intervals of the streaks of both the Scrapes, I saw the distance that conteined forty five spaces of the one, conteined thirty of the other: which, indeed, is the Form attributed to the Diapente. But here, before I proceed any farther, I will tell you, that of the three manners of rendring a Sound Acute, that which you refer to the flendernesse or finenesse of the Chord, may with more truth be ascribed to the Weight. For the alteration taken from the thicknesse, answereth, when the Chords are of the same matter; and so a Gut-string to make an Eighth, ought to be four times thicker than the other Gut-string; and one of Wier four times thicker than another of Wier. But if I would make an Eighth with one of Wier to one of Gut-string, I am not to make it four times thicker, but four times graver, so that, as to thicknesse, this of Wier shall not be four times thicker; but quadruple in Gravity, for some times it shall be more small than its respondent to the Acuter Eighth, that is of Gut-string. Hence it commeth to passe that, stringing an Instrument with Chords of Gold, and another with Chords of Brasse, if they shall be of the same length, thicknesse, and Tention, Gold being almost twice as heavy, the Strings shall prove about a Fifth more Grave. And here it is to be noted, that the Gravity of the Moveable more re-

sisteth

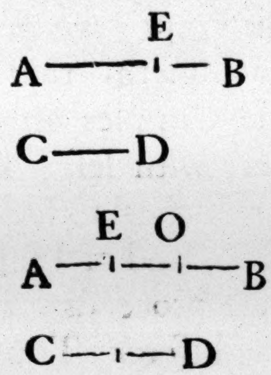
ſiſteth the Velocity, than the thickneſſe doth; contrary to what others at the firſt would think: for indeed, in appearance, its more reaſonable, that the Velocity ſhould be retarded by the Reſiſtance of the *Medium* againſt Opening in a Moveable thick and light, than in one grave and ſlender: and yet in this caſe it happeneth quite contrary. But purſuing our firſt Intent, I ſay, That the neereſt and immediate reaſons of the Forms of Muſical Intervals, is neither the length of the Chord, nor the Tention, nor the thickneſſe, but the proportion of the numbers of the Vibrations, and Percuſſions of the Undulations of the Air that beat upon the Drum of our Ear, which it ſelf alſo doth tremulate under the ſame meaſures of Time. Having eſtabliſhed this Point, we may, perhaps, aſſign a very apt reaſon, whence it commeth, that of thoſe Sounds that are different in Tone, ſome Couples are received with great delight by our Sence, others with leſs, and others occaſion in us a very great diſturbance; which is to ſeek a reaſon of the Conſonances more or leſſe perfect, and of Diſſonances. The moleſtation and harſhneſſe of theſe proceeds, as I believe, from the diſcordant Pulſations of two different Tones, which diſproportionally ſtrike the Drum of our Ear: and the Diſſonances ſhall be extreme harſh, in caſe the Times of the Vibrations were incommenſurable. For one of which take that, when of two Chords ſet to an Uniſon, one is ſounded, and ſuch a part of another, as is the Side of the Square of its Diameter; a Diſſonance like to the * Tritone, or Semi-diapente. Conſonances, and with pleaſure received, ſhall thoſe Couples of Sounds be, that ſhall ſtrike in ſome order upon the Drum; which order requireth, firſt, that the Pulſations made in the ſame Time be commenſurable in number, to the end, the Cartillage of the Drum, may not ſtand in the perpetual Torment of a double inflection of allowing and obeying the ever diſagreeing Percuſſions. Therefore the firſt and moſt grateful Conſonance ſhall be the Eighth, being, that for every ſtroke, that the Grave-ſtring or Chord giveth upon the Drum, the Acute giveth, two; ſo that both beat together in every ſecond Vibration of the Acute Chord; and ſo of the whole number of ſtrokes, the one half accord to ſtrike together, but the ſtrokes of the Chords that are Uniſons, alwayes joyn both together, and therefore they are, as if they were of the ſame Chord, nor make they a Conſonance. The Fifth delighteth likewiſe, in regard, that for every two ſtroaks of the Grave Chord, the Acute giveth three: from whence it followeth, that numbering the Vibrations of the Acute Chord, the third part of that number will agree to beat together; that is, two Solitary ones interpoſe between every couple of Conſonances; and in the Diateſſeron there interpoſe three. In the ſecond, that is in the *Seſquioctave*

* Or a falſe Fifth.

quioctave Tone for every nine Pulsations, one onely strikes in Consort with the other of the Graver Chord; all the rest are Discords, and received upon the Drum with regret, and are judged Dissonances by the Ear.

SIMP. I could wish this Discourse were a little explained.

SALV. Suppose this line A B the Space, and dilating of a Vibration of the Grave Chord; and the line C D that of the Acute Chord, which with the other giveth the Eighth: and let A B be divided in the midst in E. It is manifest, that the Chords begining to move at the terms A and C, by that time the Acute Vibration shall be come to the term D, the other shall be distended onely to the half E, which not being the bound or term of the Motion, it strikes not: but yet a stroak is made in D. The Vibrations afterwards returning from D to C, the other passeth from E to B, whereupon the two Percussions of B and C strike both together upon the Drum: and so continuing to reiterate the like subsequent Vibrations; one shall see, that the union of the Percussions of the Vibrations C D with those of A B, happen alternately every other time: but the Pulsations of the terms A B are alwayes accompanied with one of C D, and that alwayes the same: which is manifest, for supposing that A and C strike together, in the time that A is passing to B, C goeth to D, and returneth back to C: so that the stroaks at B and C are also together. But now let the two Vibrations A B and C D be those that produce the Diapente, the times of which are in proportion Sesquialter, and divide A B of the Grave Chord, in three equal parts in E and O; And suppose the Vibrations to begin at the same moment from the terms A and C: It is manifest, that at the stroke that shall be made in D, the Vibration of A B shall have got no farther than O, the Drum therefore receiveth the Pulsation D onely: again in the return from D to C, the other Vibration passeth from O to B, and returneth to O, making the Pulsation in B, which likewise is solitary, and in Counter-time, (an accident to be considered:) for we having supposed the first Pulsations to be made at the same moment in the terms A and C, the second, which was onely by the term D, was made as long after as the time of the transition C D, that is A O, imports; but that which followeth, made in B, is distant from the other onely so much as is the time O B, which is the half: afterwards continuing the Recursion from O to A, whilst the other goeth from C to D, the two Pulsations come to be made both at once in A and D. There afterwards follow other Periods like to these, that



is,

is, with the interposition of two single and solitary Pulsations of the Acute Chord, and one of the Grave Chord, likewise solitary, is interposed between the two solitary strokes of the Acute. So that if we did but suppose the Time divided into Moments, that is, into small equal Particles: supposing that in the two first moments, I passed from the Concordant Pulsations made in A and C to O and D, and that in D, I make a Percussion: and that in the third and fourth moment I return from D to C, striking in C, and that from O, I past to B, and returned to O, striking in B; and that lastly in the fifth and sixth moment from O and C, I past to A and D striking in both: we shall have the Pulsations distributed with such order upon the Drum, that supposing the Pulsations of the two Chords in the same instant, it shall two moments after receive a solitary Percussion, in the third moment another, solitary likewise, in the fourth another single one, and two moments after, that is, in the sixth, two together; and here ends the Period, and, if I may so say, Anomaly; which Period is oft-times afterwards replicated.

SAGR. I can hold no longer, but must needs expresse the content I take in hearing reasons so appositely assigned of effects that have so long time held me in darknesse and blindnesse. Now I know why the Unison differeth not at all from a single Tone: I see why the Eighth is the principal Consonance, but withal so like to an Unison, that, as an Unison, it is taken and cojoyned with others: it resembleth an Unison, for that whereas the Pulsations of Chords set to an Unison, keep time in striking, these of the Grave Chord in an Eighth alwayes keep time with those of the Acute, and of these one interposeth alone, and in equal distances, and as, one may say, without any variety, whereupon that Consonance is over sweet. But the Fifth, with those its Counter-times, and with the interposures of two solitary Pulsations of the Acute Chord, and one of the Grave Chord, between the Couples of Discordant Pulsations, and those three solitary ones, with an interval of time, as great as the half of that which interposeth between each Couple, and the solitary Percussions of the Acute Chord, maketh such a Titillation and Tickling upon the Cartillage of the Drum of the Ear, that allaying the Dulcity with a mixture of Acrimony, it seemeth at one and the same time to kisse and bite.

SALV. It is convenient, in regard I see, that you take such delight in these Novelties, that I shew you the way whereby the Eye also, and not the Ear alone, may recreate it self in beholding the same sports that the Ear feeleth. Suspend Balls of Lead or other heavy matter on three strings of different lengths, but in such proportion, that while the longer maketh two Vibrations,

the

the ſhorter may make four, and the middle one three; which will happen, when the longeſt containeth ſixteen feet, or other meaſures, of which the middle one containeth nine, and the ſhorteſt four: and removing them all together from Perpendicularity, and then letting them go, you ſhall ſee a pleaſing Intermixtion of the ſaid *Pendulums* with various encounters, but ſuch,that, at every fourth Vibration of the longeſt, all the three will concurre in one and the ſame term together, and then a gain will depart from it, reiterating anew the ſame Period: the which commixture of Vibrations, is the ſame, that being made by the Chords, preſents to the Ear an Eighth, with a Fifth in the midſt. And if youqualifie the length of other ſtrings in the like diſpoſure, ſo that their Vibrations anſwer to thoſe of other Muſical, but Conſonant Intervals, you ſhall ſee other and other Interweavings, and alwaies ſuch, that in determinate times, and after determinate numbers of Vibrations, all the ſtrings (be they three, or be they four) will agree to joyn in the ſame moment, in the term of their Recurſions, and from thence to begin ſuch another Period: but if the Vibrations of two or more ſtrings are either Incommenſurable, ſo,that they never return harmoniouſly to terminate determinate numbers of Vibrations, or though they be not Incommenſurable, yet if they return not till after a long time, and after a great number of Vibrations, then the ſight is confounded in the diſorderly order of irregular Intermixtures, and the Ear with warineſſe and regret receiveth the intemperate Impulſes of the Airs Tremulations, that without Order or Rule, ſucceſſively beat upon its Drum.

But whither, my Maſters, have we been tranſported for ſo many hours by various Problems, and unlook't for Diſcourſes? We have made it Night, and yet we have handled few or none of the points propounded; nay we have ſo loſt our way, that I ſcarſe remember our firſt entrance, and that ſmall Introduction, which we laid down, as the Hypotheſis and beginning of the future Demonſtrations.

SAGR It will be convenient, therefore, that we break up our Conference for this time, giving our Minds leave to compoſe themſelves in the Nights Repoſe, that we may to Morrow (if you pleaſe ſo far to favour us) reaſſume the Diſcourſes deſired, and chiefly intended.

SALV. I ſhall not fail to be here to Morrow at the uſual hour, to ſerve and enjoy you.

The End of the Firſt Dialogue.

GALILEUS, HIS DIALOGUES OF MOTION.

The Second Dialogue.

INTERLOCUTORS,

SALVIATUS, SAGREDUS, and SIMPLICIUS.

SAGREDUS.

Simplicius, and I, staid expecting your coming, and we have been trying to recall to memory our last Consideration, which, as the Principle and Supposition, on which you ground the Conclusions that you intended to Demonstrate to us, was that Resistance, that all Bodies have to *Fraction*, depending on that Cement, that connects and glutinates the parts, so, as that they do not separate and divide without a powerful attraction: and our enquiry hath been, what might be the Cause of that Coherence, which in some Solids is very vigorous; propounding that of *Vacuum* for the principal, which afterwards occasioned so many Digressions as held us the whole day, and far from the

matter at first proposed, which was the Contemplation of the Resistances of Solids to Fraction.

SALV. I remember all that hath been said, and returning to our begun discourse; What ever this Resistance of Solids to breaking by a violent attraction, is supposed to be, it is sufficient, that it is to be found in them: which, though it be very great against the strength of one that draweth them streight out, it is observed to be lesse in forcing them transversely, or sidewaies: and thus we see, for example, a rod of Steel, or Glasse to sustain the length-waies a weight of a thousand pounds, which, fastned at Right-Angles into a Wall, will break if you hang upon it but only fifty. And of this second Resistance we are to speak, enquiring, according to what proportions it is found in Prismes, and Cylinders of like and unlike figure, length, and thicknesse, and, withal, of the same matter. In which Speculation, I take for a known Principle, that which in the Mechanicks is demonstrated amongst the Passions of the Vectis, which we call the Leaver: namely, That in that use of the Leaver, the Force is to the Resistance in Reciprocal proportion, as the Distances from the Fulciment to the said Force and the Resistance.

SIMP. This *Aristotle*, in his Mechanicks, demonstrated before any other man.

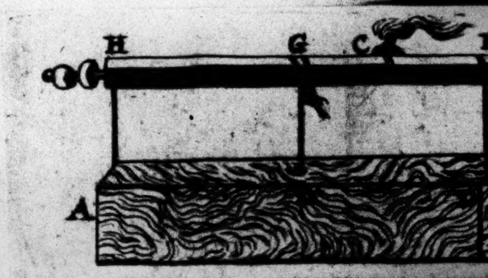
SALV. I am content to grant him the precedency in time, but for the firmnesse of Demonstration, I think, that *Archimedes* ought to be preferred far before him, on one sole Proposition of whom, by him demonstrated in his Book, *De Equiponderantium*, depend the Reasons, not only of the Leaver, but of the greater part of the other Mechanick Instruments.

SAGR. But since that this Principle is the foundation of all that which you intend to demonstrate to us, it would be very requisite, that you produce us the proof of this same Supposition, if it be not too long a work, giving us a full and perfect information thereof.

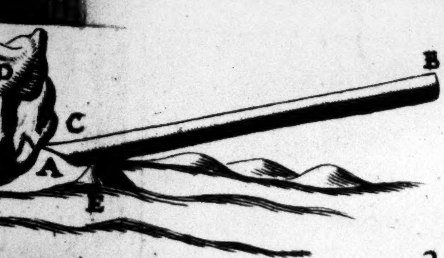
SALV. Though I am to do this, yet it will be better, that I lead you into the field of all our future Speculations, by an enterance somewhat different from that of *Archimedes*; and that, supposing no more, but only that equal Weights, put into a Ballance of equal Arms, make an *Equilibrium*, (a Principle likewise supposed by *Archimedes* himself.) I come, in the next place, to demonstrate to you, that not only it is as true as the other, That unequal Weights make an *Equilibrium* in a Stiliard of Armes unequal, according to the proportion of those Weights Reciprocally suspended, but that it is one and the same thing to place equal Weights at equal distances, as to place unequal Weights at distances that are in Reciprocal Proportion to the Weights. Now for a plain

Demon-

Demonſtration of what I ſay, deſcribe a Solid Priſm or Cylinder A B, [*as in* Figure 1. *at the end of this Dialogue*,] ſuſpended by its ends at the Line H I, and ſuſtained by two Cords, H A, and I B. It is manifeſt, that if I ſuſpend the whole by the Cord C, placed in the middle of the Beam or Ballance H I, the Priſm A B will be equilibrated, one half of its weight, being on one ſide, and the other half on the other ſide of the Point of Suſpenſion C by the Principle that we preſuppoſed. Now let the Priſm be divided into unequal parts by the Line D, and let the part D A be greater, and D B leſſer; and to the end, that ſuch diviſion being made, the Parts of the Priſm may reſt in the ſame ſcituation and conſtitution, in reſpect of the Line H I, let us help it with a Cord E D, which, being faſtened in the Point E, ſuſtaineth the parts A D, and D B: It is not to be doubted, but that there being no local mutation in the Priſm, in reſpect of the Ballance H I, it ſhall remain in the ſame ſtate of Equilibration. But it will reſt in the ſame Conſtitution likewiſe, if the Part of the Priſm, that is now ſuſpended at the two extreams, or ends with Cords A H and D E, be hanged at one ſole Cord G L, placed in the midſt: and likewiſe the other part D B, will not change ſtate, if ſuſpended by the middle, and ſuſtained by the Cord F M. So that the Cords H A, E D, and I B being untied, and only the two Cords G L, and F M being left, the *Equilibrium* will ſtill remain, the Suſpenſion being ſtill made at the Point C. Now, here let us conſider, that we have two Grave Bodies A D, and D B, hanging at the terms G and F of a Beam G F, in which the *Equilibrium* is made at the Point C: in ſuch manner, that the diſtance of the ſuſpenſion of the Weight A D from the Point C, is the Line C G, and the other part C F, is the diſtance at which the other Weight D B hangeth. It remaineth, therefore, only to be demonſtrated, that thoſe Diſtances have the ſame proportion to one another, as the Weights themſelves have, but reciprocally taken: that is, that the diſtance G C is to the diſtance C F, as the Priſm D B to the Priſm D A, which we prove thus. The Line G E being the half of E H, and E F the half of E I, all G F ſhall be equall to all H I, and therefore equal to C I: and taking away the common part C F, the remainder G C ſhall be equal to the remainder F I, that is, to F E: and C E taken in common, the two Lines G E and C F ſhall be equal: and, therefore, as G E, is to E F, ſo is F C, to C G: but as G C is to E F, ſo is the double to the double; that is H E to E I; that is, the Priſm A D to the Priſm D B. Therefore by Equality of proportion, and by Converſion, as the diſtance G C is to the diſtance C F, ſo is the Weight B D to the Weight D A: which is that that I was to demonſtrate. If you underſtand this, I believe that you will not ſcruple to admit, that the two Priſmes A D, and D B make an



Equilibrium in th Point C, for the half of the whole Solid A B is on the right hand of the Suspension C, and the other half on the left; and that in this manner there are represented two equal Weights, disposed and distended at two equal distances. Again, that the two Prismes A D, and D B, being reduced into two Dice, or two Balls, or into any two other Figures, (provided that they keep the same Suspensions G and F) do continue to make their *Equilibrium* in the Point C, I believe none can deny, for that it is most manifest, that Figures change not weight, where the same quantity of matter is retained. From which we may gather the general Conclusion, That two Weights, whatever they be, make an *Equilibrium* at Distances reciprocally answering to their Gravities. This Principle, therefore, being established, before we pass any farther, I am to propose to Consideration, how these Forces, Resistances, Moments, Figures, may be considered in Abstract, and separate from Matter, as also in Concrete and conjoyned with Matter; and in this manner those Accidents that agree with Figures, considered as Immaterial, shall receive certain Modifications, when we shall come to add Matter to them, and consequently Gravity. As for example, if we take a Leaver, as for instance B A [*as in* Fig. 2.] which, resting upon the Fulciment E, we apply to raise the heavy Stone D: It is manifest by the Principle demonstrated, that the Force placed at the end B, shall suffice to equal the Resistance of the Weight D, if so be, that its Moment have the same proportion to the Moment of the said D, that the Distance A C hath to the Distance C B: and this is true, if we consider no other Moments than those of the simple Force in B, and of the Resistance in D, as if the said Leaver were immaterial, and void of Gravity. But if we bring to account the Gravity also of the Instrument or Leaver it self, which hapneth sometimes to be of Wood, and sometimes of Iron; it is manifest, that the weight of the Leaver, being added to the Force in B, it will alter the proportion, which it will be requisite to deliver in other terms. And therefore before we passe any farther, it is necessary, that we distinguish between these two waies of Consideration, calling that a taking it absolutely, when we suppose the Instrument to be taken in Abstract, that is, disjunct from the Gravity of its own Matter; but conjoyning the Matter, as also the Gravity, with simple and absolute Figures, we will phrase the Figures conjoyn'd with the Matter, Moment, or Force compounded.



2

SAGR. I must of necessity break the Resolution I had taken, not to give occasion of digressing, for I should not be able to set my self to hear what remaines with attention, if a certain scruple were not removed that cometh into my head; and it is this, That I guesse you make comparison between the Force placed in B, and the

the total Gravity of the Stone D, of which Gravity me thinks, that one, and that, very probably, the greater part, resteth upon the Plane of the Horizon : so that——

SALV. I have rightly apprehended you, so that you need say no more, but only take notice, that I named not the total Gravity of the Stone, but spake of the Moment that it hath, and exerciseth at the Point A, the extream term of the Leaver B A, which is ever less than the entire weight of the Stone ; and is variable according to the Figure of the Stone, and according as it hapneth to be more or lesse elevated.

SAGR. I am satisfied in that particular, but I have one thing more to desire, namely, that for my perfect information, you would demonstrate to me the way, if there be one, how I may find what part of the total weight that is, which cometh to be born by the subjacent Plane, and what that which gravitates upon the Leaver at the extream A.

SALV. Because I can give you satisfaction in few words, I will not fail to serve you : therefore, describing a slight Figure thereof, be pleased to suppose, that the Weight, whose Center of Gravity is A, [*as in* Fig. 3.] resteth upon the Horizon with the term B, and at the other end is born up by the Leaver C G, on the Fulciment N, by a Power placed in G : and that from the Center A, and t[illegible] C, Perpendiculars be let fall to the Horizon, A O, and C F. I[illegible] That the Moment of the whole Weight shall have to the Mom[illegible] of the whole Power in G, a proportion compounded of the [illegible] stance G N to the Distance N C, and of F B to B O. Now, as the Line F B is to B O, so let N C be to X. And the whole Weight A being born by the two Powers placeed in B and C, the Power B is to C, as the distance F O to O B : and by Composition, the two Powers B and C together, that is, the total Moment of the whole Weight A, is to the Power in C, as the Line F B is to the Line B O ; that is, as N C to X : But the Moment of the Power in C is to the Moment of the Power in G, as the Distance G N is to N C : Therefore, by Perturbation of proportion, the whole Weight A is to the Moment of the Power in G, as G N to X : But the proportion of G N to X is compounded of the proportion G N to N C, and of that of N C to X ; that is, of F B to B O : Therefore the Weight A is to the Power that bears it up in G, in a proportion compounded of G N to N C, and of that of F B to B O : which is that that was to be demonstrated. Now returning to our first intended Argument, all things hitherto declared being understood, it will not be hard to know the reason, whence it cometh to passe that

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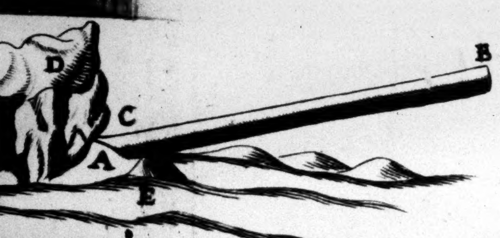

2

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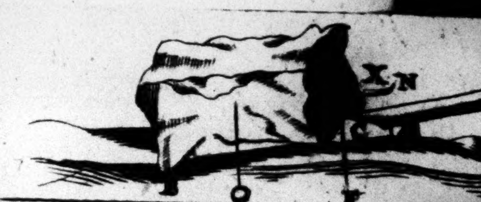

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PROPOSITION I.

A Solid Prism or Cylinder of Glasse, Steel, Wood, or other Frangible Matter, that being suspended lengthwaies, will sustain a very great Weight hanged Thereat, will, Sidewaies, (as we said even now) be broken in pieces by a far lesser Weight, according as its length shall exceed its thicknes.

VVHerefore let us describe the Solid Prism A B C D, fixed into a Wall by the Part A B, and in the other extream suppose the Force of the Weight E; (alwaies understanding the Wall to be erect to the Horizon, and the Prism or Cylinder fastened in the Wall at Right-Angles) it is manifest, that being to break, it will be broken in the place B, where the Mortace in the Wall serveth for Fulciment, and B C for the part of the Leaver in which lieth the force, and the thicknesse of the Solid B A is the other part of the Leaver, in which lieth the Resistance, which consisteth in the unfastening, or dividing, that is to be made of the part of the Solid B D, that is without the Wall from that which is within: and by what hath been declared, the Moment of the Force placed in C, is to the Moment of the Resistance that lieth in the thicknesse of the Prism, that is, in the Connection of the Base B A, with the parts contiguous to it, as the length C B is to the half of B A: And therefore the absolute Resistance against Fraction that is in the Prism B D, (which absolute Resistance is that which is made by drawing it downwards, for at that time the motion of the Mover is the same with that of the Body Moved) against the fracture to be made by help of the Leaver B C,

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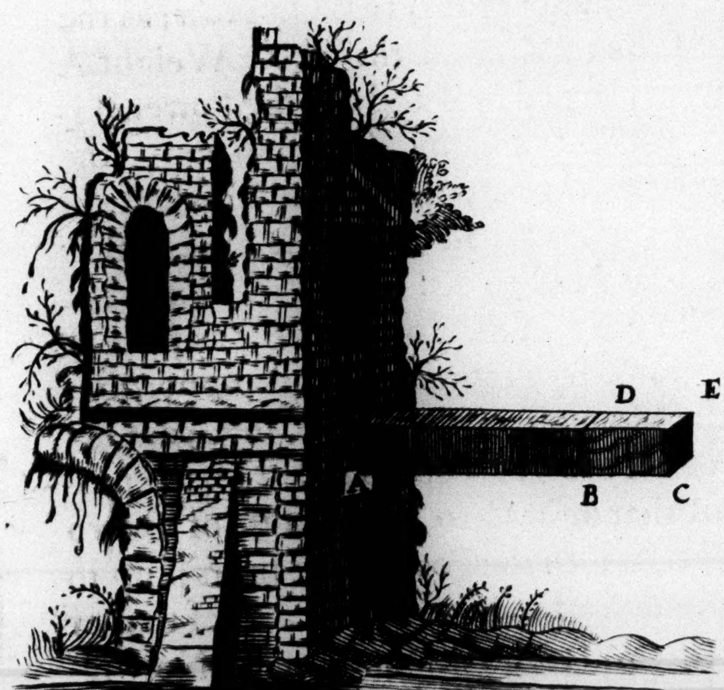


B C, is as the Length B C to the half of A B in the Prism, which in the Cylinder is the Semidiameter of its Base. And this is our first Proposition. And observe, that what I have said ought to be understood, when the Consideration of the proper Weight of the Solid B D is removed: which Solid I have taken as weighing nothing. But in case we would bring its Gravity to account, conjoyning it with the Weight E, we ought to add to the Weight E the half of the Weight of the Solid B D: so that the Weight B D being *v. gr.* two pounds, and the Weight of E ten pounds, we are to take the Weight E, as if it were eleven pounds.

SIMP. And why not as if it were twelve?

SALV. The Weight E, *Simplicius*, hanging at the term C, gravitates in respect of B C, with all its Moment of ten pounds, whereas if only B D were pendent, it would weigh with its whole Moment of two pounds; but, as you see, that Solid is distributed thorow all the length B C, uniformly, so that its parts near to the extream B, gravitate lesse than the more remote: so that, in a word, compensating those with these, the weight of the whole Prism is brought to operate under the Center of its Gravity, which answereth to the middle of the Leaver B C: But a Weight hanging at the end C, hath a Moment double to that which it would have hanging at the middle: And therefore the half of the Weight of the Prism ought to be added to the Weight E, when we would use the Moment of both, as placed in the Term C.

SIMP. I apprehend you very well, and, if I deceive not my self, me thinks, that the Power of both the Weights B D and E, so placed, would have the same Moment, as if the whole Weight of B D, and the double of E were hanged in the midst of the Leaver B C.

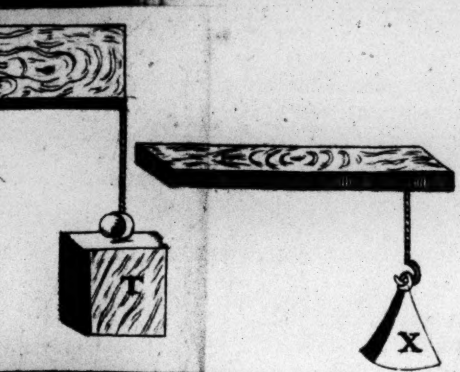
SALV. It is exactly so, and you are to bear it in mind. Here we may immediatly understand

PROPOSITION II.

How, and with what proportion, a Ruler, or Prism, more broad than thick, resisteth Fraction, better if it be forced according to its breadth, than according to its thicknesse.

FOR understanding of which, let a Prism be supposed A D: [*as in* Fig. 4.] whose breadth is A C, and its thicknes much lesser C B: It is demanded, why we would attempt to break it edge-waies, as in the first Figure it will resist the great Weight T, but placed flat-waies, as in the second Figure, it will not resist X, les-

X, lesser than T: Which is manifested, since we understand the Fulciment, one while under the Line B C, and another while under C A, and the Distances of the Forces to be alike in both Cases, to wit, the length *B* D. But in the first Case, the Distance of the Resistance from the Fulciment, which is the half of the Line C A, is greater than the Distance in the other Case, which is the half of B C: Therefore the Force of the Weight T, must of necessity be greater than X, as much as the half of the breadth C A is greater than half the thichnesse B C, the first serving for the Counter-Leaver of C A, and the second of C B to overcome the same Resistance, that is the quantity of the *Fibres*, or strings of the whole Base A B. Conclude we therefore, that the said Prism or Ruler, which is broader than it is thick, resisteth, breaking more the edge-waies than the flat-waies, according to the Proportion of the breadth to the thickness.



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It is requisite that we begin in the next place

PROPOSITION III.

To find according to what proportion the encrease of the Moment of the proper Gravity is made in a Prism or Cylinder, in relation to the proper Resistance against Fraction, whilst that being parallel to the Horizon, it is made longer and longer: Which Moment I find to encrease successively in duplicate Proportion to that of the prolongation.

FOR demonstration whereof, describe the Prism or Cylinder A D, firmly fastned in the Wall at the end A, and let it be equidistant from the Horizon, and let the same be understood to be prolonged as far as E, adding thereto the part *B* E. It is manifest, that the prolongation of the Leaver A *B* to C encreaseth, by it self alone, that is taken absolutely, the Moment of the Force pressing against the Resistance of the Separation and Rupture to be made in A, according to the proportion of C A to *B* A: but, moreover, the Weight of the Solid affixed *B* E, encreaseth the Moment of the pressing Gravity of the Weight of the Solid A *B*, according to the Proportion of the Prism A E to the Prism A *B*; which proportion is the same as that of the length A C, to the length A *B*: Therefore it is clear that

that the two augmentations of the Lengths and of the Gravities being put together, the Moment compounded of both is in double proportion to either of them. We conclude therefore, That the Moments of the Forces of Prismes and Cylinders of equal thicknesse, but of unequal length, are to one another in duplicate proportion to that of their Lengths; that is, are as the Squares of their Lengths.

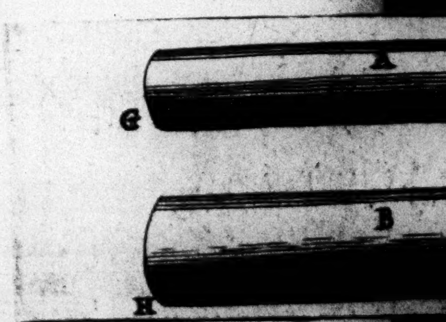
This Scheme belongs to Prop: 1st.

We will shew, in the second place, according to what proportion the Resistance of Fraction in Prismes and Cylinders encreaseth, when they continue of the same length, and encrease in thickness. And here I say, that

PROPOSITION IV.

In Prismes and Cylinders of equal length, but unequal thickness, the Resistance against Fraction encreaseth in a proportion triple to the Diameters of their Thicknesses, that is, of their Bases.

LET the two Cylinders be these A and *B*, [*as in* Fig. 5.] whose equal lengths are D G, and F H, the unequal *Bases* the Circles, whose Diameters are C D, and E F. I say, that the Resistance of the Cylinder *B* is to the Resistance of the Cylinder A against Fraction, in a proportion triple to that which the Diameter F E hath to the Diameter D C. For if we consider the absolute and simple Resistance that resides in the *Bases*, that is, in the Circles E F, and D C to breaking, offering them violence by pulling them end-waies, without all doubt, the Resistance of the Cylinder *B*, is so much greater than that of the Cylinder A, by how much the Circle E F is greater than C D; for the Fibres, Filaments, or tenacious parts, which hold together the Parts of the Solid, are so many the more. *But* if we consider, that in offering



them violence transversly we make use of two Leavers; of which the Parts or Distances,at which the Forces are applied are the Lines D G, and F H, the Fulciments are in the Points D and F; but the other Parts or Distances, at which the Resistances are placed, are the Semidiameters of the Circles D C and E F, because the Filaments dispersed thorow the whole Superficies of the Circles are as if they were all reduced into the Centers : considering, I say, those Leavers,we would be understood to intend, that the Resistance in the Center of the Base E F against the Force of H, is so much greater than the Resistance of the Base C D, against the Force placed in G, (and the Forces in G and H are of equal Leavers D G, and F H) as the Semidiameter F E is greater than the Semidiameter D C, the Resistance against Fraction, therefore, in the Cylinder B, encreaseth above the Resistance of the Cylinder A, according to both the proportions of the Circles E F and D C, and of their Semidiameters, or, if you will, Diameters : But the proportion of the Circles is double of that of the Diameters; Therefore the proportion of the Resistances, which is compounded of them, is in triplicate proportion of the said Diameters: Which is that which I was to prove. But because also the Cubes are in triplicate proportion to their Sides, we may likewise conclude, *That the Resistances of Cylinders of equal Length, are to one another as the Cubes of their Diameters.*

From that which we have Demonstrated we may likewise conclude, that

COROLARY.

The Resistances of Prisms, and Cylinders of equal length are in Sesquialter proportion to that of the said Cylinders.

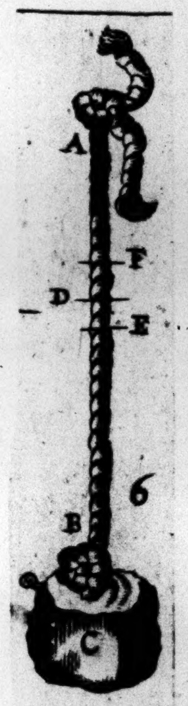
THE which is manifest, because the Prisms and Cylinders, equal in height, are to one another, in the same proportion as their Bases; that is, the double of the Sides or Diameters of the said Bases : But the Resistances (as hath been demonstrated) are in triplicate proportion to the said Sides or Diameters: Therefore the proportion of the Resistances is Sesquialter to the proportion of the said Solids, and, consequently, to the Weights of the said Solids.

SIMP. It is convenient, that, before we proceed any farther, I be resolved of a certain Doubt, and this it is, That I have not hitherto heard proposed to Consideration another certain kind of Resistance, that, in my opinion, is successively diminished in Solids, according as they are more and more prolonged, and not only in using them sidelongs, but also lengthwaies, in the self same

manner

manner just as we see a very long Cord to be much lesse apt to sustain a great weight, than if it were short: so that I believe, that a Ruler of Wood or Iron will sustain a much greater weight, if it shall be short, than if it shall be very long; understanding it alwaies to be used lengthwaies, and not transversly; and also its own weight being accounted for, which in the longer is greater.

Salv. I fear, *Simplicius*, that in this Point you, with many others, are deceived, if so be, that I have rightly apprehended your meaning, so that you would say, that a Cord *v. gr.* forty yards long cannot sustain so much, as if use were made but of one or two yards of the same Rope.



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Simp. That is it, which I would have said, and as yet it seemeth a very probable Proposition.

Salv. But I hold it not only improbable, but false: and think that I can very easily reclaim you from your Errour. Therefore let us suppose this Rope A B, [*as in* Fig. 6.] fastned on high by the end A, and by the other end let there hang the Weight C, by the force of which, the said Rope is to be broken. Do you assign me the particular place, *Simplicius*, where the Rupture is to happen.

Simp. Let it be in the place D.

Salv. I ask what is the cause why it should break in D.

Simp. The reason thereof is, because the Rope was not strong enough in that part, to sustain *v. gr.* an hundred pounds of weight, for so much is the Rope D B with the Stone C.

Salv. Therefore when ever such a Rope shall come to be violently stretched by those hundred pounds of weight, it shall break in that place.

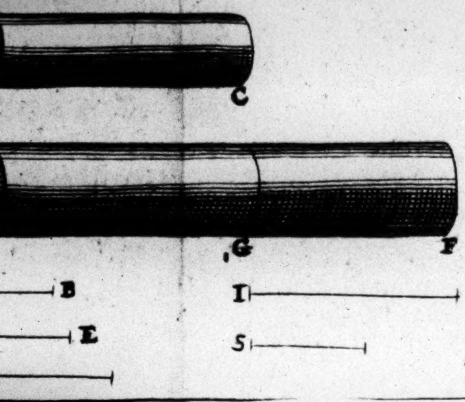
Simp So I think.

Salv. But tell me now; if one did hang the same Weight, not at the end of the Rope *B*, but near to the point D, as for instance, in E, or else did tye the Rope not at the height A, but very near, and almost at the Point D it self, as in F, tell me, I say, whether the Point D would feel the same weight of an hundred pounds.

Simp. It would so, still joyning the piece of Rope E *B* to the Stone C.

Salv. If then the Rope in the Point D commeth to be drawn by the said hundred pounds of weight, it will break by your concession. And yet F E, is a small piece of the length A *B*: why do you say then, that the long Rope is weaker than the short one? *B*e content, therefore, to suffer your self to be reclaimed from an Errour, in which you have had many Companions, and those in other things very knowing. And let us go on: and having demonstrated, that Prisms and Cylinders encrease their Moments above

their Resistances,according to the Squares of their Lengths(alwaies provided, that they retain the same thicknesse) and that likewise, these that are equally long, but different in thicknesse, encrease their Resistances according to the proportion of the Cubes of the Sides or Diameters of their Bases, we may enquire what befalleth to those Solids,being different in length and thickness,in which I observe, that



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PROPOSITION V.

Prisms and Cylinders,of different length and thickness, have their Resistances against Fraction, in a proportion compounded of the proportion of the Cubes of the Diameters of their Bases, and of the proportion of their lengths reciprocally taken.

Let these two A B C, and D E F, [*as in* Fig. 7.] be such Cylinders. I say, the Resistance of the Cylinder A C shall be to the Resistance of the Cylinder D F, in a proportion compounded of the proportion of the Cube of the Diameter A B, to the Cube of the Diameter D E, and of the proportion of the Length E F to the Length B C. Suppose E G equal to B C, and to the Lines A B, and D E, let C H be a third proportional, and I, a fourth; and as E F is to B C, so let I be to S. And because the Resistance of the Cylinder A C is to the Resistance of the Cylinder D G, as the Cube A B to the Cube D E; that is, as the Line A B to the Line I: and the Resistance of the Cylinder G D is to the Resistance of the Cylinder D F, as the Length F E is to the Length E G; that is, as the Line I is to S: Therefore by Equality of proportion, as the Resistance of the Cylinder A C is to the Resistance of the Cylinder D F, so is the Line A B to S: But the Line A B is to S, in a proportion compounded of A B to I, and of I to S: Therefore the Resistance of the Cylinder A C is to the Resistance of the Cylinder D F, in a proportion compounded of A B to I, that is, as the Cube of A B to the Cube of D E, and of the proportion of the Line I to S; that is, of the Length E F to the Length B C: Which was to be demonstrated.

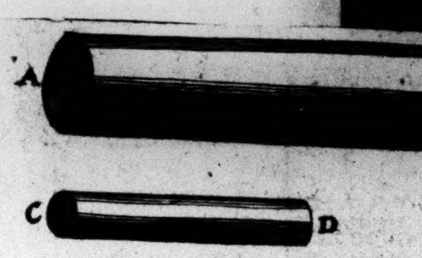
After the Proposition last demonstrated, we will consider what hapneth between like Cylinders and Prisms, of which we will demonstrate, how that

PRO-

PROPOSITION VI.

Of like Cylinders and Prisms the Moments compounded, that is to say, resulting from their Gravities, and from their Lengths, which are, as it were, Leavers, have to one another a proportion Sesquialter to that which is between the Resistances of their same Bases.

FOR demonstration of which let us describe the two like Cylinders A B, and C D, [*as in* Fig. 8.] I say, that the Moment of the Cylinder A B, to overcome the Resistance of its Base B, hath to the Moment of C D, to overcome the Resistance of its Base C, a proportion Sesquialter to that which the same Resistance of the Base B, hath to the Resistance of the Base D: And because the Moments of the Solids A B, and C D, to overcome the Resistances of their Bases B and D, are compounded of their Gravities, and of the Forces of their Leavers, and the Force of the Leaver A B is equal to the Force of the Leaver C D, and that because the length A B hath the same proportion to the Semidiameter of the Base B, (by the similitude of the Cylinders) that the Length C D hath to the Semidiameter of the Base D; it remaineth, that the total Moment of the Cylinder A B, be to the total Moment of C D, as the sole Gravity of the Cylinder A B is to the sole Gravity of the Cylinder C D; that is, as the said Cylinder A B is to the said C D: But these are in triplicate proportion to the Diameters of their Bases *B* and D; and the Resistances of the same *B*ases, being to one another as the said *B*ases, they are consequently in duplicate proportion to their same *B*ases: Therefore the Moments of Cylinders are in Sesquialter proportion to the Resistances of their *B*ases.

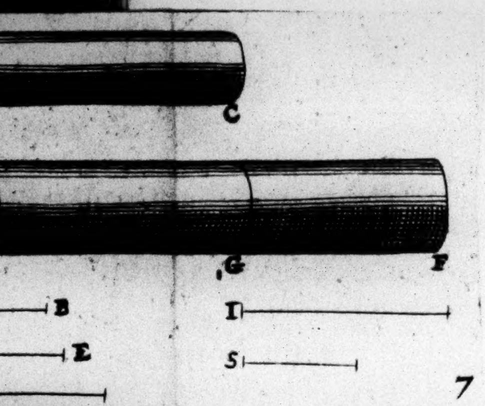


SIMP. This Proposition, indeed, is not only new, but unexpected to me, and at first sight, very remote from the judgment that I should have conjecturally past upon it: for in regard, that these Figures are in all other respects alike, I should have thought that their Moments likewise should have retained the same proportion towards their proper Resistances.

SAGR. This is the Demonstration of that Proposition, that in the beginning of our Discourses, I said, I thought------ I had some glimps of.

SALV. That which now befalleth, *Simplicius*, hapned for some time

their Resistances, according to the Squares of their Lengths (alwaies provided, that they retain the same thicknesse) and that likewise, these that are equally long, but different in thicknesse, encrease their Resistances according to the proportion of the Cubes of the Sides or Diameters of their Bases, we may enquire what befalleth to those Solids, being different in length and thickness, in which I observe, that



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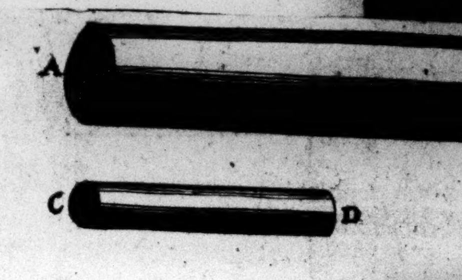


SIMP. This Proposition, indeed, is not only new, but unexpected to me, and at first sight, very remote from the judgment that I should have conjecturally past upon it: for in regard, that these Figures are in all other respects alike, I should have thought that their Moments likewise should have retained the same proportion towards their proper Resistances.

SAGR. This is the Demonstration of that Proposition, that in the beginning of our Discourses, I said, I thought------ I had some glimps of.

SALV. That which now befalleth, *Simplicius*, hapned for some time

time to my ſelf, believing, that the Reſiſtances of like Solids were alike, till that a certain, and that no very fixed or accurate Obſervation ſeemed to repreſent unto me, that Solids do not contain an equal tenure in their Toughneſs, but that the bigger are leſſe apt to ſuffer violent accidents, as luſty men are more damnified by their falls than little children; and, as in the begining we ſaid, we ſee a great *B*eam or Column break to pieces falling from the ſame height, and not a ſmall Priſin or little Cylinder of Marble. This ſame Obſervation gave me the hint for finding of that which I am now about to demonſtrate; a Quality truly admirable, for that amongſt the infinite Solid-like Figures, there are not ſo much as two, whoſe Moments retain the ſame proportion towards their proper Reſiſtances.

SIMP. Now you put me in mind of ſomething inſerted by *Ariſtotle* amongſt his Mechanical Queſtions, where he would give a Reaſon, whence it is, that *B*eams the longer they are, they are by ſo much the more weak, and bend more and more, although the ſhort ones be the ſlendereſt, and the long ones thickeſt: and, if I well remember, he reduceth the Reaſon to the ſimple Leaver.

SALV. It is very true, and becauſe the Solution ſeemeth not wholly to remove the cauſe of doubting *Monſignore di Guevara*, who, the truth is, with his moſt learned *Commentaries* hath highly enobled and illuſtrated that Work, enlargeth himſelf with other accute Speculations for the obviating all difficulties, yet himſelf alſo remaining perplexed in this point, whether, the lengths and thickneſſes of ſuch Solid Figures, encreaſing with the ſelf ſame proportion, they ought to retain the ſame tenure in their Toughneſſes and Reſiſtances againſt their breaking, and likewiſe againſt their bending. After I had long conſidered thereon, I have, in this manner found, that which I am about to tell you. And firſt I will demonſtrate that

PRO-

PROPOSITION VII.

Of like and heavy Prisms or Cylinders there is one only, and no more, that is reduced (being charged with its own weight) to the ultimate state between breaking and holding it self together: so that every greater, as being unable to resist its own weight, will break, and every lesser resisteth some Force that is employed against it to break it.

LET the heavy Prism be A B [*as in* Fig. 9.] reduced to the utmost length of its Consistance, so that being lengthned never so little more it will break: I say, that this is the only one amongst all those that are like unto it, (which yet are infinite) that is capable of being reduced to that dubious and ticklish state so that every greater being oppressed with its own weight wil break, and every lesser not, nay, will be able to resist some addition of a new violence, over and above that of its own weight First, take the Prism C E, like to, and greater than A B. I say, that this cannot consist, but will break, being overcome by its own Gravity. Suppose the part C D as long as A B. And because the Resistance C D is to that of A B, as the Cube of the thicknesse of C D to the Cube of the thickness of A B; that is, as the Prism C E to the Prism A B (being alike:) Therefore the Weight of C E is the greatest that can be sustained at the length of the Prism C D: But the Length C E is greater: Therefore the Prism C E will break. But let F G be lesser: it shall be demonstrated likewise (supposing F H equal to B A) that the Resistance of F G is to that of A B, as the Prism F G is to the Prism A B, in case that the Distance A B, that is F H, were equal to F G, but it is greater: Therefore the Moment of the Prism F G, placed in G, doth not suffice to break the Prism F G.

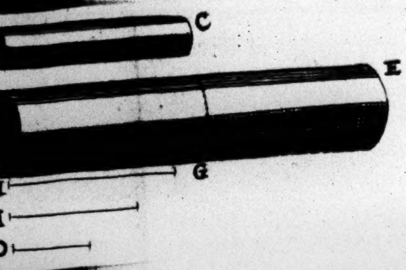
SAGR. A most manifest and brief Demonstration, inferring the truth and necessity of a Proposition that at first sight seemeth far from probability. It would be requisite, therefore, to alter much the proportion betwixt the Length and Thicknesse of the greater Prism by making it thicker or shorter, to the end it might be reduced to that nice state of indifferency between holding and breaking; and the Investigation of that same State, as I think, would be no lesse ingenuous.

SALV. Nay, rather more, as it is also more laborious: and I am sure

sure I have spent no small time to find it; and I will now impart it to you: Therefore

PROP. VIII. PROBL. I.

A Cylinder or Prism of the utmost length not to be broken by its own weight, and also a greater length, being given, to find the thicknesse of another Cylinder or Prism that under given length is the only one, and biggest, that can resist its own weight.



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LET the Cylinder B C [*as in* Fig. 10.] be the biggest that can resist its own weight, and let D E be a Length greater than A C; it is required to find the Thicknesse of the Cylinder, that under the Length D E is the greatest resisting its own weight. Let I be a third proportional to the Lengths D E, and A C; and as D E is to I, so let the Diameter F D be to the Diameter B A: and make the Cylinder F E. I say, that this is the biggest, and only one amongst all that are like to it that resisteth its own weight. To the Lines D C and I let M be a third proportional, and O a fourth. And suppose F G equal to A C. And because the Diameter F D is to the Diameter A B, as the Line D E to I, and O is a fourth proportional to D E and I, the Cube of F D shall be to the Cube of B A as D E is to O: But as the Cube of F D is to the Cube of B A, so is the Resistance of the Cylinder D G to the Resistance of the Cylinder B C: Therefore the Resistance of the Cylinder D G is to that of the Cylinder B C, as the Line D F is to O. And because the Moment of the Cylinder B C is equal to its Resistance, if we shew that the Moment of the Cylinder F E is to the Moment of the Cylinder B C, as the Resistance D F to the Resistance B A; that is, as the Cube of F D to the Cube of B A; that is, as the Line D E to O, we shall have our intent: that is, that the Moment of the Cylinder F E is equal to the Resistance placed in F D. The Moment of the Cylinder F E is to the Moment of the Cylinder D G, as the Square of D E is to the Square of A C; that is, as the Line D E to I: But the Moment of the Cylinder D G is to the Moment of the Cylinder B C, as the Square D F to the Square B A; that is, as the Square of D E to the Square of I; that is, as the Square of I to the Square of M; that is, as I to O: Therefore, by Equality of proportion, as the Moment of the Cylinder F E is to the Moment of the Cylinder B C, so is the Line D E to O; that is, the Cube D F to the Cube B A; that is, the Resistance of the Base D F to the Resistance of

of the Base B A: Which is that that was sought.

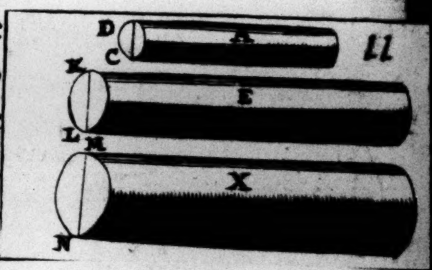
SAGR This, *Salviatus*, is a long Demonstration, and very hard to be born in mind at the first hearing, therefore I could wish, that you would please to repeat it.

SALV. I will do what you shall command; but haply it would be better to produce one more concise and short: but then it will be requisite to describe a Figure somewhat different.

SAGR. The favour will then be the greater: and bestow upon me the draught of that already explained, that I may at my leasure consider it again.

SALV. I will not fail to serve you. Now, suppose a Cylinder A, [*as in* Fig. 11.] the Diameter of whose Base let be the Line D C, and let this A be the greatest that can sustain it self and not break, than which we will find a bigger, which likewise shall be the biggest also, and the only one that sustaineth it self. Let us desire one like to the said A, and as long as the assigned Line, and let this be *v. gr.* E, the Diameter of whose Base let be K L; and to the two Lines D C, and K L let M N be a third proportional; which let be the Diameter of the Base of the Cylinder X, in length equal to E. I say, that this X is that which we seek. And because the Resistance D C is to the Resistance K L, as the Square D C to the Square K L; that is, as the Square K L to the Square M N; that is, as the Cylinder E to the Cylinder X; that is, as the Moment E to the Moment X: But the Resistance K L is to M N, as the Cube of K L is to the Cube of M N; that is, as the Cube B C to the Cube K L; that is, as the Cylinder A to the Cylinder E; that is, as the Moment A to the Moment E: Therefore, by Perturbation of proportion, as the Resistance D C is to M N, so is the Moment A to the Moment X: Therefore the Prism X, is in the same Constitution of Moment and Resistance as the Prism A.

The last Problem performed another way.



But let us make the Problem more general, and let the Proposition be this:

The Cylinder A C *being given, and its Moment towards its Resistance being supposed at pleasure, and any Length* D E *being assigned, to find the Thicknesse of the Cylinder whose Length is* D E, *and whose Moment towards its Resistance retaineth the same proportion, that the Moment of the Cylinder* A C *doth to its Resistance.*

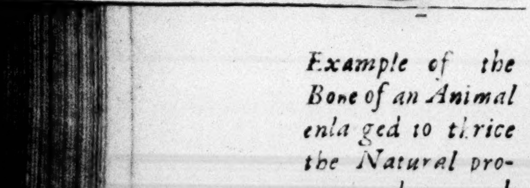
The last Proposition made more general.

Reassuming the abovesaid Figure and almost the same Method, we will say: Because the Moment of the Cylinder F E hath the same proportion to the Moment of the part D G, that the Square E D hath to the Square F G; that is, that the Line D E hath to I: and because the Moment of the Cylinder F G is to the Moment of the Cylinder A C, as the Square F D to the Square A B; that is, as the Square D E to the Square I; that is, as the Square I to the Square M; that is, as the Line I to O: Therefore, *ex æquali*, the Moment of the Cylinder F E hath the same proportion to the Moment of the Cylinder A C, that the Line D E hath to the Line O; that is, that the Cube D E hath to the Cube of I; that is, that the Cube of F D hath to the Cube of A B; that is, that the Resistance of the Base F D hath to the Resistance of the Base A B: Which was to be performed.

Now, let it be observed, that from the things hitherto demonstrated, we may plainly gather, how Impossible it is, not only for Art, but for Nature her self to encrease her Machines to an immense Vastnesse: so that it would be impossible by Art to build extraordinary vast Ships, Palaces, or Temples, whose * Oars, Sail-yards, Beams, Iron Bolts, and, in a word, their other parts should consist or hold together: neither again could Nature make Trees of unmeasurable greatnesse, for that their Arms or Bows being oppressed with their own weight would at last break: and likewise it would be impossible for her to make structures of Bones for Men, Horses, or other Animals, that might subsist, and proportionatly perform their Offices, when those Animals should be augmented to immense heights, unlesse she should take Matter much more hard and Resisting than that which she commonly useth, or else should deform those Bones by augmenting them beyond their due Symetry, and making the Figure or shape of the Animal to become monstrously big: Which haply was hinted by my most Witty Poet, where describing an huge Giant, he saith,

* Oares are used in the Ships or Gallies of the Mediterrane, upon which our Author was a Coaster.

Bones of Animals magnified beyond their natural size, would not subsist, if it be required to retain the same proportion of thicknesse and hardnesse in them that is in those of Natural Animals.



12

Non si puo compartir quanto sia lungo,
Si smisuratamente è tutto grosso.

And for a short example of this that I say, [*as in* Fig. 12.] I have heretofore drawn the Figure of a Bone only trebled in Length, and augmented in Thicknesse in such proportion, as that it may in its great Animal perform the office proportionate to that of the lesser Bone in a smaller Animal, and the Figures are these: whereby you see what a disproportionate Figure that of the augmented Bone becometh. Whence it is manifest, that he that would in an huge Giant keep the proportions that the Members have in an

Example of the Bone of an Animal enlarged to thrice the Natural proportion, how much thicker it ought to be to perform its office.

an ordinary Man, must either find Matter much more hard and resisting to make Bone of, or else must admit that its Strength is in proportion much more weak than in Men of middle Stature: otherwise, encreasing the Giant to an immeasurable height he would be oppressed, and fall under his own weight. Whereas on the contrary, in diminishing of Bodies we do not see the Strength and Forces to diminish in the same proportion, nay, in the lesser the Robustiousnesse encreaseth with a great proportion. So that I believe, that a little Dog could carry on his back two or three Dogs equal to himself, but I do not think that an Horse could carry so much as one single Horse of his own size.

SIMP. But if it be so, I have great reason to doubt the Immense bulks that we see in Fishes, for (if I rightly understand you) a Whale shall be as big as ten Elephants, and yet they sustain themselves.

SALV. Your doubt, *Simplicius*, prompts me with another Condition which I perceived not before, which is also able to make Giants and other very big Animals to consist, and act themselves no lesse than smaller, and this will happen when not only Strength is added to the Bones and other Parts, whose office it is to sustain their own and the supervenient weight; but the structure of the Bones being left with the same proportions, the same Fabricks would just in the same manner, yea, with much more ease, consist, when the Gravity of the matter of those Bones, or that of the Flesh, or whatever else is to rest it self upon the Bones is diminished in that proportion: and of this second Artifice, Nature hath made use in the framing of Fishes, making their Bones, and Pulps, not only very light, but without any Gravity.

SIMP. I see very well, *Salviatus*, whither your Discourse tendeth: you will say, that because the Element of Water is the Habitation of Fishes, which by its Corpulence, or, as others will, by its Gravity diminisheth the weight of Bodies demerged in it, for that reason the Matter of Fishes, not weighing any thing, may be sustained without surcharging their Bones: but this doth not suffice, for although the rest of the substance of the Fish weigh not, yet without doubt the matter of their Bones hath its weight: and who will say, that the Rib of a Whale that is as big as a Beam doth not weigh very much, and in Water sinketh to the Bottom? These therefore should not be able to subsist in so vast a Bulk.

SALV. You argue very cunningly; and for an answer to your Doubt, tell me, whether you have observed Fishes to stand immoveable under water at their pleasures, and not to descend towards the Bottom, or raise themselves towards the top without making some motion with their Fins?

SIMP. This is a very manifest Observation.

The Cause why Fishes do equilibrate themselves in the Water.

SALV. This power therefore that the Fishes have to stay themselves, as if they were immoveable in the midst of the Water, is a most infallible argument, that the Composition of their Corporeal Masse equalleth the Specifick Gravity of the Water, so that if there be found in them some parts that are more grave than the Water, it is necessarily requisite that they have others so much lesse grave, so that the *Equilibrium* may be ballanced. If therefore the Bones be more grave, it is necessary that the Pulps, or other Matters that are in them, be more light; and these will with their lightnesse counterpoise and compensate the weight of the Bones. So that in Aquatick Animals the quite contrary hapneth to that which befals the Terrestrial, namely, that in the latter it is the office of the Bones to sustain their own weight, and the weight of the Flesh; and in the former, the *Flesh* [*if one may so call it*] beareth up its own weight, and that of the Bones. And therefore cease to wonder how there may be most vast Animals in the Water, but not on the Earth, that is, in the Air.

Aquatick Animals greater than the Terrestrial, and for what Reason.

SIMP. I am satisfied, and moreover observe, that these which we call Terrestrial Animals, ought with more reason to be called Aerial; because in the Air they really live, and by the Air they are environ'd, and of the Air they breath.

SAGR. The Discourse of *Simplicius* pleaseth me, as also his Doubt and its Solution. And farthermore I comprehend very easily, that one of these huge Fishes being haul'd on shore, could not perchance be able to sustain it self for any time; but that the Connections of the Bones being relaxed, its Masse would be crush'd under its own weight.

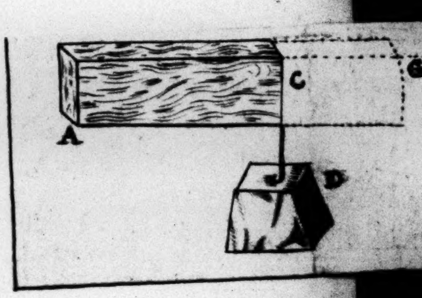
SALV. For the present, I encline to the same Opinion: nor am I far from thinking that the same would happen to that huge Ship, which floating in the Sea is not dissolved by its weight, and the burden of its Lading and Artilery, but on dry ground, and environed with Air, it perhaps would fall in pieces. But let us pursue our businesse, and demonstrate, that

PROP.

PROP. IX. PROBL. II.

A Prisme or Cylinder with its weight, and the greatest Weight sustained by it being given, to find the greatest Length, beyond which being prolonged, it would break under its own Weight.

LEt there be given the Prisme A C (*as in* Fig. 13.) with its weight, and likewise let the Weight D be given, the greatest that can be sustained by the extreme C: it is required to finde the greatest Length unto which the said Prisme may be prolonged, without breaking. As the weight of the Prisme A C is to the Compound of the weights A C, with the double of the Weight D, so let the length C A be to C A H: between which let A G be a Mean-Proportional. I say that A G is the Length sought. For the depressing Moment of the Weight D in C, is equal to the Moment of the double weight D, if it be placed in the middle of A C, where is also the Center of the Moment of the Prisme A C: The Moment, therefore, of the Resistance of the Prisme A C, which resides in A, is equivalent to the gravitation of the double of the Weight D with the weight A C, but hanged in the midst of A C. And because it hath been made, that as the Moment of the said Weights so situated, that is, of the double of D, with A C, is to the Moment of A C, so is H A to A C, between which A G is a Mean Proportional: Therefore the Moment of D doubled with the Moment of A C, is to the Moment A C, as the Square G A to the Square A C: But the pressing Moment of the Prisme G A, is to the Moment of A C, as the Square G A to the Square A C: Therefore the Length A G is the greatest that was sought, namely, that unto which the Prisme A C being prolonged, it would sustain it self, but beyond it would break.



Hitherto we have considered the Moments and Resistances of solid Prismes and Cylinders, one end of which is supposed immoveable, and to the other onely the Force of a pressing weight is applyed, considering it by it self alone, or joyned with the Gravity of the same Solid, or else the sole Gravity of the said Solid. Now I desire that we may speak something of those same Prismes or Cylinders, in case they were sustained at both ends, or did rest upon one sole point taken between the ends. And first, I say that,

PROP.

PROPOSITION X.

The Cylinder that being charged with its own Weight shall be reduced to its greatest Length, beyond which it would not sustain it self, whether it be born up in the middle by one sole Fulciment, or else by two at the ends, may be double in length to that which should be fastned in the Wall, that is sustained at but one end.

WHich of it self is very obvious; for if we shall suppose of the Cylinder which I describe A B C, its half A B to be the utmost Length that is able to be sustained, being fastened at the end B, it shall be sustained in the same manner, if being laid upon the Fulciment G, it shall be counterpoised by its other half B C. And likewise, if of the Cylinder D E F, the Length shall be such that onely one half of it can be sustained, being fastened at the end D, and consequently the other E F, fixed at the end F; it is manifest, that placing the Fulciments H and I under the ends D and F, every Moment of Force or of Weight that is added in E, will there make the Fracture.

That which requireth a more subtil Speculation is, when substracting from the proper Gravity of such Solids, it were propounded to us

PROP. XI. PROBL. III.

To find whether that Force or Weight, that being applied to the middle of a Cylinder sustained at the ends, would suffice to break it, could do the same, applied in any other place, neerer to one end than to the other.

AS for Example, whether we desiring to break a Staffe and took it with the ends in our hands, and setting our knee, to the midst of it, the same Force that should suffice to break it in that manner, would also suffice in case the knee were

were set, not in the midst, but neerer to one of the ends.

SAGR. I think the Problem is toucht upon by *Aristotle* in his *Mechanical Questions*.

SALV. The Question of *Aristotle* is not precisely the same, for he seeks no more, but to render a reason why lesse labour is required to break the Staffe, holding the hands at the ends of it, that is, far distant from the Knee, than if we held them neerer: and he giveth a general Reason of the same, reducing the cause of it to the Leavers, which are longer when the Arms are extended, grasping the ends. Our Question addeth something more, seeking whether, setting the Knee in the midst, or in another place, but alwayes keeping the hands at the ends, the same Force serveth in all situations.

SAGR. At first apprehension it should seem that it doth, for that the two Leavers retain in a certain fashion the same Moment, seeing that as the one is shortned, the other is lengthened.

SALV. Now you see, how easie it is to make Equivocations, and with what caution and circumspection we are to walk, least we run into them. This that you say, and which indeed at the first sight carrieth with it so much of probability, is in the strictnesse of it so false, that whether the Knee, which is the Fulciment of the two Leavers, be placed or not placed in the midst, it maketh such alteration, that of that Force which would suffice to make the Fracture in the midst, it being to be made in some other place, it will not suffice to apply four times so much, nor ten, nor an hundred, no nor a thousand. Upon this we will make some general Consideration, and then we will come to the Specifick Determination of the Proposition, according to which, the Forces for making of Fractures gradually vary more in one point than in another.

Let us first designe this Truncheon A B to be broken in the midst upon the Fulciment C, and neer unto that let us designe it again, but under the Characters D E, to be broken on the Fulciment F, remote from the middle. First it is manifest, that the Distances A C and C B being equal, the Force shall be shared equally in the ends B and A. Again, according as the Distance D F groweth lesse than the Distance A C, the Moment of the Force placed in D groweth lesse than the Moment in A, that is placed at the Distance C A, and lesseneth according to the proportion of the Line D F to A C; and consequently, it is requisite to encrease it to equalize or exceed the Resistance of F: But the Distance D F may diminish *in infinitum*, in relation to the Distance A C: Therefore it is requisite, that it be possible for the Force to be applyed in D, to encrease *in infinitum*, that it may countervail the Resistance in F. But, on the contrary, according

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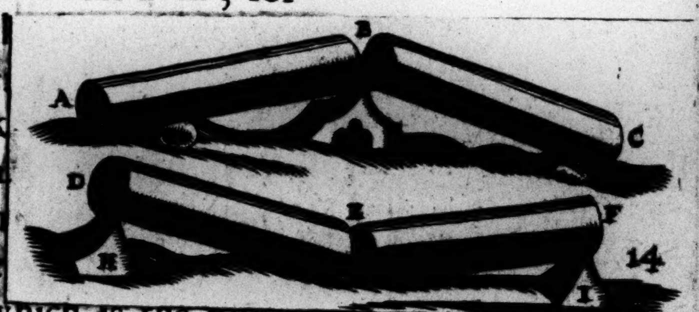

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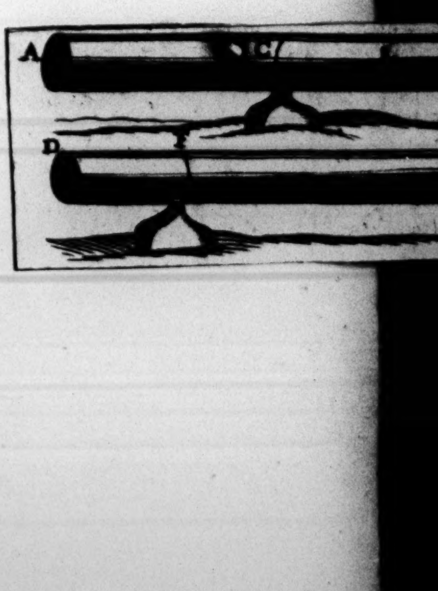
Let us first designe this Truncheon AB to be broken in the midst upon the Fulciment C, and neer unto that let us designe it again, but under the Characters DE, to be broken on the Fulciment F, remote from the middle. First it is manifest, that the Distances AC and CB being equal, the Force shall be shared equally in the ends B and A. Again, according as the Distance DF groweth lesse than the Distance AC, the Moment of the Force placed in D groweth lesse than the Moment in A, that is placed at the Distance CA, and lesseneth according to the proportion of the Line DF to AC; and consequently, it is requisite to encrease it to equalize or exceed the Resistance of F: But the Distance DF may diminish *in infinitum*, in relation to the Distance AC: Therefore it is requisite, that it be possible for the Force to be applyed in D, to encrease *in infinitum*, that it may countervail the Resistance in F. But, on the contrary, ac-



cording

cording as the Distance F E encreaseth above C B, it is requisite to diminish the Force in E, that it may compensate the Resistance in F: But the Distance F E in relation to C B, cannot encrease *in infinitum*, by drawing the Fulciment F towards the end D, no nor yet to the double: Therefore, the Force in E, that it may compensate the Resistance in F, shall be alwayes more than half of the Force in B. We may comprehend, therefore, the necessity of augmenting the Moments of the Collected Forces in E and D infinitely to equalize or exceed the Resistance placed in F, according as the Fulciment F shall approach neerer and neerer to the end D.

Sagr. What will *Simplicius* say to this? Must he not confesse the Virtue of Geometry to be a more powerful instrument than all others, to sharpen the Wit, and dispose it to discourse and speculate well? and that *Plato* had great reason to desire that his Scholars should be well grounded in the Mathematicks? I have very well understood the nature of the Leaver, and how that its Length encreasing or decreasing, the Moment of the Force and of the Resistance augmented or diminished, and yet in the determination of the present Problem I deceived my self, and that not a little, but infinitely much.

Simp. The truth is, I begin to see that Logick, although it be a most apposite Instrument to regulate our Discourse, doth not attain, as to the prompting of the Mind with Invention, unto the acutenesse of Geometry.

Sagr. In my conceit, Logick giveth us to understand, whether the Discourses and Demonstrations already made and found are concluding, but that it teacheth us how to finde concluding Discourses and Demonstrations; the truth is, I do not believe: But it will be better, that *Salviatus* shew us according to what proportion the Moments of the Forces do go increasing, to overcome the Resistance of the same Piece of Wood, according to the several places of the Fracture.

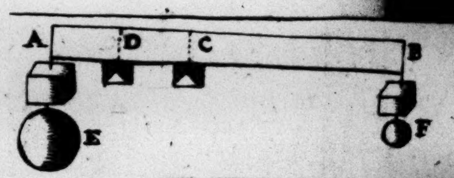
Salv. The proportion that you seek, proceedeth after such a manner, that

PROP.

PROPOSITION XII.

If in the length of a Cylinder we shall marke two places, upon which we would make the Fracture of the said Cylinder, the Resistances of those two places have the same proportion to each other, as have the Rectangles made by the Distances of those places reciprocally taken.

LEt the two Forces (*as in* Fig. 16.) be A and B the least, to break in C, and E and F likewise the least, to break in D. I say the Forces A and B have the same proportion to the Forces E and F, that the Rectangle A D B hath to the Rectangle A C B. For the Forces A and B, have to the Forces E and F, a proportion compounded of the Forces A and B, to the Force B, of B to F, and of F to E and E: But as the Forces A and B are to the Force B, so is the Length B A to A C; and as the Force B is to F, so is the Line D B to B C; and as the Force F is to F and E, so is the Line D A to A B: Therefore the Forces A and B have to the Forces E and F a proportion compounded of these three, namely, of B A to A C, of D B to B C, and of D A A B. But of the two proportions D A to A B, and A B to A C, is compounded the proportion of D A to A C: Therefore the Forces A and B have to the Forces E and F, the proportion compounded of this D A to A C, and of the other D B to D C. But the Rectangle A D B hath to the Rectangle A C B, a proportion compounded of the same D A to A C, and of D B to B C: Therefore the Forces A and B are to the Forces E and F, as the Rectangle A D B is to the Rectangle A C B; which is as much as to say, the Resistance against Fraction in C, hath the same proportion to the Resistance against Fraction in D, that the Rectangle A D B hath to the Rectangle A C B: Which was to be demonstrated.

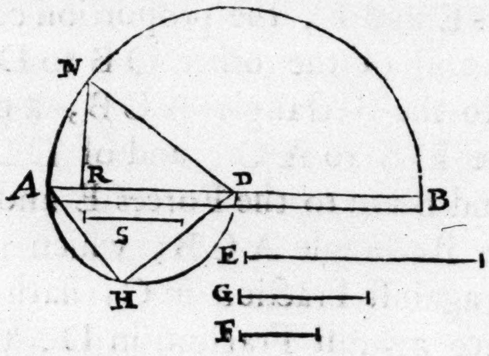


In consequence of this Theorem we may resolve a Problem of great Curiosity; and it is this:

PROP. XIII. PROBL. IV.

There being given the greatest weight that can be supported at the middle of a Cylinder or Prisme, where the Resistance is least; and there being given a Weight greater than that, to find in the said Cylinder, the point at which the given greater Weight may be supported as the greatest Weight.

LEt the given weight greater than the greatest weight that can be supported at the middle of the Cylinder A B, have unto the said greatest weight, the proportion of the line E to F: it is required to find the point in the Cylinder at which the said given weight commeth to be supported as the biggest. Between E and F let G be a Mean-Proportional; and as E is to G, so let A D be to S, S shall be lesser than A D. Let A D be the Diameter of the Semicircle A H D: in which suppose A H equal to S; and joyn together H and D, and take D R equal to it. I say that R is the point sought, at which the given weight, greater than the greatest that can be supported at the middle of the Cylinder D, would become as the greatest weight. On the length B A describe the Semicircle A N B, and raise the Perpendicular R N, and conjoyn N and D: And because the two Squares N R and R D are equal to the Square N D; that is, to the Square A D; that is, to the two A H and and H D; and H D is equal to the Square D R: Therefore the Square N R, that is, the Rectangle A R B shall be equal to the Square A H; that is, to the Square S: But the Square S is to the Square A D, as F to E; that is, as the greatest supportable Weight at D to the given greater Weight: Therefore this greater shall be supported at R, as the greatest that can be there sustained. Which is that that we sought.

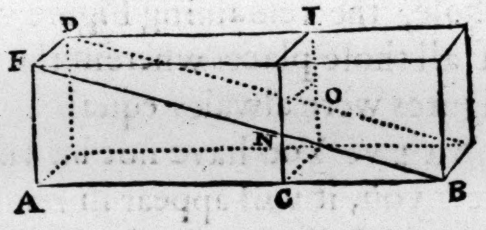


SAGR. I understand you very well, and am considering that the Prisme A B having alwayes more strength and resistance against Pression in the parts that more and more recede from the middle, whether in very great and heavy Beams one may take

away

away a pretty big part towards the end with a notable alleviation of the weight; which in Beams of great Rooms would be commodious, and of no small profit. And it would be pretty, to find what Figure that Solid ought to have, that it might have equal Resistance in all its parts; so as that it were not with more ease to be broken by a weight that should presse it in the midst, than in any other place.

SALV. I was just about to tell you a thing very notable and pleasant to this purpose. I will assume a brief Scheme for the better explanation of my meaning. This Figure D B is a Prism, whose Resistance against Fraction in the term A D by a Force pressing at the term B, is lesse than the Resistance that would be found in the place C I, by how much the length C B is lesser than B A; as hath already been demonstrated. Now suppose the said Prisme to be sawed Diagonally according to the Line F B, so that the opposite Surfaces may be two Triangles, one of which towards us is F A B. This Solid obtains a quality contrary to the Prisme, to wit, that it lesse resisteth Fraction by the Force placed in B at the term C than at A, by as much the Length C *B* is lesse than *B* A; Which we will easily prove: For imagining the Section C N O parallel to the other A F D, the Line *F* A shall be to C N in the Triangle F A *B* in the same proportion, as the Line A *B* is to *B* C: and therefore if we suppose the Fulciment of the two Leavers to be in the Points A and C, whose Distances are *B* A, A F, B C, and C N, these, I say, shall be like: and therefore that Moment which the *F*orce placed at *B* hath at the Distance *B* A above the Resistance placed at the Distance A *F*, the said *F*orce at *B* shall have at the Distance *B* C above the same Resistance, were it placed at the Distance C N: *B*ut the Resistance to be overcome at the *F*ulciment C, being placed at the Distance C N, from the *F*orce in *B* is lesser than the Resistance in A so much as the Rectangle C O is lesse than the Rectangle A D; that is, so much as the Line C N is less than A *F*; that is, C *B* than BA: Therefore the Resistance of the part O C B against *F*raction in C is so much less than the Resistance of the whole D A O against *F*racture in O, as the Length C B is less than A B. We have therefore from the Beam or Prisme D B, taken away a part, that is half, cutting it Diagonally, and left the Wedge or triangular Prism F B A; and they are two Solids of contrary Qualities, namely, that more resists the more it is shortned, and this in shortning loseth its toughness as fast. Now this being granted,



it ſeemeth very reaſonable, nay, neceſſary, that one may give it a cut, by which taking away that which is ſuperfluous, there remaineth a Solid of ſuch a *Figure*, as in all its parts hath equal Reſiſtance.

SIMP. It muſt needs be ſo; for where there is a tranſition from the greater to the leſſer, one meeteth alſo with the equal.

SAGR. But the buſineſſe is to find how we are to guide the Saw for making of this Section.

SIMP. This ſeemeth to me as if it were a very eaſie buſineſſe; for if in ſawing the Priſm diagonally, taking away half of it, the Figure that remains retaineth a contrary quality to that of the whole Priſm, ſo as that in all places wherein this acquireth ſtrength, that as faſt loſeth it, me thinks, that keeping the middle way, that is, taking only the half of that half, which is the fourth part of the whole, the remaining Figure will not gain or loſe ſtrength in any of all thoſe places wherein the loſſe and the gain of the other two Figures were alwaies equal.

SALV. You have not hit the mark, *Simplicius*; and as I ſhall ſhew you, it will appear in reality, that that which may be cut off from the Priſm, and taken away without weakening it is not its fourth part, but the third. Now it remaineth (which is that that was hinted by *Sagredus*)

PROP. XIV. PROBL. V.

To find according to what Line the Section is to be made; Which I will prove to be a Parabolical Line.

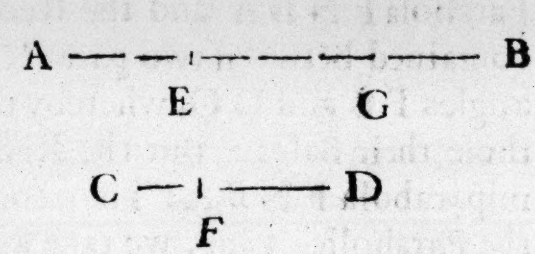
But firſt it is neceſſary to demonſtrate a certain Lemma, which is this:

LEMMA I.

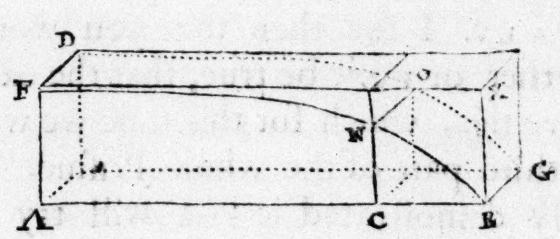
If there ſhall be two Ballances or Leavers divided by their Fulciments in ſuch ſort that the two Diſtances, whereat the Forces are to be placed, have to each other double the proportion of the Diſtances at which the Reſiſtances ſhall be, which Reſiſtances are to each other as their Diſtances, the ſuſtaining Powers ſhall be equal.

LEt A B and C D be two Leavers divided upon their Fulciments E and F, in ſuch ſort that the Diſtance E B hath to F D a proportion double to that which the Diſtance E A hath to F C. I ſay, the

the Powers that in B D shall sustain the Resistances A and C shall be equal to each other. Let E G be supposed a Mean-Proportional between E B and F D; therefore as B E is to E G, so shall G E be to F D, and A E to C *F*; and so is supposed the Resistance of A to the Resistance of C. And because that as E G is to *F* D, so is A E to C *F*; by Permutation as G E is to E A, so shall D *F* be to *F* C: And therefore (in regard that the two Leavers D C and G A are divided proportionally in the Points *F* and E) in case the Power that being placed at D compensates the Resistance of C were at G, it would countervail the same Resistance of C placed in A: But by what hath been granted, the Resistance of A hath the same proportion to the Resistance of C, that A E hath to C *F*; that is, B E hath to E G: Therefore the Power G, or if you will D, placed at B will sustain the Resistance placed at A: Which was to be demonstrated.



This being understood: in the Surface *F* B of the Prisme D B, let the Parabolical Line *F* N B be drawn, whose Vertex is B, according to which let the said Prisme be supposed to be sawed, the Solid comprised between the Base A D, the Rectangular Plane A G, the Right Line B G, and the Superficies D G B *F* being left incurvated according to the Curvity of the Parabolical Line *F* N B. I say, that that Solid is throughout of equal Resistance. Let it be cut by the Plane C O parallel to A D; and imagine two Leavers divided and supported upon the Fulciments A and C; and let the Distances of one be B A and A F, and of the other B C, and C N. And because in the Parabola *F B* A, A *B* is to B C, as the Square of *F* A to the Square of C N, it is manifest, that the Distance *B* A of one Leaver, hath to the Distance *B* C of the other a proportion double to that which the other Distance A *F* hath to the other C N, And because the Resistance that is to be equal by help of the Leaver *B* A hath the same proportion to the Resistance that is to be equal by help of the Leaver *B* C, that the Rectangle D A hath to the Rectangle O C; which is the same that the Line A *F* hath to N C, which are the other two Distances of the Leavers; it is manifest by the foregoing Lemma, that the same Force that being



applyed

applyed to the Line *B* G will equal the Resistance D A, will likewise equal the Resistance C O. And the same may be demonstrated, if one cut the Solid in any other place: therefore that Parabolical Solid is throughout of equal Resistance. In the next place, that cutting the Prisme according to the Parabolical Line F N B, the third part of it is taken away, appeareth, For that the Semi-Parabola F N *B* A and the Rectangle F *B* are Bases of two Solids contained between two parallel Planes, that is, between the Rectangles F B and D G, whereby they retain the same Proportion, as those their Bases: But the Rectangle F *B* is Sesquialter to the Semiparabola F N *B* A: Therefore cutting the Prisme according to the Parabolick Line, we take away the third part of it. Hence we see, that *B*eams may be made with the diminution of their Weight more than thirty three in the hundred, without diminishing their Strength in the least; which in great Ships, in particular, for bearing the Decks may be of no small benefit; for that in such kind of Fabricks Lightnesse is of infinite importance.

SAGR. The Commodities are so many, that it would be tedious, if not impossible, to mention them all. *B*ut I, laying aside these, would more gladly understand that the alleviation is made according to the assigned proportions. That the Section, according to the Diagonal Line, cuts away half of the weight I very well know: but that the other Section according to the Parabolical Line takes away the third part of the Prisme I can believe upon the word of *Salviatus*, who evermore speaks the truth, but in this Case Science would better please me than Faith.

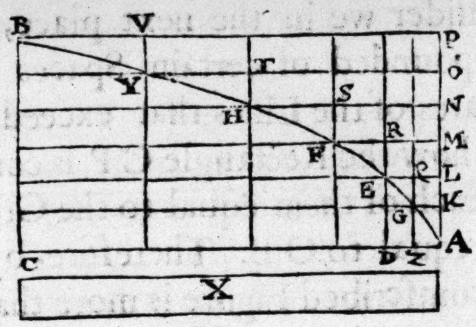
SALV. I see then that you would have the Demonstration, whether or no it be true, that the excesse of the Prisme over and above this, which for this time we will call a Parabolical Solid, is the third part of the whole Prisme. I am certain that I have formerly demostrated it; I will try now whether I can put the Demonstration together again: to which purpose I do remember that I made use of a Certain Lemma of *Archimedes*, inserted by him in his *B*ook *de Spiralibus*, and it is this:

LEMMA II.

If any number of Lines at pleasure shall exceed one another equally, and the excesses be equal to the least of them, and there be as many more, each of them equal to the greatest; the Squares of all these shall be lesse than the triple of the Squares of those that exceed one another: but they shall be more than triple to those others that remain, the Square of the greatest being substracted.

This

THis being granted: Let the Parabolick Line A *B* be inscribed in this Rectangle A C *B* P: we are to prove the Mixt Triangle *B* A P, whose sides are *B* P and P A, and *B*ase the Parabolical Line *B* A, to be the third part of the whole Rectangle C P. For if it be not so, it will be either more than the third part, or lesse. Let it be supposed that it may be lesse, and to that which is wanting suppose the Space X to be equal. Then dividing the Rectangle continually into equal parts with Lines parallel to the Sides *B* P and C A, we shall in the end arrive at



such parts, as that one of them shall be lesse than the Space X. Now let one of them be the Rectangle O *B*, and by the Points where the other Parallels intersect the Parabolick Line, let the Parallels to A P passe: and here I will suppose a Figure to be circumscribed about our Mixt-Triangle, composed of Rectangles, which are *B* O, I N, H M, F L, E K, G A; which Figure shall also yet be less than the third part of the Rectangle C P, in regard that the excesse of that Figure over and above the Mixed Triangle is much lesse than the Rectangle *B* O, which yet again is lesse than the Space X.

SAGR. Softly, I pray you, for I do not see how the excesse of this circumscribed Figure above the Mixt Triangle is considerably lesser than the Rectangle *B* O.

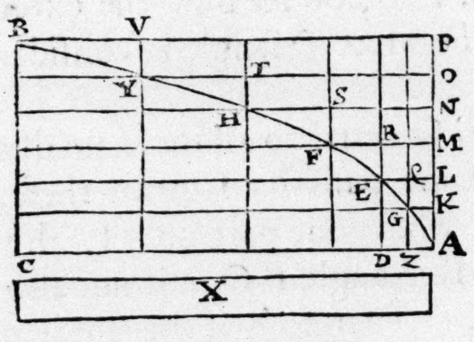
SALV. Is not the Rectangle *B* O equal to all these small Rectangles by which our Parabolical Line passeth; I mean these, *B* I, I H, H F, F E, E G, and G A, of which one part only lyeth without the Mixt Triangle? And the Rectangle *B* O, is it not also supposed to be lesse than the Space X? Therefore if the Triangle together with X did, as the Adversary supposeth, equalize the third part of the Rectangle C P the circumscribed Figure that adjoyns to the Triangle so much lesse than the Space X, will remain even yet lesse than the third part of the said Rectangle C P. *B*ut this cannot be, for it is more than a third part, therefore it is not true that our Mixt Triangle is lesse than one third of the Rectangle.

SALV. I understand the Solution of my Doubt. *B*ut it is requisite now to prove unto us, that the Circumscribed Figure is more than a third part of the Rectangle C P; which, I believe, will be harder to do.

SALV. Not at all. For in the Parabola the Square of the Line D E hath the same proportion to the Square of Z G, that the Line *The Quadrature of the Parabola shewn by one single Demonstration.*

D A

D A hath to A Z; which is the same that the Rectangle K E hath to the Rectangle A G, their heights A K and K L being equal. Therefore the proportion that the Square E D hath to the Square Z G; that is, the Square L A hath to the Square A K, the Rectangle K E hath likewise to the Rectangle K Z. And in the self-same manner we might prove that the other Rectangles L F, M H, N I, O B are to one another as the Squares of the Lines M A, N A, O A, P A. Consider we in the next place, how the Circumscribed Figure is compounded of certain Spaces that are to one another as the Squares of the Lines that exceed with Excesses equal to the least, and how the Rectangle C P is compounded of so many other Spaces each of them equal to the Greatest, which are all the Rectangles equal to O B. Therefore, by the Lemma of *Archimedes*, the Circumscribed Figure is more than the third part of the Rectangle C P: But it was also lesse, which is impossible: Therefore the Mixt-Triangle is not lesse than one third of the Rectangle C P. I say likewise, that it is not more: For if it be more than one third of the Rectangle C P, suppose the Space X equal to the excesse of the Triangle above the third part of the said Rectangle C P, and the division and subdivision of the Rectangle into Rectangolets, but alwaies equal, being made, we shall meet with such as that one of them is lesser than the Space X; which let be done: and let the Rectangle B O be lesser than X; and, having described the Figure as before, we shall have inscribed in the Mixt-Triangle a Figure compounded of the Rectangles V O, T N, S M, N L, Q K, which yet shall not be less than the third part of the great Rectangle C P, for the Mixt Triangle doth much lesse exceed the Inscribed Figure than it doth exceed the third part of the Rectangle C P; Because the excesse of the



Triangle above the third part of the Rectangle C P is equal to the Space X which is greater than the Rectangle B O, and this also is considerably greater than the excesse of the Triangle above the Inscribed Figure: For to the Rectangle B O, all the Rectangolets A G, G E, E F, F H, H I, I B are equal, of which the Excesses of the Triangle above the Inscribed Figure are lesse than half: And therefore the Triangle exceeding the third part of the Rectangle C P, by much more (exceeding it by the Space X) than it exceedeth its inscribed Figure, that same Figure shall also be greater than the third part of the Rectangle C P: But it is lesser, by the Lemma presupposed: For that the Rectangle C P, as being the

the Aggregate of all the biggest Rectangles, hath the same proportion to the Rectangles compounding the Inscribed *Figure*, that the Aggregate of of all the Squares of the Lines equal to the biggest, hath to the Squares of the Lines that exceed equally, substracting the Square of the biggest: And therefore (as it hapneth in Squares) the whole Aggregate of the biggest (that is the Rectangle C P) is more than triple the Aggregate of the exceeding ones, the biggest deducted, that compound the Inscribed Figure. Therefore the Mixt-Triangle is neither greater nor lesser than the third part of the Rectangle C P: It is therefore equal.

SAGR. A pretty and ingenuous Demonstration: and so much the more, in that it giveth us the Quadrature of the Parabola, shewing it to be *Sesquitertial* of the Triangle inscribed in the same; proving that which *Archimedes* demonstrateth by two very different, but both very admirable, methods of a great number of Propositions. As hath likewise been demonstrated lately by *Lucas Valerius*, another second *Archimedes* of our Age, which Demonstration is set down in the Book that he writ of the Center of the Gravity of Solids.

SALV. A Treatise which verily is not to come behind any one that hath been written by the most *F*amous Geometricians of the present and all past Ages: which when it was read by our *Academick*, it made him desist from prosecuting his Discoveries that he was then proceeding to write on the same Subject: in regard he saw the whole business so happily found and demonstrated by the said *Valerius*.

SAGR. I was informed of all these things by our *Academick*; and have besought him withall that he would one day let me see his Demonstrations that he had found at the time when he met with the *B*ook of *Valerius*: but I never was so happy as to see them.

SALV. I have a Copy of them, and will impart them to you, for you will be much pleased to see the variety of Methods, which these two Authors take to investigate the same Conclusions, and their Demonstrations: wherein also some of the Conclusions have different Explanations, howbeit in effect equally true.

SAGR. I shall be very glad to see them, therefore when you return to our wonted Conferences you may do me the favour to bring them with you. *B*ut in the mean time, this same of the Resistance of the Solid taken from the Prisme by a Parabolick Section, being an Operation no lesse ingenuous than beneficial in many Mechanical Works, it would be good that Artificers had some easie and expedite Rule how they may draw the said Parabolick Line upon the Plane of the Prisme.

SALV. There are several waies to draw those Lines, but two that are more expedite than all the rest, I will describe unto you. *Several waies to describe a Parabola.*

One of which is truly admirable, ſince that thereby, in leſſe time than another can with Compaſſes ſlightly draw upon a paper four or ſix Circles of different ſizes, I can deſign thirty or forty Parabolick Lines no leſſe exact, ſmall; and ſmooth than the Circumferences of thoſe Circles. I have a Ball of Braſſe exquiſitely round, no bigger than a Nut, this thrown upon a Steel Mirrour held, not erect to the Horizon, but ſomewhat inclined, ſo that the Ball in its motion may run along preſſing lightly upon it, leaveth a Parabolical Line finely and ſmoothly deſcribed, and wider or narrower according as the Projection ſhall be more or leſs elevated. Whereby alſo we have a clear and ſenſible Experiment that the Motion of Projects is made by Parabolick Lines: an Effect obſerved by none before our *Academick*, who alſo layeth down the Demonſtration of it in his Book of Motion, which we will joyntly peruſe at our next meeting. Now the Ball, that it may deſcribe by its motion thoſe Parabola's, muſt be rouled a little in the hands that it may be warmed, and ſomewhat moyſtned, for by this means it will leave its track more apparent upon the Mirrour. The other way to draw the Line that we deſire upon the Priſme is after this manner. Let two Nailes be faſtned on high in a Wall, at an equal diſtance from the Horizon, and remote from one another twice the breadth of the Rectangle upon which we would trace the Semiparabola, and to theſe two Nails tye a ſmall thread of ſuch a length that its doubling may reach as far as the length of the Priſme; this ſtring will hang in a Parabolick Figure: ſo that tracing out upon the Wall the way that the ſaid String maketh on it, we ſhall have a whole Parabola deſcribed: which a Perpendicular that hangeth in the midſt between theſe two Nailes will divide into two equal parts. And for the transferring or ſetting off of that Line afterwards upon the oppoſite Surfaces of the Priſme it is not difficult at all, ſo that every indifferent Artiſt will know how to do it. The ſame Line might be drawn upon the ſaid Surface of the Priſme by help of the Geometrical Lines delineated upon the *Compaſſe* of our *Friend*, without any more ado.

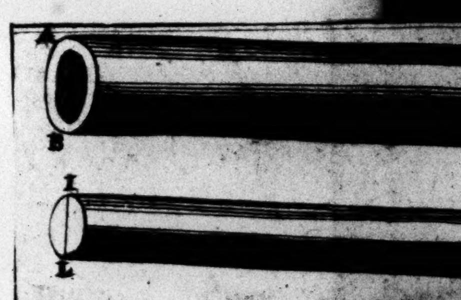
We have hitherto demonſtrated ſo many Concluſions touching the Contemplation of theſe Reſiſtances of Solids againſt Fraction by having firſt opened the way unto the Science with ſuppoſing the direct Reſiſtance for known; that we may in purſuance of them proceed forwards to the finding of other, and other Concluſions, with their Demonſtrations of thoſe which in Nature are infinite. Only at preſent, for a final concluſion of this daies Conferences, I will add the Speculation of the Reſiſtances of the Hollow Solids which Art, and chiefly Nature, uſeth in an hundred Operations, when without encreaſing the weight ſhe greatly augmenteth the ſtrength: as is ſeen in the Bones of Birds, and in many Canes that

are light and of great Resistance against bending and breaking. For if a Wheat Straw that supports an Ear that is heavier than the whole Stalk, were made of the same quantity of matter but were massie or solid, it would be much lesse repugnant to Fraction or Flection. And with the same Reason Art hath observed, and Experience confirmed, that an hollow Cane, or a Trunk of Wood or Metal, is much more firm and tough than if being of the same weight and length it were solid, which consequently would be more slender, and therefore Art hath contrived to make Lances hollow within when they are desired to be strong and light. We will shew therefore, that

PROPOSITION XV.

The Resistances of two Cylinders, equall, and equally long, one of which is Hollow, and the other Massie, have to each other the same proportion, as their Diameters.

LET the Cane or Hollow Cylinder be A E, [*as in* Fig. 17.] and the Cylinder I N Massie, and equall in weight and length. I say, the Resistance of the Cane A E hath the same proportion to the Resistance of the solid Cylinder, as the Diameter A B hath to the Diameter I L. Which is very manifest; For the Cane and the Cylinder I N being equal, and of equal lengths, the Circle I L that is Base of the Cylinder shall be equal to the Ring A B that is Base of the Cane A E, (I call the Superficies that remaineth when a lesser Circle is taken out of a greater that is Concentrick with it a Ring:) and therefore their Absolute Resistances shall be equal: but because in breaking crosse-waies we make use in the Cylinder I N of the length L N for a Leaver, and of the point L for a Fulciment, and of the Semidiameter or Diameter L I for a Counter-Leaver; and in the Cane the part of the Leaver, that is the Line B E is equal to L N; but the Counter-Leaver at the Fulciment B is the Diameter or Semidiameter A B: It is manifest therefore that the Resistance of the Cane exceedeth that of the Solid Cylinder as much as the Diameter A B exceeds the Diameter I L: Which is that that we sought. Toughness therefore is acquired in the hollow Cane above the Toughness of the solid Cylinder according to the proportion of the Diameters: provided alwaies that they be both of the same matter, weight, and length.

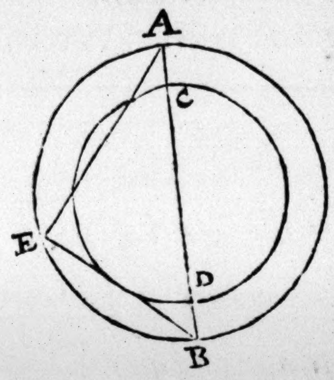


It would be well, that in consequence of this we try to investigate that which hapneth in other Cases indifferently between all Canes and solid Cylinders of equal length, although unequal in quantity of weight, and more or less evacuated. And first we will demonstrate, that

PROP. XVI. PROBL. VI.

A Trunk or Hollow Cane being given, a Solid Cylinder may be found equal to it.

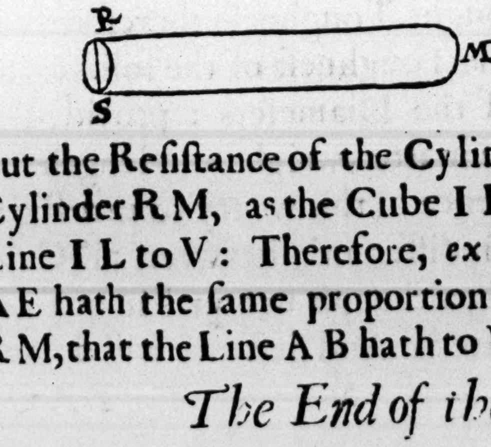
THis Operation is very easie. For let the Line A B, be the Diameter of the Cane, and C D the Diameter of the Hollow or Cavity. Let the Line A E be set off upon the greater Circle equal to the Diameter C D, and conjoyn E B. And because in the Semicircle A E B the Angle E is Right-Angle, the Circle whose Diameter is A B shall be equall to the two Circles of the Diameters A E and E B: But A E is the Diameter of the Hollow of the Cane: Therefore the Circle whose Diameter is E B, shall be equal to the Ring A C B D: And therefore the solid Cylinder, the Circle of whose Base hath the Diameter E B shall be equal to the Cane, they being of the same length. This demonstrated, we may presently be able



PROP. XVII. PROBL. VII.

To find what proportion is betwixt the Resistances of any whatsoever Cane and Cylinder, their lengths being equal.

LET the Cane A B E, and the Cylinder R S N, be of equal length: it is required to find what proportion the Resistances have to each other. By the precedent let the Cylinder I L N be found equal to the Cane, and of the same length; and to the Lines I L and R S (Diameters of the Bases of the Cylinders I N and R M) let the Line V be a fourth Proportional. I say, the Resistance of the Cane A E is to the Resistance of the Cylinder R M, as the Line A B is to V. For the Cane A E being equal to, and of the same length with the Cylinder I N, the Resistance of the Cane shall be to the Resistance of the Cylinder, as the Line A B is to I L: But the Resistance of the Cylinder I N is to the Resistance of the Cylinder R M, as the Cube I L is to the Cube R S; that is, as the Line I L to V: Therefore, *ex æquali*, the Resistance of the Cane A E hath the same proportion to the Resistance of the Cylinder R M, that the Line A B hath to V: Which is that that was sought.



The End of the Second Dialogue.

GALILEUS, HIS DIALOGUES OF MOTION.

The Third Dialogue.

INTERLOCUTORS,

SALVIATUS, SAGREDUS, and SIMPLICIUS.

OF LOCAL MOTION.

W*E promote a very new Science, but of a very old Subject. There is nothing in Nature more antient than* MOTION*, of which many and great Volumns have been written by Philosophers: But yet there are sundry Symptomes and Properties in it worthy of our Notice, which I find not to have been hitherto observed, much lesse demonstrated by any. Some slight particulars have been noted: as that the Natural Motion of Grave Bodies continually accelle-rateth,*

rateth, as they descend towards their Center: but it hath not been as yet declared in what proportion that Acceleration is made. For no man, that I know, hath ever demonstrated, That there is the same proportion between the Spaces, thorow which a thing moveth in equal Times, as there is between the Odde Numbers which follow in order after a Vnite. It hath been observed that Projects [or things thrown or darted with violence] make a Line that is somewhat curved; but that this line is a Parabola, none have hinted: Yet these, and sundry other things, no lesse worthy of our knowledg, will I here demonstrate: And which is more, I will open a way to a most ample and excellent Science, of which these our Labours shall be the Elements: into which more subtil and piercing Wits than mine will be better able to dive.

We divide this Treatise into three parts. In the first part we consider such things as respect Equable or Uniforme Motion. In the second we write of Motion naturally accelerate. In the third we treat of Violent Motion, or De Projectis.

OF EQVABLE MOTION.

Concerning Equable or Uniform Motion we have need of onely one Definition, which I thus deliver.

DEFINITION.

By an Equable or Uniform Motion, I understand that by which a Moveable in all equal Times passeth thorow equal Spaces.

ADVERTISEMENT.

I thought good to add to the old Definition (which simply termeth that an Equable Motion, whereby equal Spaces are past in equal Times) this Particle All, *that is, any whatsoever Times that are equal: for it may happen, that a Moveable may passe thorow equal Spaces in certain equal Times, though the Spaces be not equal which it hath gone in lesser, though equal parts of the same Time. From this our Definition follow these four Axiomes:* scilicet,

AXIOME I.

In the same Equable Motion that Space is greater which is passed in a longer Time; and that lesser which is past in a shorter.

AXIOME

AXIOME II.

In the ſame Equable Motion, the greater the Space is that hath been gone thorow, the longer was the Time in which the Moveable was going it.

AXIOME III.

The Space which a greater Velocity paſſeth in any Time, is greater than the Space which a leſſer Velocity paſſeth in the ſame Time.

AXIOME IV.

The Velocity which paſſeth a greater Space, is greater than the Velocity which paſſeth a leſſer Space in the ſame Time.

THEOR. I. PROP. I.

If a Moveable moving with an Equable Motion, and with the ſame Velocity paſſe two ſeveral Spaces, the Times of the Motion ſhall be to one another as the ſaid Spaces.

Let the Moveable by an Equable Motion with the ſame Velocity paſs the two Spaces A B *and* B C: *and let* D E *be the Time of the Motion thorow* A B; *and let the Time of the Motion thorow* B C *be* E F *I ſay that the Time* D E *to the Time* E F, *is as the Space* A B *to the Space* B C. *Protract the Spaces and Times on both ſides, towards* G H *and* I K, *and in* A G *take any number of Spaces equal to* A B,

and in D I *the like number of Times equal to* D E. *Again, in* C H *take any number of Spaces equal to* B C, *and in* F K *take the ſame number of Times equal to the Time* E F. *This done, the Space* B G *will contain juſt as many Spaces equal to* B A, *as the Time* E I *containeth Times equal to* E D, *equimultiplied according to what ever Rate; And likewiſe the Space* B H *will contain as many Spaces equal to* B C, *as the*

the Time KE *containeth Times equal to* FE, *at what ever rate equimultiplied. And forasmuch as* DE *is the Time of the Motion thorow* AB, *the whole Time* EI, *shall be the Time of the whole Space of the Motion thorow* BG, *by reason that the Motion is Equable, and that the number of the Times in* EI *equal to* DE, *is the same with the number of Spaces in* BG, *equal to* BA: *For the same reason* EK *is the Time of the Lation thorow* HB. *Now in regard the Motion is Equable, if the Space* GB *were equal to* HB, *the Time* IE *would be equal to* EK: *and if* GB *be greater than* BH, IE *shall likewise be greater than* EK: *and if lesser, lesser. They are therefore four Magnitudes;* AB *the first,* BC *the second,* DE *the third, and* EF *the Fourth; and the first and third, to wit, the Space* AB, *and Time* DE, *there were taken the Time* IE, *and the Space* GB *equimultiple, according to any multiplication; and it hath been demonstrated that these do at once either equal, or fall short of, or else exceed the Time* EK, *and Space* BH, *which are equimultiple of the second and fourth: Therefore the first hath to the second, to wit the Space* AB *to the Space* BC, *the same proportion that the third hath to the fourth, to wit, the Time* DE *to the Time* EF. *Which was to be demonstrated.*

THEOR. II. PROP. II.

If a Moveable in equal Times passe thorow two Spaces, the said Spaces will be to each other, as the Velocities. And if the Spaces are to each other as the Velocities, the Times will be equal.

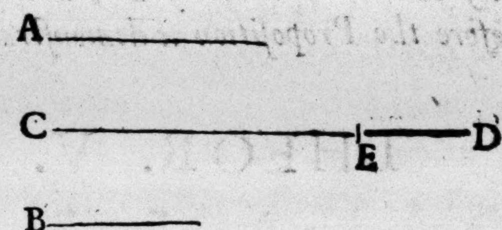
L*Et us suppose* AB *and* BC *in the former Figure, to be two Spaces past, by the Moveable in equal times; the Space* AB *with the Velocity* DE, *and the Space* BC *with the Velocity* EF. *I say, that the Space* AB *is to the Space* BC, *as the Velocity* DE *is to the Velocity* EF: *and thus I prove it. Take as before, of the Spaces and Velocities equi-multiples, accordieg to any what ever Rate, scilicet* GB *and* IE, *of* AB *and* DE, *and likewise* HB *and* KE, *of* BG *and* EF: *It may be concluded as above, that* GB *and* IE *are both at once either equal to, or fall short of, or else exceed the equi-multiples of* DH *and* EK. *Therefore the Proposition is proved.*

THEOR.

THEOR. III. PROP. III.

The Times in which the same Space is past thorow by unequal Velocities, have the same proportion to each other as their Velocities contrarily taken.

Let the two unequal Velocities be A *the greater, and* B *the lesse: and according to both these let a Motion be made thorow the same Space* CD. *I say the Time in which the Velocity* A *passeth the Space* CD, *shall be to the Time in which the Velocity* B *passeth the said Space, as the Velocity* B *to the Velocity* A. *As* A *is to* B, *so let* CD *be to* CE: *Then, by the former Proposition, the Time in which the Velocity* A *passeth* CD, *shall be the same with the Time in which* B *passeth* CE. *But the Time in which the Velocity* B *passeth* CE, *is to the Time in which it passeth* CD, *as* CE *is to* CD: *Therefore the Time in which the Velocity* A *passeth* CD, *is to the Time in which the Velocity* B *passeth the same* CD, *as* CE *is to* CD; *that is, the Velocity* B *is to the Velocity* A: *Which was to be proved.*

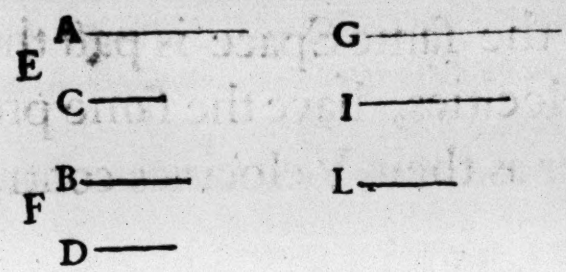


THEOR. IV. PROP. IV.

If two Moveables move with an Equable Motion, but with unequal Velocities, the Spaces which they passe in unequal Times, are to each other in a proportion compounded of the proportion of the Velocities, and of the proportion of the Times.

Let the two Moveables moving with an Equable Motion, be E *and* F: *And let the proportion of the Velocity of the Moveable* E *be to the Velocity of the Moveable* F, *as* A *is to* B: *And let the Time in which* E *is moved, be unto the Time in which* F *is moved, as* C *is to* D. *I say the Space passed by* E, *with the Velocity* A *in the Time* C, *is to the Space passed by* F, *with the Velocity* B *in the Time* D, *in a proportion compounded of the proportion of the Velocity* A *to the Velocity* B; *and of*

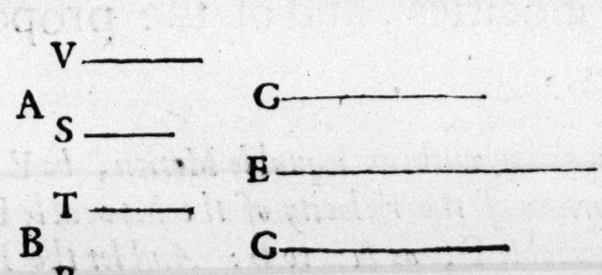
the proportion of the Time C *to the Time* D. *Let the Space passed by the Moveable* E, *with the Velocity* A *in the Time* C, *be* G: *And as the Velocity* A *is to the Velocity* B, *so let* G *be to* I: *And as the Time* C *is to the Time* D, *so let* I *be to* L: *It is manifest, that* I *is the Space passed by* F *in the same Time in which* E *passeth thorow* G; *seeing that the Spaces* G *and* I *are as the Velocities* A *and* B; *and seeing that as the Time* C *is to the Time* D, *so is* I *unto* L; *and since that* I *is the Space passed by the Moveable* F *in the Time* C: *Therefore* L *shall be the Space that* F *passeth in the Time* D, *with the Velocity* B: *But the proportion of* G *to* L, *is compounded of the proportions of* G *to* I, *and of* I *to* L; *that is, of the proportions of the Velocity* A *to the Velocity* B, *and of the Time* C *to the Time* D: *Therefore the Proposition is demonstrated.*



THEOR. V. PROP. V.

If two Moveables move with an Equable Motion, but with unequal Velocities, and if the Spaces passed be also unequal, the Times shall be to each other in a proportion compounded of the proportion of the Spaces, and of the proportion of the Velocities contrarily taken.

L*Et* A *and* B *be the two Moveables, and let the Velocity of* A *be to the Velocity of* B, *as* V *to* T, *and let the Spaces passed, be as* S *to* R. *I say the proportion of the Time in which* A *is moved to the Time in which* B *is moved, shall be compounded of the proportions of the Velocity* T *to the Velocity* V, *and of the Space* S *to the Space* R. *Let* C *be the Time of the Motion* A; *and as the Velocity* T *is to the Velocity* V, *so let the Time* C *be to the Time* E: *And forasmuch as* C *is the Time in which* A *with the Velocity* V *passeth the Space* S; *and that the Time* C *is to the Time* E, *as the Velocity* T *of the Moveable* B *is to the Velocity* V, E *shall be the Time in which the Moveable* B *would passe the*

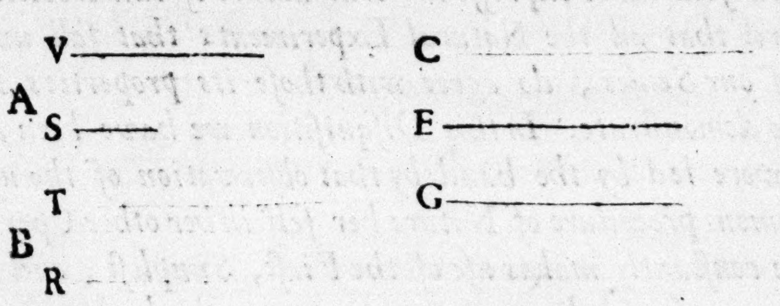


the same Space S. *Again as the Space* S *is to the Space* R, *so let the Time* E *be to the Time* G: *Therefore* G *is the Time in which* B *would passe the Space* R. *And because the proportion of* C *to* G *is compounded of the proportions of* C *to* E. *and of* E *to* G; *And since the proportion of* C *to* E *is the same with that of the Velocities of the Moveables* A *and* B *contrarily taken*; *that is, with that of* T *and* V; *And the proportion of* E *to* G *is the same with the proportion of the Spaces* S *and* R: *Therefore the Proposition is demonstrated.*

THEOR. VI. PROP. VI.

If two Moveables move with an Equable Motion, the proportion of their Velocities shall be compounded of the proportion of the Spaces passed, and of the proportion of the Times contrarily taken.

LEt A *and* B *be the two Moveables moving with an Equable Motion*; *and let the Spaces by them passed, be as* V *to* T; *and let the Times be as* S *to* R. *I say that the proportion of the Velocity of the Moveable* A, *to that of the Velocity of* B, *shall be compounded of the proportions of the Space* V *to the Space* T, *and of the Time* R *to the Time* S. *Let* C *be the Velocity with which the Moveable* A *passeth the Space* V *in the Time* S: *And let the Velocity* C *be to the Velocity* E, *as the Space* V *is to the Space* T; *And* E *shall be the Velocity with which the Moveable*



B *passeth the Space* T *in the Time* S: *Again, let the Velocity* E *be to the other Velocity* G, *as the Time* R *is to the Time* S; *And* G *shall be the Velocity with which the Moveable* B *passeth the Space* T *in the Time* R. *We have therefore the Velocity* C, *wherewith the Moveable* A *passeth the Space* V *in the Time* S; *and the Velocity* G, *wherewith the Moveable* B *passeth the Space* T *in the Time* R: *And the proportion of* C *to* G *is compounded of the proportions of* C *to* E *and of* E *to* G: *But the proportion of* C *to* E, *is supposed the same with that of the Space* V *to the Space* T; *and the proportion of* E *to* G, *is the same with that of* R *to* S: *Therefore the Proposition is manifest.*

* That is the Academick, *i. e.* *Galileus*.

SALV. This that we have read, is what our * *Author* hath written of the Equable Motion. We will pass therefore to a more subtil and new Contemplation touching the Motion Naturally Accelerate: and behold here the Title and Introduction.

OF MOTION NATVRALLY ACCELERATE.

IN the former Book we have considered the Accidents which accompany Equable Motion; we are now to treat of another kind of Motion which we call Accelerate. And first it will be expedient to find out and explain a Definition best agreeing to that which Nature makes use of. For though it be not nconvenient to feign a Motion at pleasure, and then to consider the Accidents that attend it (as those have done, who having framed in their imagination Helixes and Conchoides, which are Lines arising from certain Motions, although not used by Nature, and upon that Supposition have laudably demonstrated the Symptomes thereof) yet in regard that Nature maketh use of a certain kind of Acceleration in the descent of Grave Bodies, we are resolved to search out and contemplate the passions thereof, and see whether the Definition that we are about to produce of this our Accelerate Motion, doth aptly and congruously sute with the Essence of Motion Naturally Accelerate. After many long and laborious Studies we have found out a Definition which seemeth to expresse the true nature of this Accelerate Motion, in regard that all the Natural Experiments that fall under the Observation of our Senses, do agree with those its properties that we intend anon to demonstrate. In this Disquisition we have been assisted, and as it were led by the hand by that observation of the usual Method and common procedure of Nature her self in her other Operations, wherein she constantly makes use of the First, Simplest, and Easiest Means that are: for I believe that no man can think that Swimming or flying can be performed in a more simple or easie way, than that which Fishes and Birds do use out of a Natural Instinct. Why therefore shall not I be perswaded, that, when I see a Stone to acquire continually new additions of Velocity in its descending from its Rest out of some high place, this encrease made in the simplest easiest and most obvious manner that we can imagine? Now if we seriously examine all the ways that can be devised, we shall find no encreases, no acquisitions lesse intricate or more intelligible than that which ever encreaseth or makes its additions after the same manner. This appeareth by the great Affinity that is between Time and Motion. For as the Equability or Uniformity of Motion is defined and expressed by the Equability of the

Times

Times and Spaces, (for we call that Motion or Lation Equable, by which equal Spaces are past in equal Times) so by the same Equability of the parts of Time, we may perceive, that the encrease of Celerity in the Natural Motion of Grave Bodies, is made after a Simple and plain manner; conceiving in our Mind that their Motion is continually accelerated uniformly and at the same Rate, whilst equal additions of Celerity are conferred upon them in all equal Times. So that taking any equal particles of Time beginning from the first Instant in which the Moveable departeth from Rest, and entereth upon its Descent, the Degree of Velocity acquired in the first and second Particles of Time, is double the degree of Velocity that the Moveable acquired in the first Particle: and the degree of Velocity that it acquireth in three Particles, is triple, and that in four quadruple to the same Degree of the first Time: As, for our better understanding, if a Moveable should continue its Motion according to the degree or moment of Velocity acquired in the first Particle of Time, and should extend its course equably with that same Degree; this Motion would be twice as slow as that which it would obtain according to the degree of Velocity acquired in two Particles of Time: So that it will not be improper if we understand the Intention of the Velocity, to proceed according to the Extension of the Time. From whence we may frame this Definition of the Motion of which we are about to treat.

DEFINITION.

Motion Accelerate in an Equable or Uniform Proportion, I call that which departing from Rest, superaddeth equal moments of Velocity in equal Times.

SAGR. Though it were Irrational for me to oppose this or any other Definition assigned by any whatsoever Author, they being all Arbitrary, yet I may very well, without any offence, question whether this Definition, which is understood and admitted in Abstract, doth sute, agree, and hold true in that sort of Accelerate Motion, which Grave Bodies descending naturally do exercise. And because the Authour seemeth to promise us, that the Natural Motion of Grave Bodies is such as he hath defined it, I could wish that some Scruples were removed that trouble my mind; that so I might apply my self afterwards with greater attention to the Proportions and Demonstrations which are expected.

SALV. I like well, that you and *Simplicius* do propound Doubts as they come in the way: which I do imagine will be the same

ſame that I my ſelf did meet with when I firſt read this Treatiſe, and that, either were reſolved by conferring with the Author, or removed by my own conſidering of them.

SAGR. Whilſt I am fancying to my ſelf a Grave Deſcending Moveable to depart from Reſt, that is from the privation of all Velocity, and to enter into Motion, and in that to go encreaſing, according to the proportion after which the Time encreaſeth from the firſt inſtant of the Motion; and to have *v. gr.* in eight Pulſations, acquired eight degrees of Velocity, of which in the fourth Pulſation it had gained four, in the ſecond two, in the firſt one, Time being ſubdiviſible *in infinitum*, it followeth, that the Antecedent Velocity alwayes diminiſhing at that Rate, there will be no degree of Velocity ſo ſmall, or, if you will, of Tardity ſo great, in which the ſaid Moveable is not found to be conſtituted, after its departure from infinite Tardity, that is, from Reſt. So that if that degree of Velocity which it had at four Pulſations of Time, was ſuch, that maintaining it Equable, it would have run two Miles in an hour, and with the degree of Velocity that it had in the ſecond Pulſation, it would have gone one mile an hour, it muſt be granted, that in the Inſtants of Time neerer and neerer to its firſt Inſtant of moving from Reſt, it is ſo ſlow, as that (continuing to move with that Tardity) it would not have paſſed a Mile in an hour, nor in a day, nor in a year, nor in a thouſand; nay, nor have gone one ſole foot in a greater time? An accident to which me thinks the Imagination but very uneaſily accords, ſeeing that Senſe ſheweth us, that a Grave Falling Body commeth down ſuddenly, and with great Velocity.

SALV. This is one of thoſe Doubts that alſo fell in my way upon my firſt thinking on this affair, but not long after I removed it: and that removal was the effect of the ſelf ſame Experiment which at preſent ſtarts it to you. You ſay, that in your opinion, Experience ſheweth that the Moveable hath no ſooner departed from Reſt, but it entereth into a very notable Velocity: and I ſay, that this very Experiment proves it to us; that the firſt Impetus's of the Cadent Body, although it be very heavy, are moſt ſlack and ſlow. Lay a Grave Body upon ſome yielding matter, and let it continue upon it till it hath preſſed into it as far as it can with its ſimple Gravity; it is manifeſt, that raiſing it a yard or two, and then letting it fall upon the ſame matter, it ſhall with its percuſſion make a new preſſure, and greater than that made at firſt by its meer weight: and the effect ſhall be cauſed by the falling Moveable conjoyned with the Velocity acquired in the Fall: which impreſſion ſhall be greater and greater, according as the Percuſſion ſhall come from a greater height; that is, according as the Velocity of the Percutient ſhall be greater. We

may

may therefore without mistake conjecture the quantity of the Velocity of a falling heavy Body; by the quality and quantity of the Percussion. But tell me Sirs, that Beetle which being let fall upon a Stake from an height of four yards, driveth it into the ground, *v. gr.* four inches, comming from an height of two yards, shall drive it much lesse, and lesse from an height of one, and lesse from a foot; and lastly lifting it up an inch, what will it do more than if without any blow it were laid upon it? Certainly but very little, and the operation would be wholly imperceptible, if it were raised the thicknesse of a leaf. And because the effect of the Percussion is regulated by the Velocity of the Percutient, who will question but that the Motion is very slow, and the Velocity extreme small, where its operation is imperceptible? See now of what power Truth is, since the same Experiment that seemed at the first blush to hold forth one thing, being better considered, ascertains us of the contrary. But without having recourse to that Experiment (which without doubt is most perswasive) me-thinks that it is not hard to penetrate such a Truth as this by meer Discourse. We have an heavy stone sustained in the Air at Rest: let it be disengaged from its upholder, and set at liberty; and, as being more grave than the Air, it goeth descending downwards, and that not with a Motion Equable, but slow in the beginning, and continually afterwards accelerate: and seeing that the Velocity is Augmentable and Diminishable *in infinitum*, what Reason shall perswade me, that that Moveable departing from an infinite Tardity (for such is Rest) entereth immediately into ten degrees of Velocity, rather than in one of four, or in this more than in one of two, of one, of half one, or of the hundredth part of one; and to be short, in all the infinite lesser? Pray you hear me. I do not think that you would scruple to grant me, that the acquist of the Degrees of Velocity of the falling Stone may be made with the same Order as is the Diminution and losse of the same degrees, when with an impellent Virtue it is driven upwards to the same height: But if that be so, I do not see how it can be supposed that in the diminution of the Velocity of the ascendent Stone, spending it all, it can come to the state of Rest before it hath passed thorow all the degrees of Tardity.

SIMP. But if the greater and greater degrees of Tardity are infinite, it shall never spend them all; so that the ascendent Grave will never attain to Rest, but will move *ad infinitum*, still retarding: a thing which we see not to happen.

SALV. This would happen, *Simplicius*, in case the Moveable should stay for some time in each degree: but it passeth thorow them, without staying longer than an instant in any of them.

And

And becauſe in every quantitative Time, though never ſo ſmall, there are infinite Inſtants, therefore they are ſufficient to anſwer to the infinite degrees of Velocity diminiſhed. And that the aſcendent Grave Body perſiſts not for any quantitative Time in one and the ſame degree of Velocity, may thus be made out: Becauſe, a certain quantitative Time being aſſigned it in the firſt inſtant of that Time, and likewiſe in the laſt, the Moveable ſhould be found to have one and the ſame degree of Velocity, it might by this ſecond degree be likewiſe driven upwards ſuch another Space, like as from the firſt it was tranſported to the ſecond; and by the ſame reaſon it would paſſe from the ſecond to the third, and, in ſhort, would continue its Motion Uniform *ad infinitum*.

SAGR. From this Diſcourſe, as I conceive, one might derive a very apposite Reaſon of the Queſtion controverted amongſt Philoſophers, Touching what ſhould be the Cauſe of the acceleration of the Natural Motion of Grave Moveables. For when I conſider in the Grave Body driven upwards, its continual Diminution of that Virtue impreſſed upon it by the Projicient, which ſo long as it was ſuperiour to that other contrary one of Gravity, forced it upwards, this and that being come to an *Equilibrium*, the Moveable ceaſeth to riſe any higher, and paſſeth thorow the ſtate of Reſt, in which the *Impetus* impreſſed is not annihilated, but onely that exceſſe is ſpent, which it before had above the Gravity of the Moveable, whereby prevailing over the ſame, it did drive it upwards. And the Diminution of this forrein *Impetus* continuing, and conſequently the advantage beginning to be on the part of the Gravity, the Deſcent alſo beginneth but ſlow, in regard of the oppoſition of the Virtue impreſſed, a conſiderable part of which ſtill remaineth in the Moveable: but becauſe it doth go continually diminiſhing, and is ſtill with a greater and greater proportion overcome by the Gravity, hence ariſeth the continual Acceleration of the Motion.

SIMP. The conceit is witty, but more ſubtil than ſolid: for in caſe it were concludent, it ſalveth onely thoſe Natural Motions to which a Violent Motion preceded, in which part of the extern Virtue ſtill remains in force: but where there is no ſuch remaining impulſe, as where the Moveable departeth from a long Quieſcence, the ſtrength of your whole Diſcourſe vaniſheth.

SAGR. I believe that you are in an Errour, and that this Diſtinction of Caſes which you make, is needleſſe, or, to ſay better, *Null*. Therefore tell me, whether may there be impreſſed on the Project by the Projicient ſometimes much, and ſometimes little Vertue; ſo as that it may be ſtricken upwards an hundred yards, and alſo twenty, or four, or one?

SIMP.

SIMP. No doubt but there may.

SAGR. And no lesse possible is it, that the said Virtue impressed shall so little seperate the Resistance of the Gravity, as not to raise the Project above an inch: and finally the Virtue of the Projicient may be onely so much, as just to equalize and compensate the Resistance of the Gravity, so as that the Moveable is not driven upwards, but onely sustained. So that when you hold a Stone in your hand, what else do you, but impresse on it so much Virtue impelling upwards, as is the faculty of its Gravity drawing downwards? And this your Virtue, do you not continue to keep it impressed on the Stone all the time that you hold it in your hand? What say you, is it diminished by your long holding it? And this sustention which impedeth the Stones descent, what doth it import, whether it be made by your hand, or by a Table, or by a Rope, that suspends it? Doubtlesse nothing at all. Conclude with your self therefore, *Simplicius*, that the precedence of a long, a short, or a Momentary Rest to the Fall of the Stone, makes no alteration at all, so that the Stone should not alwaies depart affected with so much Virtue contrary to Gravity, as did exactly suffice to have kept it in Rest.

SALV. I do not think it a seasonable time at present to enter upon the Disquisition of the Cause of the Acceleration of Natural Motion: touching which sundry Philosophers have produced sundry opinions: some reducing it to the approximation unto the Center others to the lesse parts of the *Medium* successively remaining to be perforated; others to a certain Extrusion of the Ambient *Medium*, which in reuniting upon the back of the Moveable, goeth driving and continually thrusting it; which Fancies, and others of the like nature, it would be necessary to examine, and with small benefit to answer. It serveth our Authours turn at the present, that we understand that he will declare and demonstrate to us some Passions of an Accelerate Motion (be the Cause of its Acceleration what it will) so as that the Moments of its Velocity do go encreasing, after its departure from Rest with that most simple proportion wherewith the Continuation of the Time doth encrease: which is as much as to say, that in equal Times there are made equal additaments of Velocity. And if it shall be found, that the Accidents that shall hereafter be demonstrated, do hold true in the Motion of Naturally Descendent and Accelerate Grave Moveables, we may account, that the assumed Definition taketh in that Motion of Grave Bodies, and that it is true, that their Acceleration doth encrease according as the Time and Duration of the Motion encreaseth.

SAGR. By what as yet is set before my Intellectuals, it appears to me that one might with (haply) more plainnesse define, and yet

never alter the Conceit; ſaying that, A Motion uniformly accelerate is that in which the Velocity goeth encreaſing according as the Space encreaſeth that is paſſed thorow: So that, for example, the degree of Velocity acquired by the Moveable in a deſcent of four yards ſhould be double to that that it would have after it had deſcended a Space of two, and this double to that acquired in the Space of the firſt Yard. For I do not think that it can be doubted, but that that Grave Moveable which falleth from an height of ſix yards hath, and percuſſeth with an *Impetus* double to that which it had when it had deſcended three yards, and triple to that which it had at two, and ſextuple to that had in the Space of one.

SALV. I comfort my ſelf in that I have had ſuch a Companion in my Errour: and I will tell you farther, that your Diſcourſe hath ſo much of likelihood and probability in it, that our Author himſelf did not deny unto me, when I propoſed it to him, that he likewiſe had been for ſome time in the ſame miſtake. But that which I afterwards extreamly wondred at, was to ſee in four plain words, diſcovered, not only the falſity, but impoſſibility of two Propoſitions that carry with them ſo much of ſeeming truth, that having propounded them to many, I never met with any one but did freely admit them to be ſo.

SIMP. Certainly I ſhould be of the number, and that the Deſcendent Grave Moveable *vires acquirat eundo*, encreaſing its Velocity at the rate of the Space, and that the Moment of the ſame Percutient is double, coming from a double height, ſeem to me Propoſitions to be granted without any hæſitation or controverſie.

SALV. And yet they are as falſe and impoſſible, as that Motion is made in an inſtant. And hear a clear proof of the ſame. In caſe the Velocities have the ſame proportion as the Spaces paſſed, or to be paſſed, thoſe Spaces ſhall be paſſed in equal Times: if therefore the Velocities with which the falling Moveable paſſeth the Space of four yards, were double to the Velocities with which it paſſeth the two firſt yards (like as the Space is double to the Space) then the Times of thoſe Tranſitions are equal: but the ſame Moveable's paſſing the four yards, and the two in one and the ſame Time, hath place only in Inſtantaneous Motion. But we ſee, that the falling grave Body maketh its Motion in Time, and paſſeth the two yards in a leſſer than it doth the four. Therefore it is falſe that its Velocity encreaſeth as its Space. The other Propoſition is demonſtrated to be falſe with the ſame perſpicuity. For that which percuſſeth being the ſame, the difference and Moment of the Percuſſion cannot be determined but by the difference of Velocity; If therefore the percutient, coming from a double height, make a Percuſſion with a double Moment, it is neceſſary that it ſtrike with a double Velocity: But the double Velocity paſſeth the double Space in the

the same Time; and we see the Time of the Descent from the greater altitude to be longer.

Sagr. This is too great an Evidence, too great a Facility wherewith you manifest abstruce Conclusions: this extream easiness rendreth them of lesse value than they were whilst they lay hid under contrary appearances. I believe that the Generality of men little presse those Notions which are easily obtained, in comparison of those about which men make so long and inexplicable altercations.

Salv. To those which with great brevity and clarity shew the fallacies of Propositions that have been commonly received for true by the generality of people, it would be a very tolerable injury to return them only slighting instead of thanks: but there is much displeasure and molestation in another certain affection sometimes found in some men, that pretending in the same Studies at least Parity with any whomsoever, do see that they have let pass such and such for true Conclusions, which afterwards by another, with a short and easie disquisition, have been detected and convicted for false. I will not call that affection Envy, that is accustomed to convert in time to hatred and despite against the discoverers of such Fallacies, but I will call it an itch, and a desire to be able rather to maintain their inveterate Errours, than to permit the reception of new-discovered Truths. Which humour sometimes induceth them to write in contradiction of those truths which are but too perfectly known unto themselves only to keep the Reputation of others low in the opinion of the numerous and ill-informed Vulgar. Of such false Conclusions received for true, and very easie to be confuted, I have heard no small number from our *Academick*, of some of which I have kept account.

Sagr. And you must not deprive us of them; but in due time impart them to us, when a particular Meeting shall be appointed for them. For the present, continuing the discourse we are about, I think that by this time we have established the Definition of Motion uniformly Accelerate, treated of in the ensuing discourses, and it is this;

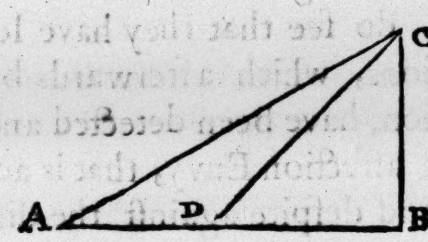
A Motion Equable, or Uniformly Accelerate, we call that which departing from Rest superadds equal Moments of Velocity in equal Times.

Salv. That Definition being confirmed, the Author asketh and supposeth but one only Principle to be true, namely:

SVPPOSITION.

I suppose that the degrees of Velocity acquired by the same Moveable upon Planes of different inclinations are equal then, when the Elevations of the said Planes are equal.

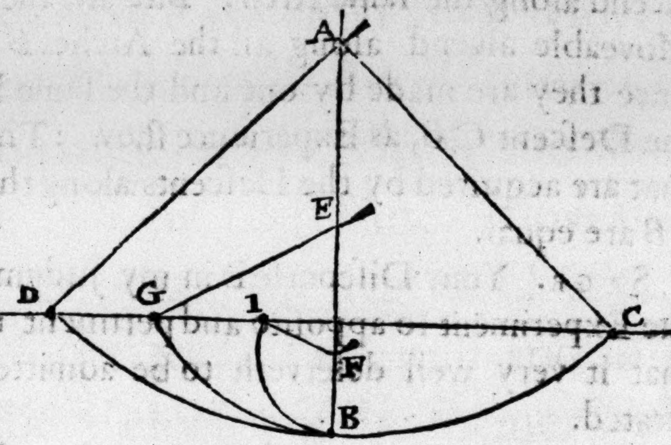
BY the Elevation of an inclined Plane he meaneth the Perpendicular, which from the higher term of the said Plane, falleth upon the Horizontal Line produced along by the lower term of the said Plane inclined; as for better understanding, the Line A B being parallel to the Horizon, upon which let the two Planes C A, and C D be inclined: the Perpendicular C B falling upon the Horizontal Line B A the Author calleth the Elevation of the Planes C A and C D; and supposeth that the degrees of Velocity of the same Moveable descending along the inclined Planes C A and C D, acquired in the Terms A and D are equal, for that their Elevation is the same C B. And so great also ought the degree of Velocity be understood to be which the same Moveable falling from the Point C would acquire in the term B.



SAGR. The truth is, this Supposition hath in it so much of probability, that it deserveth to be granted without dispute, alwaies presupposing that all accidental and extern Impediments are removed, and that the Planes be very Solid and Terse, and the Moveable in Figure most perfectly Rotund, so that neither the Plane, nor the Moveable have any unevenness. All Contrasts and Impediments, I say, being removed, the light of Nature dictates to me without any difficulty, that a Ball heavy and perfectly round descending by the Lines C A, C D, and C B would come to the terms A D, and B with equal *Impetus's*.

SALV. You argue very probably; but over and above the probability, I will by an Experiment so increase the likelihood, as that it wants but little of being equal to a very necessary Demonstration. Imagine this leafe of Paper to be a Wall erect at Right-angles to the Horizon, and at a Nail, fastned in the same, hang a Ball or Plummet of Lead, weighing an ounce or two, suspended by the small thread A B, two or three yards long, perpendicular to the Horizon: and on the Wall draw an Horizontal Line D C, cutting the

the Perpendicular A B at Right angles, which A B must hang two Inches, or thereabouts, from the Wall: Then transferring the string A B with the Ball into C, let go the said Ball; which you will see first to descend describing C B D, and to pass so far beyond the Term B, that running along the Arch B D it will rise almost as high as the designed Parallel C D, wanting but a very small matter of reaching to it, the precise arrival thither being denied it by the Impediment of the Air, and of the Thread.



From which we may truly conclude, that the *Impetus* acquired in the point B by the Ball in its descent along the Arch C B, was so much as sufficed to carry it upwards along such another Arch B D unto the same height: having made, and often reiterated this Experiment, let us drive into the Wall, along which the Perpendicular A B passeth, another Nail, as in E or in F, which is to stand out five or six Inches; and this to the end that the thread A B, returning as before to carry back the Ball C along the Arch C B, when it is come to B, the Thread stopping at the Nail E may be constrained to move along the Circumference B G, described about the Center E: by which we shall see what that same *Impetus* is able to do, which before, being conceived in the same term B, carried the same Moveable along the Arch B D unto the height of the Horizontal Line C D. Now, Sirs, you shall with delight see the Ball carried unto the Horizontal Line in the Point G; and the same will happen if the stop be placed lower, as in F, where the Ball would describe the Arch B I, evermore terminating its ascent exactly in the Line C D: and in case the Check were so low that the overplus of the thread beneath it cannot reach to the height of C D, (which would happen if it were nearer to the point B than to the intersection of A B with the Horizontal Line C D) then the thread would whirle and twine about the Nail. This experiment leaveth no place for our doubting of the truth of the Supposition: for the two Arches C B and D B being equall, and scituate alike, the acquist of Moment made along the Descent in the Arch C B, is the same with that made along the Descent in the Arch D B. But the Moment acquired in *B*, along the Arch C *B*, is able to carry the same Moveable upwards along the Arch *B* D: Therefore the Moment acquired in the Descent D *B* is equall to that which driveth

the

the ſame Moveable along the ſame Arch from *B* to D: So that generally every Moment acquired along the Deſcent of an Arch is equall to that which hath power to make the ſame Moveable reaſcend along the ſame Arch: *B*ut all the Moments that make the Moveable aſcend along all the Arches *B* D, *B* G, *B* I are equal, ſince they are made by one and the ſame Moment acquired along the Deſcent C *B*, as Experience ſhews: Therefore all the Moments that are acquired by the Deſcents along the Arches D *B*, G *B*, and I *B* are equal.

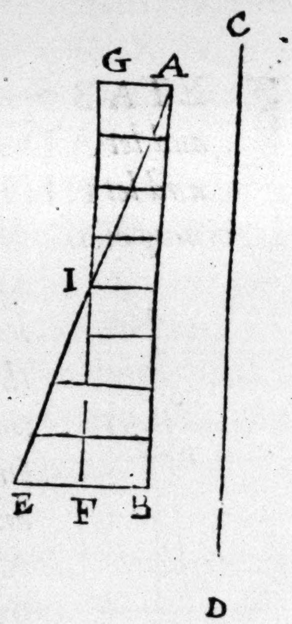
SAGR. Your Diſcourſe is in my Judgment very Rational, and the Experiment ſo appoſite and pertinent to verifie the *Poſtulatum*, that it very well deſerveth to be admitted as if it were Demonſtrated.

SALV. I will not conſent, *Sagredus*, that we take more to our ſelves than we ought; and the rather for that we are chiefly to make uſe of this Aſſumption in Motions made upon ſtreight and not curved Superficies; in which the Acceleration proceedeth with degrees very different from thoſe wherewith we ſuppoſe it to proceed in ſtreight Planes. Inſomuch, that although the Experiment alledged ſhews us, that the deſcent along the Arch C *B* conferreth on the Moveable ſuch a Moment, as that it is able to re-carry it to the ſame height along any other Arch *B* C, *B* G, and *B* I, yet we cannot with the like evidence ſhew, that the ſame would happen in caſe a moſt exact *B*all were to deſcend by ſtreight Planes inclined according to the inclinations of the Chords of theſe ſame Arches: yea, it is credible, that Angles being formed by the ſaid Right Planes in the term *B*, the *B*all deſcended along the Declivity according to the Chord C *B*, finding a ſtop in the Planes aſcending according to the Chords *B* D, *B* G, and *B* I, in juſtling againſt them, would loſe of its *Impetus*, and could not be able in riſing to attain the height of the Line C D. *B*ut the Obſtacle being removed, which prejudiceth the Experiment, I do believe, that the underſtanding may conceive, that the *Impetus* (which in effect deriveth vigour from the quantity of the Deſcent) would be able to remount the Moveable to the ſame height. Let us therefore take this at preſent for a *Poſtulatum* or Petition, the abſolute truth of which will come to be eſtabliſhed hereafter by ſeeing other Concluſions raiſed upon this Hypotheſis to anſwer, and exactly jump with the Experiment. The Author having ſuppoſed this only Principle, he paſſeth to the Propoſitions, demonſtratively proving them; of which the firſt is this;

THEOR. I. PROP. I.

The time in which a Space is passed by a Moveable with a Motion Uniformly Accelerate, out of Rest, is equal to the Time in which the same Space would be past by the same Moveable with an Equable Motion, the degree of whose Velocity is subduple to the greatest and ultimate degree of the Velocity of the former Uniformly Accelerate Motion.

LET us by the extension AB *represent the Time, in which the Space* CD *is passed by a Moveable with a Motion Uniformly Accelerate, out of Rest in* C: *and let the greatest and last degree of Velocity acquired in the Instants of the Time* AB *be represented by* EB; *and constitute at pleasure upon* AB *any number of parts, and thorow the points of division draw as many Lines, continued out unto the Line* AE, *and equidistant to* BE, *which will represent the encrease of the degrees of Velocity after the first Instant* A. *Then divide* BE *into two equall parts in* F, *and draw* FG *and* AG *parallel to* BA *and* BF: *The Parallelogram* AGFB *shall be equall to the Triangle* AEB, *its Side* GF *dividing* AE *into two equall parts in* I: *For if the Parallels of the Triangle* AEB *be continued out unto* IGF, *we shall have the Aggregate of all the Parallels contained in the Quadrilatural Figure equal to the Aggregate of all the Parallels comprehended in the Triangle* AEB; *For those in the Triangle* IEF *are equal to those contained in the Triangle* GIA, *and those that are in the* Trapezium *are in common. Now since all and singular the Instants of Time do answer to all and singular the Points of the Line* AB; *and since the Parallels contained in the Triangle* AEB *do represent the degrees of Acceleration or encreasing Velocity, and the Parallels contained in the Parallelogram do likewise represent as many degrees of Equable Motion or unencreasing Velocity: It appeareth, that as many Moments of Velocity passed in the Accelerate Motion according to the encreasing Parallels of the Triangle* AEB, *as in the Equable Motion according to the Parallels of the Parallelogram* GB: *Because what is wanting in the first half of the*



Accelerate

Accelerate Motion of the Velocity of the Equable Motion (which deficient Moments are represented by the Parallels of the Triangle AGI*) is made up by the moments represented by the Parallels of the Triangle* IEF. *It is manifest, therefore, that those Spaces are equal which are in the same Time by two Moveables, one whereof is moved with a Motion uniformly Accelerated from Rest, the other with a Motion Equable according to the Moment subduple of that of the greatest Velocity of the Accelerated Motion: Which was to be demonstrated.*

THEOR. II. PROP. II.

If a Moveable descend out of Rest with a Motion uniformly Accelerate, the Spaces which it passeth in any whatsoever Times are to each other in a proportion Duplicate of the same Times; that is, they are as the Squares of them.

LET AB *represent a length of Time beginning at the first Instant* A; *and let* AD *and* AE *represent any two parts of the said Time; and let* HI *be a Line in which the Moveable out of* H, *as the first beginning of the Motion descendeth uniformly accelerating; and let the Space* HL *be passed in the first Time* AD; *and let* HM *be the Space that it shall descend in the Time* AE. *I say, the Space* MH *is to the Space* HL *in duplicate proportion of that which the Time* EA *hath to the Time* AD: *Or, if you will, that the Spaces* MH *and* HL *are to one another in the same proportion as the Squares* EA *and* AD. *Draw the Line* AC *at any Angle with* AB, *and from the points* D *and* E *draw the Parallels* DO *and* PE: *of which* DO *will represent the greatest degree of Velocity acquired in the Instant* D *of the Time* AD; *and* P *the greatest degree of Velocity acquired in the Instant* E *of the Time* AE. *And because we have demonstrated in the last Proposition concerning Spaces, that those are equal to one another, of which two Moveables have past in the same Time, the one by a Moveable out of Rest with a Motion uniformly Accelerate, and the other by the same Moveable with an Equable Motion, whose Velocity is subduple to the greatest acquired by the Accelerate Motion: Therefore* MH *and* HL *are the Spaces that two Equable Motions, whose Velocities should be as the half of* PE, *and half*

half of OD, *would passe in the Times* EA *and* DA. *If it be proved therefore that these Spaces* MH *and* LH *are in duplicate proportion to the Times* EA *and* DA; *We shall have done that which was intended. But in the fourth Proposition of the First Book we have demonstrated: That the Spaces past by two Moveables with an Equable Motion are to each other in a proportion compounded of the proportion of the Velocities and of the proportion of the Times: But in this case the proportion of the Velocities and the proportion of the Times is the same (for as the half of* PE *is to the half of* OD, *or the whole* PE *to the whole* OD, *so is* AE *to* AD: *Therefore the proportion of the Spaces passed is double to the proportion of the Times. Which was to be demonstrated.*

Hence likewise it is manifest, that the proportion of the same Spaces is double to the proportions of the greatest degrees of Velocity: that is, of the Lines PE *and* OD: *because* PE *is to* OD, *as* EA *to* DA.

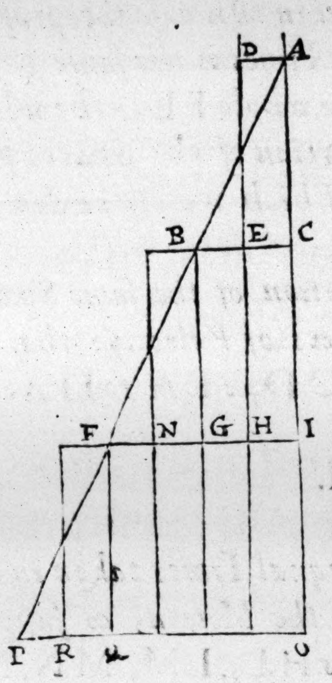
COROLARY. I.

Hence it is manifest, that if there were many equal Times taken in order from the first Instant or beginning of the Motion, as suppose AD, DE, EF, FG, *in which the Spaces* HL, LM, MN, NI *are passed, those Spaces shall be to one another as the odd numbers from an Unite:* scilicet, *as* 1, 3, 5, 7. *For this is the Rate or proportion of the excesses of the Squares of Lines that equally exceed one another, and the excesse of which is equal to the least of them, or, if you will, of Squares that follow one another, beginning* ab Unitate. *Whilst therefore the degree of Velocity is encreased according to the simple Series of Numbers in equal Times, the Spaces past in those Times make their encrease according to the Series of odd Numbers from an Unite.*

SAGR. Be pleased to stay your Reading, whilst I do paraphrase touching a certain Conjecture that came into my mind but even now; for the explanation of which, unto your understanding and my own, I will describe a short Scheme: in which I fansie by the Line AI the continuation of the Time after the first Instant, applying the Right Line AF unto A according to any Angle: and joyning together the Terms IF, I divide the Time AI in half at C, and then draw CB parallel to IF. *A*nd then considering BC, as the greatest degree of Velocity which beginning from Rest in the first Instant of the Time *A* goeth augmenting according to the encrease of the Parallels to BC, drawn in the Triangle *A*BC, (which is all one as to encrease according to the encrease of the Time) I admit without dispute, upon what hath been said already, That the Space past by the falling Moveable with the

Velocity encreased in the manner aforesaid would be equal to the Space that the said Moveable would passe, in case it were in the same Time *A* C, moved with an Uniform Motion, whose degree of Velocity should be equal to E C, the half of B C. I now proceed farther, and imagine the Moveable; having descended with an *A*ccelerate Motion, to have in the Instant C the degree of Velocity B C: It is manifest, that if it did continue to move with the same degree of Velocity B C, without farther *A*cceleration, it would passe in the following Time C I, a Space double to that which it passed in the equal Time *A* C, with the degree of Uniform Velocity E C, the half of the Degree B C. But because the Moveable descendeth with a Velocity encreased alwaies Uniformly in all equal Times; it will add to the degree C B in the following Time C I, those same Moments of Velocity that encrease according to the Parallels of the Triangle B F G, equal to the Triangle *A* B C. So that adding to the degree of Velocity G I, the half of the degree F G, the greatest of those acquired in the *A*ccelerate Motion, and regulated by the Parallels of the Triangle B F G, we shall have the degree of Velocity I N, with which, with an Uniform Motion, it would have moved in the Time C I: Which degree I N, being triple the degree E C, proveth that the Space passed in the second Time C I ought to be triple to that of the first Time C *A*. *A*nd if we should suppose to be added to *A* I another equal part of Time I O, and the Triangle to be enlarged unto *A* P O; it is manifest, that if the Motion should continue for all the Time I O with the degree of Velocity I F, acquired in the *A*ccelerate Motion in the Time *A* I, that degree I F being Quadruple to E C, the Space passed would be Quadruple to that passed in the equal first Time *A* C: But continuing the encrease of the Uniform *A*cceleration in the Triangle F P Q like to that of the Triangle *A* B C, which being reduced to equable Motion addeth the degree equal to E C, Q R being added, equal to E C, we shall have the whole Equable Velocity exercised in the Time I O, quintuple to the Equable Velocity of the first Time *A* C, and therefore the Space passed quintuple to that past in the first Time *A* C. We see therefore, even by this familiar computation, That the Spaces passed in equal Times by a Moveable which departing from Rest goeth acquiring Velocity, according to the encrease of the Time, are to one another as the odd Numbers *ab*

unitate



unitate 1, 3, 5: *And* that the Spaces passed being conjunctly taken, that passed in the double Time is quadruple to that passed in the subduple, that passed in the triple Time is nonuple; and, in a word, that the Spaces passed are in duplicate proportion to their Times; that is, as the Squares of the said Times.

SIMP. I must confesse that I have taken more pleasure in this plain and clear discourse of *Sagredus*, than in the to-me-more obscure Demonstration of the *Author*: so that I am very well satisfied, that the businesse is to succeed as hath been said, the Definition of Uniformly *Accelerate* Motion being supposed, and granted. But whether this be the *Acceleration* of which Nature maketh use in the Motion of its descending Grave Bodies, I yet make a question: and therefore for information of me, and of others like unto me, me thinks it would be seasonable in this place to produce some Experiment amongst those which were said to be many, which in sundry Cases agree with the Conclusions demonstrated.

SALV. You, like a true *Artist*, make a very reasonable demand, and so it is usual and convenient to do in Sciences that apply Mathematical Demonstrations to Physical Conclusions, as we see in the Professors of Perspection, *Astronomy*, Mechanicks, Musick, and others, who with Sensible Experiments confirm those their Principles that are as the foundations of all the following Structure: and therefore I desire that it may not be thought superfluous, that we discourse with some prolixity upon this first and grand fundamental on which we lay the weight of the Immense Machine of infinite Conclusions, of which we have but a very small part set down in this Book by our *Author*, who hath done enough to open the way and door that hath been hitherto shut unto all Speculative Wits. Touching Experiments, therefore, the *Author* hath not omitted to make several; and to assure us, that the *Acceleration* of natural-descending Graves hapneth in the aforesaid proportion, I have many times in his company set my self to make a triall thereof in the following Method.

In a prisme or Piece of Wood, about twelve yards long, and half a yard broad one way, and three Inches the other, we made, upon the narrow Side or edge a Groove of little more than an Inch wide; we shot it with the Grooving Plane very straight, and to make it very smooth and sleek, we glued upon it a piece of Vellum, polished and smoothed as exactly as can be possible: and in it we have let a brazen Ball, very hard, round, and smooth, descend. Having placed the said Prisme Pendent, raising one of its ends above the Horizontal Plane a yard or two at pleasure, we have let the Ball (as I said) descend along the Grove, observing, in the manner that I shall tell you presently, the Time which it spent in running

runing it all; repeating the same observation again and again to assure our selves of the Time, in which we never found any difference, no not so much as the tenth part of one beat of the Pulse. Having done, and precisely ordered this businesse, we made the same Ball to descend only the fourth part of the length of that Grove: and having measured the time of its descent, we alwaies found it to be punctually half the other. And then making trial of other parts, examining one while the Time of the whole Length with the Time of half the Length, or with that of $\frac{2}{3}$, or of $\frac{3}{4}$, or, in brief, with any whatever other Division, by Experiments repeated near an hundred Times, we alwaies found the Spaces to be to one another as the Squares of the Times. And this in all Inclinations of the Plane, that is, of the Grove in which the Ball was made to descend. In which we observed moreover, that the Times of the Descents along sundry Inclinations did retain the same proportion to one another, exactly, which anon you will see assigned to them, and demonstrated by the Author. And as to the measuring of the Time; we had a good big Bucket full of Water hanged on high, which by a very small hole, pierced in the bottom, spirted, or, as we say, spin'd forth a small thread of Water, which we received with a small cup all the while that the Ball was descending in the Grove, and in its parts; and then weighing from time to time the small parcels of Water, in that manner gathered, in an exact pair of scales, the differences and proportions of their Weights gave justly the differences and proportions of the Times; and this with such exactnesse, that, as I said before, the trials being many and many times repeated, they never differed any considerable matter.

SIMP. I should have received great satisfaction by being present at those Experiments: but being confident of your diligence in making them, and veracity in relating them, I content my self, and admit them for true and certain.

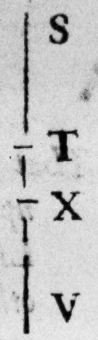
SALV. We may, then, reassume our Reading, and go on.

COROLLARY II.

It is collected in the second place, that if any two Spaces are taken from the beginning of the Motion, passed in any Times, those Times shall be unto each other as one of them is to a Space that is the Mean proportional between them.

FOR taking two Spaces ST, *and* SV *from the beginning of the Motion* S, *to which* SX *is a Mean-proportional, the Time of the descent along* ST, *shall be to the Time of the descent along* SV, *as* ST *to* SX; *or, if you will, the Time along* SV *shall be to the Time along* ST, *as*

as VS *is to* SX. *For it is demonstrated, that Spaces passed are in duplicate proportion to the Times, or,(which is the same) are as the Squares of the Times : But the proportion of the Space* VS *to the Space* ST *is double to the proportion of* VS *to* SX, *or is the same that* VS, *and* SX *squared have to one another : Therefore, the proportion of the Times of the Motion by* VS, *and* ST, *is as the Spaces or Lines* VS *to* SX.



SCHOLIUM.

That which is demonstrated in Motions *that are made Perpendicularly, may be understood also to hold true in the* Motions *made along Planes of any whatever Inclination; for it is supposed, that in them the degree of Acceleration encreaseth in the same proportion; that is, according to the encrease of the Time; or, if you will, according to the simple and primary Series of Numbers.*

SALV. Here I desire *Sagredus*, that I also may be allowed, albeit perhaps with too much tediousnesse in the opinion of *Simplicius*, to defer for a little time the present Reading, untill I may have explained what from that which hath been already said and demonstrated, and also from the knowledge of certain Mechanical Conclusions heretofore learnt of our *Academick*, I now remember to adjoyn for the greater confirmation of the truth of the Principle, which hath been examined by us even now with probable Reasons and Experiments: and, which is of more importance, for the Geometrical proof of it, let me first demonstrate one sole Elemental *Lemma* in the Contemplation of *Impetus's*.

SAGR. If our advantage shall be such as you promise us, there is no time that I would not most willingly spend in discoursing about the confirmation and thorow establishing these Sciences of Motion: and as to my own particular, I not only grant you liberty to satisfie your self in this particular, but moreover entreat you to gratifie, as soon as you can, the Curiosity which you have begot in me touching the same: and I believe that *Simplicius* also is of the same mind.

SIMP. I cannot deny what you say.

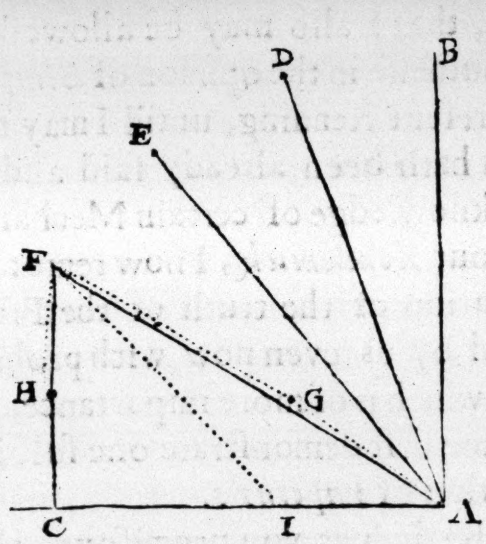
SALV. Seeing then that I have your permission, I will in the first place consider, as an Effect well known, That

PROP.

LEMMA.

That the Moments or Velocities of the same Moveable are different upon different Inclinations of Planes, and the greatest is by the Line elevated perpendicularly above the Horizon, and by the others inclined, the said Velocity diminisheth according as they more and more depart from Perpendicularity, that is, as they incline more obliquely: so that the Impetus, Talent, Energy, or, we may say, Moment of descending is diminished in the Moveable by the subjected Plane, upon which the said Moveable lyeth and descendeth.

AND the better to express my self, let the Line A B be perpendicularly erected upon the Horizon A C: then suppose the same to be declined in sundry Inclinations towards the Horizon, as in A D, A E, A F, &c. I say, that the greatest and total *Impetus* of the Grave Body in descending is along the Perpendicular B A, and less than that along D A, and yet less along E A; and successively diminishing along the more inclined F *A*, and finally is wholly extinct in the Horizontal C *A*, where the Moveable is indifferent either to Motion or Rest, and hath not of it self any Inclination to move one way or other, nor yet any Resistance to its being moved: for as it is impossible that a Grave Body, or a Compound thereof should move naturally upwards, receding from the Common Center, towards which all Grave Matters conspire to go, so it is impossible that it do spontaneously move, unless with that Motion its particular Center of Gravity do acquire Proximity to the said Common Center: so that upon the Horizontal which here is understood to be a Superficies equidistant from the said Center, and therefore altogether void of Inclination, the *Impetus* or Moment of that same Moveable shall be nothing at all. Having understood this mutation of *Impetus*, I am to explain that which, in an old Treatise of the Mechanicks, written heretofore in *Padona* by our *Academick*, only for the use of his Scholars, was diffusely and demonstratively proved, upon the occasion of considering the Original and Nature of the admirable Instrument called the Screw, and it is, With what proportion that mutation of



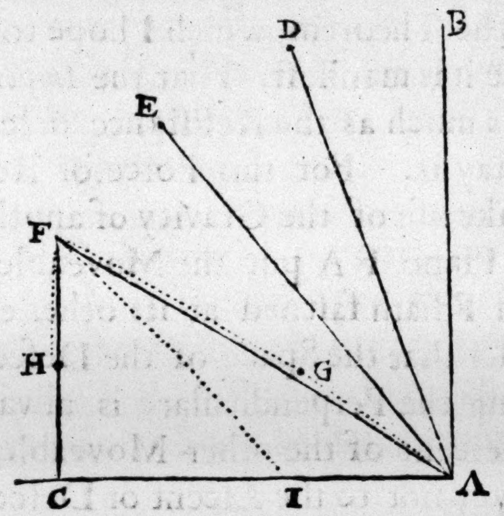
Impetus is made along ſeveral Inclinations or Declivities of Planes.

As, for example, in the inclined Plane A F, drawing its Elevation above the Horizontal, that is, the Line F C, along the which the *Impetus* of a Grave Body, and the Moment of Deſcent is the greateſt; it is ſought what proportion this Moment hath to the Moment of the ſame Moveable along the Declivity F A: Which Proportion, I ſay, is Reciprocal to the ſaid Lengths. And this is the *Lemma* that was to go before the Theorem, which I hope to be able anon to Demonſtrate. Hence it is manifeſt, That the *Impetus* of Deſcent of a Grave Body is as much as the Reſiſtance or leaſt force that ſufficeth to arreſt and ſtay it. For this Force or Reſiſtance, and its meaſure, I will make uſe of the Gravity of another Moveable. Let us now upon the Plane F A put the Moveable G tyed to a thread which ſliding over F hath faſtned at its other end the Weight H: and let us conſider that the Space of the Deſcent or Aſcent of the Weight H along the Perpendicular, is alwaies equal to the whole Aſcent or Deſcent of the other Moveable G along the * Declivity A F, but yet not to the Aſcent or Deſcent along the Perpendicular, in which only the ſaid Moveable G (like as every other Moveable) exerciſeth its Reſiſtance. Which is manifeſt: for conſidering in the Triangle A F C the Motion of the Moveable G, as for example, upwards from A to F, to be compoſed of the tranſverſe Horizontal Line A C, and of the Perpendicular C F: And in regard, that as to the Horizontal Plane along which the Moveable, as hath been ſaid, hath no Reſiſtance to moving (it not making by that Motion any loſs, nor yet acquiſt in regard of its particular diſtance from the Common Center of Grave Matters, which in the Horizon continueth ſtill the ſame) it remaineth that the Reſiſtance be only in reſpect of the *A*ſcent that it is to make along the Perpendicular C F. Whilſt therefore the Grave Moveable G, moving from *A* to F, hath only the Perpendicular Space C F to reſiſt in its *A*ſcent, and whilſt the other Grave Moveable H deſcendeth along the Perpendicular of neceſſity as far as the whole Space F *A*, and that the ſaid proportion of *A*ſcent and Deſcent maintains it ſelf alwaies the ſame, be the Motion of the ſaid Moveables little or much (by reaſon they are tyed together) we may confidently affirm, that in caſe there were an *Equilibrium*, that is Reſt, to enſue betwixt the ſaid Moveables, the Moments, the Velocities, or their Propenſions to Motion, that is the Spaces which they would paſs in the ſame Time ſhould anſwer reciprocally to their Gravities, according to that which is demonſtrated in all caſes of Mechanick Motions: ſo that it ſhall ſuffice to impede the deſcent of G, if H be but ſo much leſs grave than it, as in proportion the Space C F is leſſer than the Space F *A*. Therefore

* Or inclined Plane.

ſuppoſe

ſuppoſe that the Moveable G is to the Moveable H, as F *A* is to F C; and then the *Equilibrium* ſhall follow, that is, the Moveables H and G ſhall have equal Moments, and the Motion of the ſaid Moveables ſhall ceaſe. *A*nd becauſe we ſee that the *Impetus*, Energy, Moment, or Propenſion of a Moveable to Motion is the ſame as is the Force or ſmalleſt Reſiſtance that ſufficeth to ſtop it; and becauſe it hath been concluded, that the Grave Body H is ſufficient to arreſt the Motion of the Grave Body G: Therefore the leſſer Weight H, which in the Perpendicular F C imployeth its total Moment, ſhall be the preciſe meaſure of the partial Moment that the greater Weight G exerciſeth along the inclined Plane F *A*: But the meaſure of the total Moment of the ſaid Grave Body G, is the ſelf ſame, (ſince that to impede the Perpendicular Deſcent of a Grave Body there is required the oppoſition of ſuch another Grave Body, which likewiſe is at liberty to move Perpendicularly:) Therefore the partial *Impetus* or Moment of G along the inclined Plane F A ſhall be to the grand and total *Impetus* of the ſame G along the Perpendicular F C, as the Weight H to the Weight G: that is, by Conſtruction, as the ſaid Perpendicular F C, the Elevation of the inclined Plane, is to the ſame inclined Plane F A: Which is that that by the *Lemma* was propoſed to be demonſtrated, and which by our Author, as we ſhall ſee, is ſuppoſed as known in the ſecond part of the Sixth Propoſition of the preſent Treatiſe.



SAGR. From this that you have already concluded I conceive one may eaſily deduce, arguing *ex æquali* by perturbed Proportion, that the Moments of the ſame Moveable, along Planes variouſly inclined (as F A and F I) that have the ſame Elevation, are to each other in Reciprocal proportion to the ſame Planes.

SALV. *A* moſt certain Concluſion. This being agreed on, we will paſs in the next place to demonſtrate the *Theoreme*, namely, that

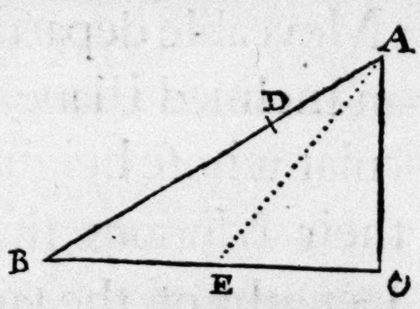
THEOR.

THEOREM.

The degrees of Velocity of a Moveable descending with a Natural Motion from the same height along Planes in any manner inclined at the arrival to the Horizon are alwaies equal, Impediments being removed.

HEre we are in the first place to advertise you, that it having been proved, that in any Inclination of the Plane the Moveable from its recession from Quiessence goeth encreasing its Velocity, or quantity of its *Impetus*, with the proportion of the Time (according to the Definition which the Author giveth of Motion naturally Accelerate) whereupon, as he hath by the precedent Proposition demonstrated, the Spaces passed are in duplicate proportion to the Times, and, consequently, to the degrees of Velocity: look what the *Impetus's* were in that which was first moved, such proportionally shall be the degrees of Velocity gained in the same Time; seeing that both these and those encrease with the same proportion in the same Time.

Now let the inclined Plane be A B, its elevation above the Horizon the Perpendicular A C, and the Horizontal Plane C B: and because, as was even now concluded, the *Impetus* of a Moveable along the Perpendicular A C is to the *Impetus* of the same along the inclined Plane A B, as A B is to A C, let there be taken in the inclined Plane A B, A D a third proportional to A B and A C: The *Impetus*, therefore, along A C is to the *Impetus* along A B, that is along A D, as A C is to A D: And therefore the Moveable in the same Time that it would pass the Perpendicular Space AC, shall likewise pass the Space A D, in the inclined Plane A B, (the Moments being as the Spaces:) And the degree of Velocity in C shall have the same proportion to the degree of Velocity in D, as A C hath to A D:



But the degree of Velocity in B is to the same degree in D, as the Time along A B is to the Time along A D, by the definition of Accelerate Motion; And the Time along A B is to the Time along A D, as the same A C, the Mean Proportional between B A and A D, is to A D, by the last Corollary of the second Proposition: Therefore the degrees of Velocity in B and in C have to the degree in D, the same Proportion as A C hath to A D; and therefore are equal: Which is the *Theorem* intended to be demonstrated.

By this we may more concludingly prove the ensuing third

Proposition of the Author, in which he makes use of this Principle; and it is, That the Time along the inclined Plane, hath to the Time along the Perpendicular, the same proportion as the said Inclined Plane and Perpendicular. For if we put the case that B A be the Time along A B, the Time along A D shall be the Mean between them, that is A C, by the second Corollary of the second Proposition: But if C A be the Time along A D, it shall likewise be the Time along *A* C, by reason that *A* D and *A* C are past in equal Times: And therefore in case B *A* be the Time along A B, *A* C shall be the Time along *A* C: Therefore, as *A* B is to A C, so is the Time along *A* B to the Time along *A* C.

By the same discourse one shall prove, that the Time along *A* C is to the Time along the inclined Plane *A* E, as *A* C is to *A* E: Therefore, *ex æquali*, the Time along the inclined Plane *AB* is, Directly, to the Time along the inclined Plane *A* E as *A B* to *A* E, *&c.*

One might also by the same application of the *Theorem*, as *Sagredus* shall very evidently see anon, immediately demonstrate the sixth Proposition of the *Author*: *B*ut let this Digression suffice for the present, which he perhaps thinketh too tedious, though indeed it is of some importance in these matters of Motion.

SAGR. You may say extreamly delightful, and most necessary to the perfect understanding of that Principle.

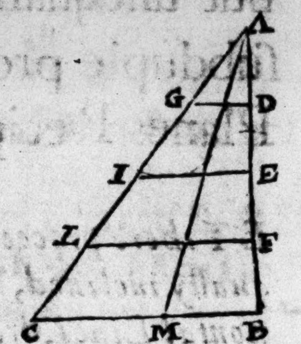
SALV. I will go on, then, in my Reading of the Text.

THEOR. III. PROP. III.

If a Moveable departing from Rest do move along an Inclined Plane, and also along the Perpendicular whose heights are the same, the Times of their Motions shall be to one another as the Lengths of the said Plane and Perpendicular.

LET the inclined Plane be A C, *and the Perpendicular* A B, *whose heights are the same above the Horizon* C B, *to wit, the self same Line* B A. *I say, that the Time of the Descent of the same* Moveable *upon the Plane* A C, *hath the same Proportion to the Time of the Descent along the Perpendicular* A B, *as the Length of the Plane* A C *hath to the Length of the said Perpendicular. For let any number of Lines* D G, E I, F L, *be drawn, Parallel to the Horizon* C B: *It is manifest from the Assumption foregoing, that the degrees of Velocity of the* Moveable, *departing from* A *the beginning of* Motion, *acquired in the Points* G *and* D *are equal,*

equal, their excesse or elevation above the Horizon being equal; and so the degrees in the Points I *and* E; *as also the degrees in* L *and* F. *And if not only these Parallels, but many more were supposed to be drawn from all the points imagined to be in the Line* A B, *untill they meet the Line* A C, *the Moments, or degrees of the Velocities along the extreams [or ends] of every one of those Parallels, shall be alwaies equal to one another: Therefore the two Spaces* A C *and* A B *are past with the same degree of Velocity: But it hath been demonstrated, that if two Spaces be passed by a Moveable with one and the same degree of Velocity, the Times of the Motions have the same proportion as those Spaces: Therefore the Time of the Motion along* A C *is to the Time along* A B, *as the Length of the Plane* A C *to the length of the Perdendicular* A B. *Which was to be demonstrated.*



SAGR. It seemeth to me, that the same might very clearly and concisely be concluded, it having first been proved that the sum of the Accelerate Motion of the Transitions along A C and A B, is as much as the Equable Motion, whose degree of Velocity is subduple to the greatest degree C B: Therefore the two Spaces A C and A B being passed with the same Equable Motion, it hath been shewn, by the First Proposition of the first, that the Times of the Transitions shall be as the said Spaces.

COROLLARY.

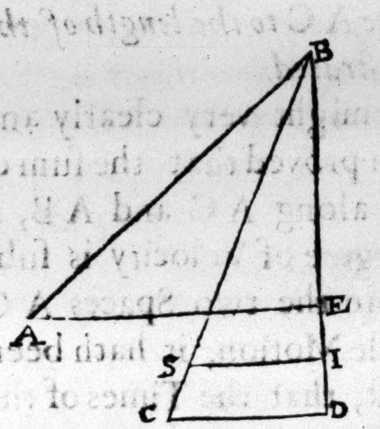
Hence is collected, that the Times of the Descents along Planes of different Inclination, but of the same Elevation, are to one another according to their Lengths.

FOR *if we suppose another Plane* A M, *coming from* A, *and terminated by the same Horizontal* C B; *it shall in like manner be demonstrated, that the Time of the Descent along* A M, *is to the Time along* A B, *as the Line* A M *to* A B: *But as the Time* A B *is to the Time along* A C, *so is the Line* A B *to* A C: *Therefore,* ex æquali, *as* A M *is to* A C, *so is the Time along* A M *to the Time along* A C.

THEOR. IV. PROP. IV.

The Times of the Motions along equal Planes, but unequally inclined, are to each other in subduple proportion of the Elevations of those Planes Reciprocally taken.

LET there proceed from the term B two equal Planes, but unequally inclined, B A and B C, and let A E and C D be Horizontal Lines, drawn as far as the Perpendicular B D: Let the Elevation of the Plane B A be B E; and let the Elevation of the Plane B C be B D: And let B I be a Mean Proportional between the Elevations D B and B E: It is manifest that the proportion of D B to B I, is subduple the proportion of D B to B E. Now I say, that the proportion of the Times of the Descents or Motions along the Planes B A and B C, are the same with the proportion of D B to B I Reciprocally taken: So that to the Time B A the Elevation of the other Plane B C, that is B D be Homologal; and to the Time along B C, B I be Homologal: Therefore it is to be demonstrated, That the Time along B A is to the Time along B C, as D B is to B I. Let I S be drawn equidistant from D C. And because it hath been demonstrated that the Time of the Descent along B A, is to the Time of the Descent along the Perpendicular B E, as the said B A is to B E; and the Time along B E is to the Time along B D, as B E is to B I; and the Time along B D is to the Time along B C, as B D to B C, or as B I to B S: Therefore, ex æquali, the Time along B A shall be to the Time along B C as B A to B S, or as C B to B S: But C B is to B S, as D B to B I: Therefore the Proposition is manifest:

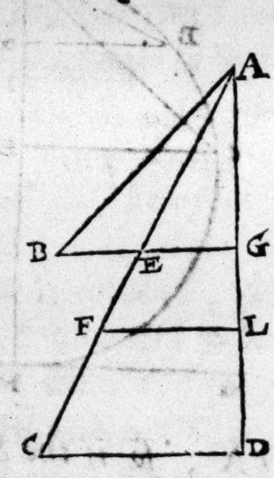


THEOR.

THEOR. V. PROP. V.

The proportion of the Times of the Descents along Planes that have different Inclinations and Lengths, and the Elivations unequal, is compounded of the proportion of the Lengths of those Planes, and of the subduple proportion of their Elevations Reciprocally taken.

Let AB *and* AC *be Planes inclined after different manners, whose Lengths are unequal, as also their Elevations. I say, the proportion of the Time of the Descent along* AC *to the Time along* AB*, is compounded of the proportion of the said* AC *to* AB*, and of the subduple proportion of their Elevation Reciprocally taken. For let the Perpendicular* AD *be drawn, with which let the Horizontal Lines* BG *and* CD *intersect, and let* AL *be a Mean-proportional between* C*A and A*E*; and from the point* L *let a Parallel be drawn to the Horizon intersecting the Plane* AC *in* F*; and* AF *shall be a Mean proportional between* CA *and* AE*. And because the Time along* AC *is to the Time along A*E*, as the Line* F*A to A*E*; and the Time along A*E *is to the Time along A*B*, as the said A*E *to the said* AB*: It is manifest that the Time along* AC *is to the Time along A*B*, as A*F *to A*B*. It remaineth, therefore, to be demonstrated, that the proportion of A*F *to A*B *is compounded of the proportion of* C*A to A*B*, and of the proportion of* G*A to A*L*; which is the subduple proportion of the Elevations* D*A and A*G *Reciprocally taken. But that is manifest,* C*A being put between* F*A and A*B*: For the proportion of* F*A to A*C *is the same as that of* L*A to A*D*, or of* G*A to A*L*; which is subduple of the proportion of the Elevations* G*A and A*D*; and the proportion of* C*A to A*B *is the proportion of the Lengths: Therefore the Proposition is manifest.*

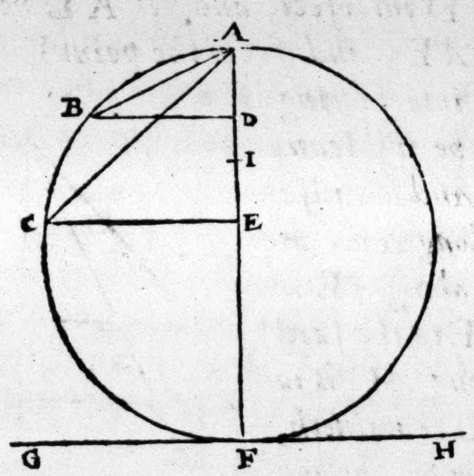


THEOR.

THEOR. VI. PROP. VI.

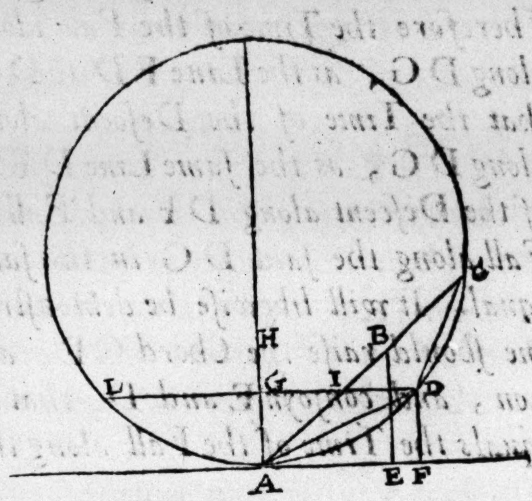
If from the highest or lowest part of a Circle, erect upon the Horizon, certain Planes be drawn inclined towards the Circumference, the Times of the Descents along the same shall be equal.

LET the Circle be erect upon the Horizon GH, *whose Diameter recited upon the lowest point, that is upon the contact with the Horizon, let be* FA, *and from the highest point* A *let certain Planes* AB *and* AC *incline towards the Circumference :* I *say that the Times of the Descents along the same are equal.* Let BD *and* CE *be two Perpendiculars let fall unto the Diameter; and let* AI *be a Mean-Proportional between the Altitudes of the Planes* EA *and* AD. *And because the Rectangles* FAE *and* FAD *are equal to the Squares of* AC *and* AB; *And also because that as the Rectangle* FAE, *is to the Rectangle* FAD, *so is* EA *to* AD. *Therefore as the Square of* CA *is to the Square of* BA, *so is the Line* EA *to the Line* AD. *But as the Line* EA *is to* DA, *so is the Square of* IA *to the Square of* AD: *Therefore the Squares of the Lines* CA *and* AB *are to each other as the Squares of the Lines* IA *and* AD: *And therefore as the Line* CA *is to* AB, *so is* IA *to* AD : *But in the precedent Proposition it hath been demonstrated that the proportion of the Time of the Descent along* AC *to the Time of the Descent by* AB, *is compounded of the proportions of* CA *to* AB, *and of* DA *to* AI, *which is the same with the proportion of* BA *to* AC : *Therefore the proportion of the Time of the Descent along* AC, *to the Time of the Descent along* AB, *is compounded of the proportions of* CA *to* AB, *and of* BA *to* AC : *Therefore the proportion of those Times is a proportion of equality : Therefore the Proposition is evident.*



The same is another way demonstrated from the Mechanicks : Namely that in the ensuing Figure the Moveable passeth in equal Times along CA *and* DA. *For let* BA *be equal to the said* DA, *and let fall the Perpendiculars* BE *and* DF : *It is manifest by the Elements of the Mechanicks:*

Mechanicks: That the Moment of the Weight elevated upon the Plane according to the Line ABC, *is to its total Moment, as* BE *to* BA; *And that the Moment of the same Weight upon the Elevation* AD, *is to its total Moment, as* DF *to* DA *or* BA: *Therefore the Moment of the said Weight upon the Plane inclined according to* DA, *is to the Moment upon the Plane inclined according to* ABC, *as the Line* DF *to the Line* BE: *Therefore the Spaces which the said Weight shall passe in equal Times along the Inclined Planes* CA *and* DA, *shall be to each other as the Line* BE *to* DF; *by the second Proposition of the First Book: But as* BE *is to* DF, *so* AC *is demonstrated to be to* DA: *Therefore the same Moveable will in equal Times passe the Lines* CA *and* DA.

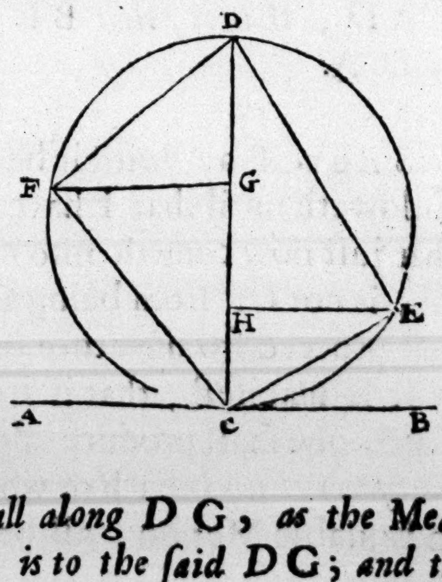


And that CA *is to* DA *as* BE *is to* DF, *is thus demonstrated.*

Draw a Line from C *to* D; *and by* D *and* B *draw the Lines* DGL, (*cutting* CA *in the point* I) *and* BH, *Parallels to* AF: *And the Angle* ADI *shall be equal to the Angle* DCA, *for that the parts* LA *and* AD *of the Circumference subtending them, are equal, and the Angle* DAC *common to them both: Therefore of the equiangled Triangles* CAD *and* DAI, *the sides about the equal Angles shall be proportional: And as* CA *is to* AD, *so is* DA *to* AI, *that is* BA *to* AI, *or* HA *to* AG; *that is,* BE *to* DF: *Which was to be proved.*

Or else the same shall be demonstrated more speedily thus.

Unto the Horizon AB, *let a Circle be erect, whose Diameter is perpendicular to the Horizon: and from the highest Term* D *let a Plane at pleasure* DF, *be inclined to the Circumference. I say that the Descent along the Plane* DF, *and the Fall along the Diameter* BC, *will be passed by the same Moveable in equal Times. For let* FG *be drawn parallel to the Horizon* AB, *which shall be perpendicular to the Diameter* DC, *and let a Line conjoyn* F *and* C: *and because the Time of the Fall along* DC, *is to the Time of the Fall along* DG, *as the Mean Proportional between* CD *and* DG, *is to the said* DG; *and the*



Mean between CD *and* DG *being* DF, *(for that the Angle* DFC *in the Semicircle, is a Right Angle, and* FG *perpendicular to* DC*:) Therefore the Time of the Fall along* DC *is to the Time of the Fall along* DG, *as the Line* FD *to* DG*: But it hath been demonstrated that the Time of the Descent along* DF, *is to the Time of the Fall along* DG, *as the same Line* DF *is to* DG*: The Times, therefore, of the Descent along* DF *and Fall along* DC, *are to the Time of the Fall along the said* DG *in the same proportion: Therefore they are equal. It will likewise be demonstrated, if from the lowest Term* C, *one should raise the Chord* CE, *and draw* EH *parallel to the Horizon, and conjoyn* E *and* D, *that the Time of the Descent along* EC *equals the Time of the Fall along the Diameter* DC.

COROLLARY I.

Hence is collected that the Times of the Descents along all the Chords drawn from the Terms C or D are equal to one another.

COROLLARY II.

It is also collected that if the Perpendicular and inclined Plane descend from the same point along which the Descents are made in equal Times, they are in a Semicircle whose Diameter is the said Perpendicular.

COROLLARY III.

Hence it is collected that the Times of the Motions along inclined Planes, are then equal, where the Elevations of equal parts of those Planes shall be to one another as their Longitudes.

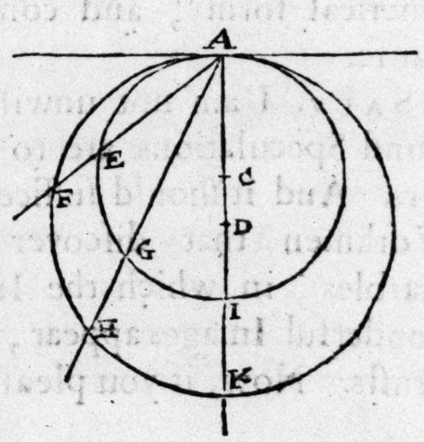
F*OR it hath been shewn that the Times* CA *and* DA *in the last Figure save one are equal, the Elevation of the part* AB *being equal to* AD, *that is, that* BE *shall be to the Elevation* DF, *as* CA *to* DA.

SAGR. Pray you Sir be pleased to stay your Reading of what followeth until that I have satisfied my self in a Contemplation that just now cometh into my mind, which if it be not a delusion, is not far from being a pleasing divertisement: as are all such that proceed from Nature or necessity.

It is manifest, that if from a point assigned in an Horizontal Plane, one shall produce along the same Plane infinite right Lines every way, upon each of which a point is understood to move with an Equable Motion, all beginning to move in the same instant

of

of Time from the aſſigned point, and the Velocities of them all being equal, there ſhall conſequently be deſcribed by thoſe moveable points Circumferences of Circles alwayes bigger and bigger, all concentrick about the firſt point aſſigned: juſt in the ſame manner as we ſee it done in the Undulations of ſtanding Water, when a ſtone is dropt into it; the percuſſion of which ſerveth to give the beginning to the Motion on every ſide, and remaineth as the Center of all the Circles that happen to be deſigned ſucceſſively bigger and bigger by the ſaid Undulations. But if we imagine a Plane erect unto the Horizon, and a point be noted in the ſame on high, from which infinite Lines are drawn inclined, according to all inclinations, along which we fancy grave Moveables to deſcend, each with a Motion naturally Accelerate with thoſe Velocities that agree with the ſeveral Inclinations; ſuppoſing that thoſe deſcending Moveables were continually viſible, in what kind of Lines ſhould we ſee them continually diſpoſed? Hence my wonder ariſeth, ſince that the precedent Demonſtrations aſſure me, that they ſhall all be alwayes ſeen in one and the ſame Circumference of Circles ſucceſſively encreaſing, according as the Moveables in deſcending go more and more ſucceſſively receding from the higheſt point in which their Fall began: And the better to declare my ſelf, let the chiefeſt point A be marked, from which Lines deſcend according to any Inclinations A F, A H, and the Perpendicular A B, in which taking the points C and D, deſcribe Circles about them that paſs by the point A, interſecting the inclined Lines in the points F, H, B, and E, G, I. It is manifeſt, by the fore-going Demonſtrations, that Moveables deſcendent along thoſe Lines departing at the ſame Time from the term A, one ſhall be in E, the other ſhall be in G, and the other in I; and ſo continuing to deſcend they ſhall arrive in the ſame moment of Time at F, A, and B: and theſe and infinite others continuing to move along the infinite differing Inclinations, they ſhall alwayes ſucceſſively arrive at the ſelf-ſame Circumferences made bigger & bigger *in infinitum*. From the two Species, therefore, of Motion of which Nature makes uſe, ariſeth, with admirable harmonious variety, the generation of infinite Circles. She placeth the one as in her Seat, and original beginning, in the Center of infinite concentrick Circles; the other is conſtituted in the ſublime or higheſt Contact of infinite Circumferences of Circles, all excentrick to one another: Thoſe proceed from Motions all equal and Equable; Theſe from Motions all al-



wayes Inequable to themselves, and all unequal to one another, that descend along the infinite different Inclinations. But we further adde, that if from the two points assigned for the Emanations, we shall suppose Lines to proceed, not onely along two Superficies Horizontal and Upright [or erect] but along all every way; like as from those, beginning at one sole point, we passed to the production of Circles from the least to the greatest, so beginning from one sole point we shall successively produce infinite Spheres, or we may say one Sphere, that shall *gradatim* increase to infinite bignesses: And this in two fashions; that is, either with placing the original in the Center, or else in the Circumference of those Spheres.

SALV. The Contemplation is really ingenuous, and adequate to the Wit of *Sagredus*.

SIMP. Though I am at least capable of the Speculation, according to the two manners of the production of Circles and Spheres, with the two different Natural Motions, howbeit I do not perfectly understand the production depending on the Accelerate Motion and its Demonstration, yet notwithstanding that licence of assigning for the place of that Emanation as well the lowest Center, as the highest Spherical Superficies, maketh me to think that its possible that some great Mistery may be contained in these true and admirable Conclusions: some Mistery I say touching the Creation of the Universe, which is held to be of Spherical form, and concerning the Residence of the First Cause.

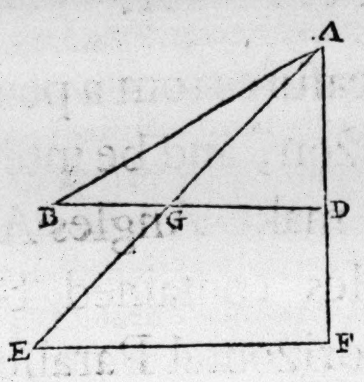
SALV. I am not unwilling to think the same: but such profound Speculations are to be expected from Sharper Wits than ours. And it should suffice us, that if we be but those lesse noble Workmen that discover and draw forth of the Quarry the Marbles, in which the Industrious Statuaries afterwards make wonderful Images appear, that lay hid under rude and mishaped Crusts. Now, if you please, we will go on.

THEOR. VII. PROP. VII.

If the Elevations of two Planes shall have a proportion double to that of their Lengths, the Motions in them from Rest shall be finished in equal Times.

Let AE *and* AB *be two unequal Planes, and unequally inclined, and let their Elevations be* FA *and* DA, *and let* FA *have the same proportion to* DA, *as* AE *hath to* AB. *I say that the Times of the Motions along the Planes* AE *and* AB, *out of Rest in* A *are equal*

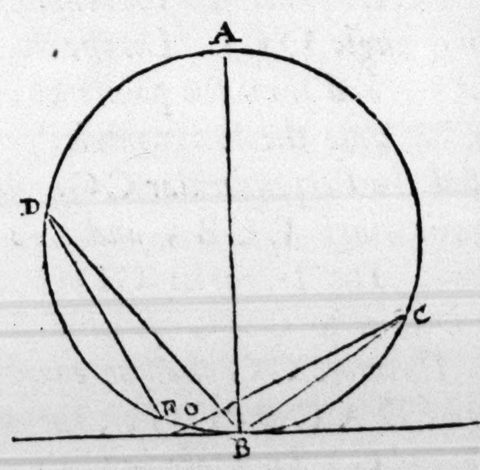
equal. Draw Horizontal Parallels to the Line of Elevation EF *and* BD, *which cutteth* AE *in* G. *And because the proportion of* FA *to* AD, *is double the proportion of* EA *to* AB; *and as* FA *to* AD, *so is* EA *to* AG: *Therefore the proportion of* EA *to* AG, *is double the proportion of* EA *to* AB: *Therefore* AB *is a Mean-Proportional between* EA *and* AG: *And because the Time of the Descent along* AB, *is to the Time of the Descent along* AG, *as* AB *to* AG; *and the Time of the Descent along* AG, *is to the Time of the Descent along* AE, *as* AG *is to the Mean-proportional between* AG *and* AE, *which is* AB: *Therefore* ex equali, *the Time along* AB *is to the Time along* AE, *as* AB *unto it self: Therefore the Times are equal: Which was to be demonstrated.*



THEOR. VIII. PROP. VIII.

In Planes cut by the same Circle, erect to the Horizon, in those which meet with the end of the erect Diameter, whether upper or lower, the Times of the Motions are equal to the Time of the Fall along the Diameter: and in those which fall short of the Diameter, the Times are shorter; and in those which intersect the Diameter, they are longer.

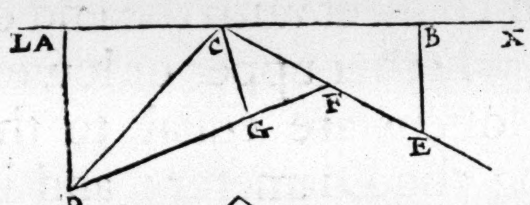
Let AB *be the Perpendicular Diameter of the Circle erect to the Horizon. That the Times of the Motions along the Planes produced out of the Terms* A *and* B *unto the Circumference are equal, hath already been demonstrated: That the Time of the Descent along the Plane* DF, *not reaching to the Diameter is shorter, is demonstrated by drawing the Plane* DB, *which shall be both longer and lesse declining than* DF. *Therefore the Time along* DF *is shorter than the Time along* DB, *that is, along* AB. *And that the Time of the Descent along the Plane that intersecteth the Diameter, as* CO *is longer, doth in the same manner appear, for that it is longer and lesse declining than* CB: *Therefore the Proposition is demonstrated.*

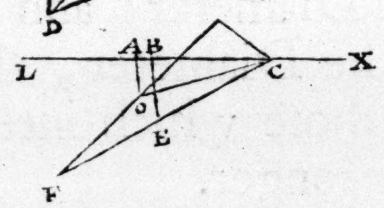


THEOR. IX. PROP. IX.

If two Planes be inclined at pleasure from a point in a Line parallel to the Horizon, and be intersected by a Line which may make Angles Alternately equal to the Angles contained between the said Planes and Horizontal Parallel, the Motion along the parts cut off by the said Line, shall be performed in equal Times.

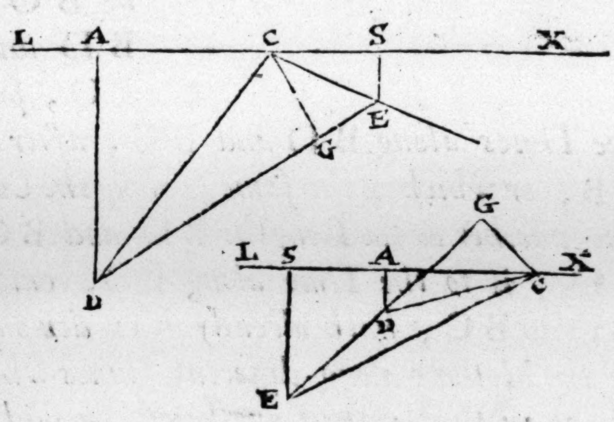
From off the point C *of the Horizontal Line* X, *let any two Planes be inclined at pleasure* C D *and* C E, *and in any point of the Line* C D *make the Angle* C D E *equal to the Angle* X C E: *and let the Line* D F *cut the Plane* C E *in* F, *in such a manner that the Angles* C D F *and* C F D *may be equal to the Angles* X C E, L C D *Alternately taken. I say, that the Times of the Descents along* C D *and* C F *are equal. And that (the Angle* C D F *being supposed equal to the Angle* X C E) *the Angle* C F D *is equal to the Angle* D C L, *is manifest. For the Common Angle* D C F *being taken from the three Angles of the Triangle* C D F *equal to two Right Angles, to which are equal all the Angles made with to the Line* L X *at the point* C, *there remains in the Triangle two Angles* C D F *and* C F D, *equal to the two Angles* X C E *and* L C D: *But it was supposed that* C D F *is equal to the Angle* X C E: *Therefore the remaining Angle* C F D *is equal to the remaining angle* D C L. *Let the Plane* C E *be supposed equal to the Plane* C D, *and from the points* D *and* E *raise the Perpendiculars* D A *and* E B, *unto the Horizontal Parallel* X L; *and from* C *unto* D F *let fall the Perpendicular* C G. *And because the Angle* C D G *is equal to the Angle* E C B; *and because* D G C *and* C B E *are Right Angles; The Triangles* C D G *and* C B E *shall be equiangled: And as* D C *is to* C G, *so let* C E *be to* E B: *But* D C *is equal to* C E: *Therefore* C G *shall be equal to* E B. *And in regard that of the Triangles* D A C *and* C G F, *the Angles* C *and* A *are equal to the Angles* F *and* G: *Therefore as* C D *is to* D A, *so shall* F C *be to* C G; *and Alternately, as* D C *is to* C F, *so is*





is DA *to* CG, *or* BE. *The proportion therefore of the Elevations of the Planes equal to* CD *and* CE, *is the same with the proportion of the Longitudes* DC *and* CE: *Therefore, by the first Corollary of the precedent Sixth Proposition, the Times of the Descent along the same shall be equal: Which was to be proved.*

Take the same another way: Draw FS *perpendicular to the Horizontal Parallel* AS *Because the Triangle* CSF *is like to the Triangle* DGC, *it shall be, that as* SF *is to* FC, *so is* GC *to* CD. *And because the Triangle* CFG *is like to the Triangle* DCA, *it shall be, that as* FC *is to* CG, *so is* CD *to* DA: *Therefore,* ex æquali, *as* SF *is to* CG, *so is* CG *to* DA: *Therefore* CG *is a Mean-proportional between* SF *and* DA: *And as* DA *is to* SF, *so is the Square* DA *unto the Square* CG. *Again, the Triangle* ACD *being like to the Triangle* CGF, *it shall be, that as* DA *is to* DC, *so is* GC *to* CF: *and, Alternately, as* DA *is to* GC, *so is* DC *to* CF; *and as the Square of* DA *is to the Square of* CG, *so is the Square of* DC *to the Square of* CF. *But it hath been proved that the Square* DA *is to the Square* CG *as the Line* DA *is to the Line* FS: *Therefore, as the Square* DC *is to the Square* CF, *so is the Line* DE *to* FS: *Therefore, by the seventh fore-going, in regard that the Elevations* DA *and* FS, *of the Planes* CD, *and* CF *are in double proportion to their Planes; the Times of the Motions along the same shall be equal.*

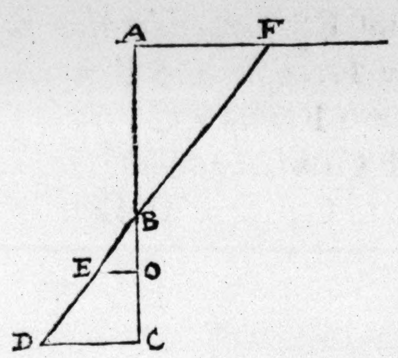


THEOR. X. PROP. X.

The Times of the Motions along several Inclinations of Planes whose Elevations are equal, are unto one another as the Lengths of those Planes, whether the Motions be made from Rest, or there hath proceeded a Motion from the same height.

L*Et the Motions be made along* ABC, *and along* ABD, *until they come to the Horizon* DC, *in such sort as that the Motion along* AB *precedeth the Motions along* BD *and* BC. *I say, that the Time of the Motion along* BD, *is to the Time along* BC, *as the*

the Length BD *is to* BC. *Let* AF *be drawn parallel to the Horizon, to which continue out* DB, *meeting it in* F; *and let* FE *be a Mean-proportional between* DF *and* FB; *and draw* EO *parallel to* DC, *and* AO *ſhall be a Mean-proportional between* CA *and* AB: *But if we ſuppoſe the Time along* AB, *to be as* AB, *the Time along* FB *ſhall be as* FB. *And the Time along all* AC, *ſhall be as the Mean-proportional* AO; *and along all* FD *ſhall be* FE: *Wherefore the Time along the remainder* BC *ſhall be* BO; *and along the remainder* BD *ſhall be* BE. *But as* BE *is to* BO, *ſo is* BD *to* BC: *Therefore the Times along* BD *and* BC, *after the Deſcent along* AB *and* FB, *or which is the ſame, along the Common part* AB, *ſhall be to one another as the Lengths* BD *and* BC: *But that the Time along* BD, *is to the Time along* BC, *out of Reſt in* B, *as the Length* BD *to* BC, *hath already been demonſtrated. Therefore the Times of the Motions along different Planes whoſe Elevations are equal, are to one another as the Lengths of the ſaid Planes, whether the Motion be made along the ſame out of Reſt, or whether another Motion of the ſame Altitude do precede thoſe Motions: Which was to be demonſtrated.*

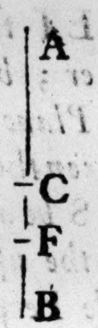


THEOR. XI. PROP. XI.

If a Plane, along which a Motion is made out of Reſt, be divided at pleaſure, the Time of the Motion along the firſt part, is to the Time of the Motion along the ſecond, as the ſaid firſt part is to the exceſſe whereby the ſame part ſhall be exceeded by the Mean-Proportional between the whole Plane and the ſame firſt part.

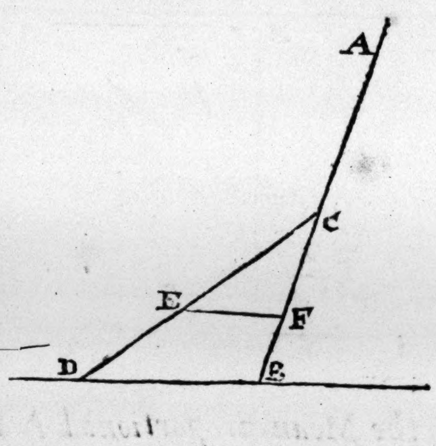
L*ET the Motion be along the whole Plane* AB, ex quiete *in* A, *which let be divided at pleaſure in* C; *and let* AF *be a Mean proportional between the whole* BA *and the firſt part* AC; CF *ſhall be the exceſſe of the Mean proportional* FA *above the part* AC. *I ſay the Time of the Motion along* AC *is to the Time of the following Motion along* CB, *as* AC *to* CF. *Which is manifeſt;*

For



For the Time along AC *is to the Time along all* AB, *as* AC *to the Mean-proportional* AF: *Therefore, by Division, the Time along* AC, *shall be to the Time along the remainder* CB *as* AC *to* CF: *If therefore the Time along* AC *be supposed to be the said* AC, *the Time along* CB *shall be* CF: *Which was the Proposition.*

But if the Motion be not made along the continuate Plane ACB, *but by the inflected Plane* ACD *until it come to the Horizon* BD, *to which from* F *a Parallel is drawn* FE. *It shall in like manner be demonstrated, that the Time along* AC *is to the Time along the reflected Plane* CD, *as* AC *is to* CE. *For the Time along* AC *is to the Time along* CB, *as* AC *is to* CF: *But the Time along* CB, *after* AC *hath been demonstrated to be to the Time along* CD, *after the said Descent along* AC, *as* CB *is to* CD; *that is, as* CF *to* CE: *Therefore,* ex æquali, *the Time along* AC *shall be to the Time along* CD, *as the Line* AC *to* CE.

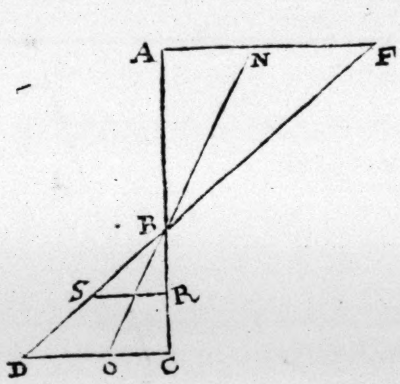


THEOR. XII. PROP. XII.

If the Perpendicular and Plane Inclined at pleasure, be cut between the same Horizontal Lines, and Mean-Proportionals between them and the parts of them contained betwixt the common Section and upper Horizontal Line be given; the Time of the Motion along the Perpendicular shall have the same proportion to the Time of the Motion along the upper part of the Perpendicular, and afterwards along the lower part of the intersected Plane, as the Length of the whole Perpendicular hath to the Line compounded of the Mean-Proportional given upon the Perpendicular, and of the excesse by which the whole Plane exceeds its Mean-Proporttonal.

Let

LET the Horizontal Lines be AF the upper, and CD the lower; between which let the Perpendicular AC, and inclined Plane DF, be cut in B; and let AR be a Mean-Proportional between the whole Perpendicular CA, and the upper part AB; and let FS be a Mean-proportional between the whole Inclined Plane DF, and the upper part BF. I say, that the Time of the Fall along the whole Perpendicular AC hath the same proportion to the Time along its upper part AB, with the lower of the Plane, that is, with BD, as AC hath to the Mean-proportional of the Perpendicular, that is AR, with SD, which is the excesse of the whole Plane DF above its Mean-proportional FS. Let a Line be drawn from R to S, which shall be parallel to the two Horizontal Lines. And because the Time of the Fall along all AC, is to the Time along the part AB, as CA is to the Mean-proportional AR, if we suppose AC to be the Time of the Fall along AC, AR shall be the Time of the Fall along AB, and RC that along the remainder BC. For if the Time along AC be supposed, as was done, to be AC it self the Time along FD shall be FD; and in like manner DS may be concluded to be the Time along BD, after FB, or after AB. The Time therefore along the whole AC, is AR, with RC; And the Time along the inflected Plane ABD, shall be AR, with SD: Which was to be proved.



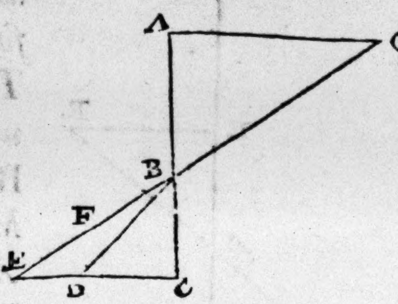
The same happeneth, if instead of the Perpendicular, another Plane were taken, as suppose NO; and the Demonstration is the same.

PROBL. I. PROP. XIII.

A Perpendicular being given, to Inflect a Plane unto it, along which, when it hath the same Elevation with the said Perpendicular, it may make a Motion after its Fall along the Perpendicular in the same Time, as along the same Perpendicular *ex quiete*.

LET the Perpendicular given be AB, to which extended to C, let the part BC be equal; and draw the Horizontal Lines CE and AG. It is required from B to inflect a Plane reaching to the Horizon CE, along which a Motion, after the Fall out of

of A, *shall be made in the same Time, as along* A B *from Rest in* A. *Let* C D *be equal to* C B, *and drawing* B D, *let* B E *be applied equal to both* B D *and* D C. *I say* B E *is the Plane required. Continue out* E B *to meet the Horizontal Line* A G *in* G; *and let* G F *be a Mean-Proportional between the said* E G *and* G B. E F *shall be to* F B, *as* E G *is to* G F; *and the Square* E F *shall be to the Square* F B, *as the Square* E G *is to the Square* G F; *that is as the Line* E G *to* G B: *But* E G *is double to* G B: *Therefore the Square of* E F *is double to the Square of* F B: *But also the Square of* D B *is double to the Square of* B C: *Therefore, as the Line* E F *is to* F B, *so is* D B *to* B C: *And by Composition and Permutation, as* E B *is to the two* D B *and* B C, *so is* B F *to* B C: *But* B E *is equal to the two* D B *and* B C: *Therefore* B F *is equal to the said* B C, *or* B A. *If therefore* A B *be understood to be the Time of the Fall along* A B, G B *shall be the Time along* G B, *and* G F *the Time along the whole* G E: *Therefore* B F *shall be the Time along the remainder* B E, *after the Fall from* G, *or from* A, *which was the Proposition.*

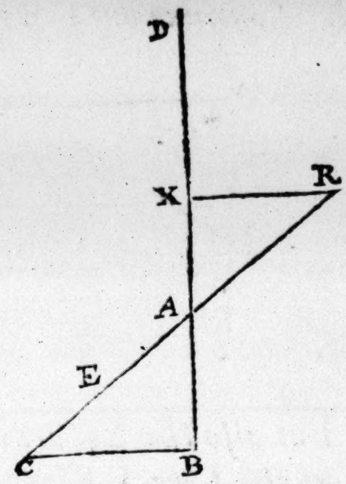


PROBL. II. PROP. XIV.

A Perpendicular and a Plane inclined to it being given, to find a part in the upper Perpendicular which shall be past *ex quiete* in a Time equal to that in which the inclined Plane is past after the Fall along the part found in the Perpendicular.

L*Et the Perpendicular be* D B, *and the Plane inclined to it* A C. *It is required in the Perpendicular* A D *to find a part which shall be past* ex quiete *in a Time equal to that in which the Plane* A C *is past after the Fall along the said part. Draw the Horizontal Line* C B; *and as* B A *more twice* A C *is to* A C, *so let* E A *be to* A R; *And from* R *let fall the Perpendicular* R X *unto* D B. *I say* X *is the point required. And because as* B A *more twice* A C *is to* A C, *so is* C A *to* A E, *by Division it shall be that as* B A *more* A C *is to* A C, *so is* C E *to* E A: *And because as* B A *is to* A C, *so is* E A *to* A R, *by Composition it shall be that as* B A *more* A C *is to* A C, *so is* E R *to* R A: *But as* B A *more* A C *is to* A C, *so is* C E *to* E A: *Therefore, as* C E *is to* E A, *so is* E R, *to* R A, *and both the Antecedents to both the Consequents, that is,* C R

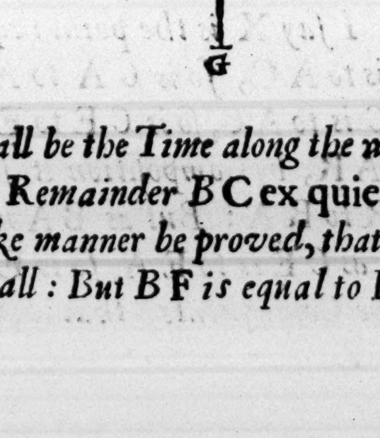
to R E: *Therefore* C R, R E, *and* R A *are Proportionals. Farthermore, because as* B A *is to* A C, *so* E A *is supposed to be to* A R, *and, in regard of the likenesse of the Triangles, as* B A *is to* A C, *so is* X A *to* A R: *Therefore, as* E A *is to* A R, *so is* X A *to* A R: *Therefore* E A *and* X A *are equal. Now if we understand the Time along* R A *to be as* R A, *the Time along* R C *shall be* R E, *the Mean-Proportional between* C R *and* R A: *And* A E *shall be the Time along* A C *after* R A *or after* X A: *But the Time along* X A *is* X A, *so long as* R A *is the Time along* R A: *But it hath been proved that* X A *and* A E *are equal: Therefore the Proposition is proved.*



PROBL. III. PROP. XV.

A Perpendicular and a Plane inflected to it being given, to find a part in the Perpendicular extended downwards which shall be passed in the same Time as the inflected Plane after the Fall along the given Perpendicular.

L*Et the Perpendicular be* A B, *and the Plane Inflected to it* B C. *It is required in the Perpendicular extended downwards to find a part which from the Fall out of* A *shall be past in the same Time as* B C *is passed from the same Fall out of* A. *Draw the Horizontal Line* A D, *with which let* C B *meet extended to* D; *and let* D E *be a Mean-proportional between* C D *and* D B; *and let* B F *be equal to* B E; *and let* A G *be a third Proportional to* B A *and* A F. *I say,* B G *is the Space that after the Fall* A B *shall be past in the same Time, as the Plane* B C *shall be past after the same Fall. For if we suppose the Time along* A B *to be as* A B, *the Time along* D B *shall be as* D B: *And because* D E *is the Mean-proportional between* B D *and* D C, *the same* D E *shall be the Time along the whole* D C, *and* B E *the Time along the Part or Remainder* B C ex quiete, *in* D, *or* * ex casu A B: *And it may in like manner be proved, that* B F *is the Time along* B G, *after the same Fall: But* B F *is equal to* B E: *Which was the Proposition to be proved.*



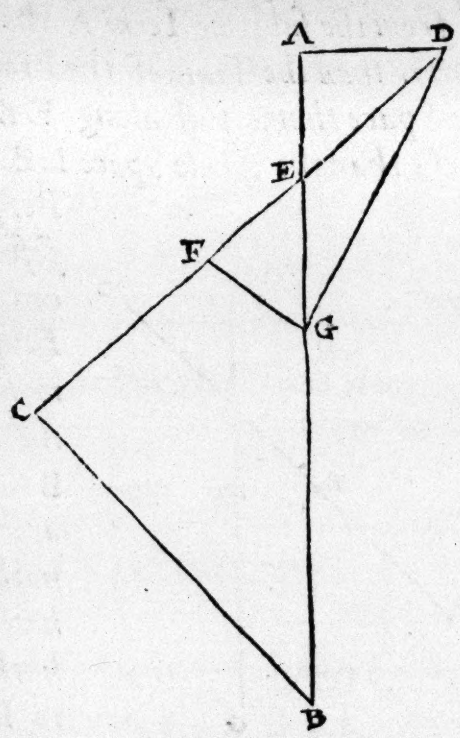
* From or after the Fall A B.

THEOR.

THEOR. XIII. PROP. XVI.

If the parts of an inclined Plane and Perpendicular, the Times of whose Motions *ex quiete* are equal, be joyned together at the same point, a Moveable coming out of any sublimer Height shall sooner passe the said part of the inclined Plane, than that part of the Perpendicular.

Let the Perpendicular *be* E B, *and the* Inclined Plane C E, *joyned at the same* Point E, *the Times of whose Motions from off Rest in* E *are equal, and in the* Perpendicular *continued out, let a sublime point A be taken at pleasure, out of which the Moveables may be let fall. I say, that the Inclined* Plane E C *shall be passed in a lesse Time than the* Perpendicular E B, *after the Fall A* E. *Draw a Line from* C *to* B, *and having drawn the Horizontal Line A* D *continue out* C E *till it meet the same in* D; *and let* D F *be a Mean-*Proportional *between* C D *and* D E; *and let A* G *be a Mean-*Proportional *between B A and A* E; *and draw* F G *and* D G. *And because the* Time *of the* Motion *along* E C *and* E B *out of Rest in* E *are equal, the Angle C shall be a Right Angle, by the second Corollary of the Sixth* Proposition; *and A is a Right Angle, and the Vertical Angles at* E *are equal:* Therefore *the Triangles A* E D *and* C E *B are equiangled, and the Sides about equal Angles are* Proportionals: *Therefore as B* E *is to* E C, *so is* D E *to* E *A. Therefore the* Rectangle B E *A is equal to the* Rectangle C E D: *And because the* Rectangle C D E *exceedeth the* Rectangle C E D, *by the Square* E D, *and the* Rectangle *B A* E *doth exceed the* Rectangle B E *A, by the Square* E *A: The excesse of the* Rectangle C D E *above the* Rectangle B *A* E, *that is of the Square* F D *above the Square A* G *shall be the same as the excesse of the Square* D E *above the Square A* E; *which excess is the Square* D *A: Therefore the Square* F D *is equal to the two Squares* G *A and A* D, *to which the Square* G D *is also equal: Therefore the*

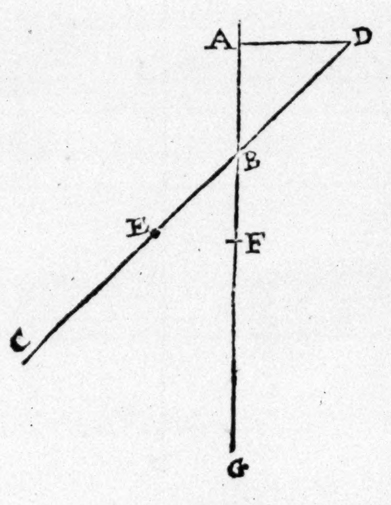


Line DF *is equal to* DG, *and the Angle* DGF *is equal to the Angle* DFG, *and the Angle* EGF *is lesse than the Angle* EFG, *and the opposite Side* EF *lesse than the Side* EG. *Now if we suppose the Time of the Fall along* AE *to be as* AE, *the Time by* DE *shall be as* DE; *and* AG *being a Mean-Proportional between* BA *and* AE, AG *shall be the Time along the whole* AB, *and the part* EG *shall be the Time along the Part* EB ex quiete *in* A. *And it may in like manner be proved that* EF *is the Time along* EC *after the Descent* DE, *or after the Fall* AE: *But* EF *is proved to be lesser than* EG: *Therefore the Proposition is proved.*

COROLLARY.

By this and the precedent it appears, that the Space that is passed along the Perpendicular after the Fall from above in the same Time in which the Inclined Plane is past, is lesse than that which is past in the same Time as in the Inclined, no fall from above preceding, yet greater than the said Inclined Plane.

For it having been proved, but now, that of the Moveables coming from the sublime Term A *the Time of the Conversion along* EC *is shorter than the Time of the Progression along* EB; *It is manifest that the Space that is past along* EB *in a Time equal to the Time along* EC *is less than the whole Space* EB. *And that the same Space along the Perpendicular is greater than* EC *is manifested by reassuming the Figure of the precedent Proposition, in which the part of the Perpendicular* BG *hath been demonstrated to be passed in the same Time as* BC *after the Fall* AB: *But that* BG *is greater than* BC *is thus collected. Because* BE *and* FB *are equal, and* BA *lesser than* BD, FB *hath greater proportion to* BA, *than* EB *hath to* BD: *And, by Composition,* FA *hath greater proportion to* AB, *than* ED *to* DB: *But as* FA *is to* AB, *so is* GF *to* FB, (*for* AF *is the Mean-Proportional between* BA *and* AG:) *And in like manner, as* ED *is to* BD, *so is* CE *to* EB: *Therefore* GB *hath greater proportion to* BF, *than* CB *hath to* BE: *Therefore* GB *is greater than* BC.

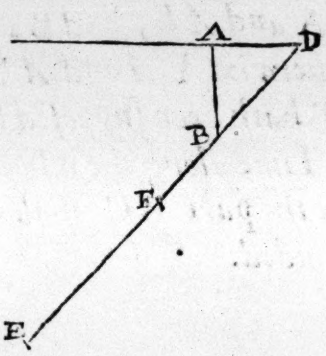


PROBL.

PROBL. IV. PROP. XVII.

A Perpendicular and Plane Inflected to it being given, to assign a part in the given Plane, in which after the Fall along the Perpendicular the Motion may be made in a Time equal to that in which the Moveable *ex quiete* passeth the Perpendicular given.

Let the Perpendicular be AB, *and a Plane Inflected to it* BE: *It is required in* BE *to assign a Space along which the Moveable after the Fall along* AB *may move in a Time equal to that in which the said Perpendicular* AB *is passed* ex quiete. *Let the Line* AD *be parallel to the Horizon, with which let the Plane prolonged meet in* D; *and suppose* FB *equal to* BA; *and as* BD *is to* DF, *so let* FD *be to* DE. *I say, that the Time along* BE *after the Fall along* AB *equalleth the Time along* AB, *out of Rest in* A. *For if we suppose* AB *to be the Time along* AB, DB *shall be the Time along* DB. *And because, as* BD *is to* DF, *so is* FD *to* DE, DF *shall be the Time along the whole Plane* DE, *and* BF *along the part* BE *out of* D: *But the Time along* BE *after* DB, *is the same as after* AB: *Therefore the Time along* BE *after* AB *shall be* BF, *that is, equal to the Time* ex quiete *in* A: *Which was the Proposition.*

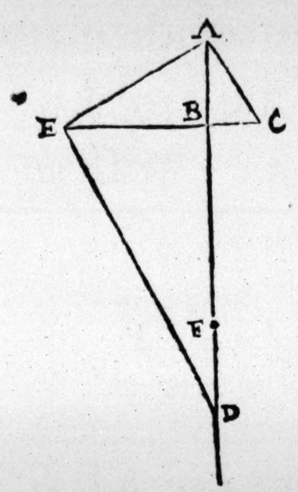


PROBL. V. PROP. XVIII.

Any Space in the Perpendicular being given from the assigned beginning of Motion that is passed in a Time given, and any other lesser Time being also given, to find another Space in the said Perpendicular that may be passed in the given lesser Time.

Let

Let the Perpendicular be AD, *in which let the Space assigned be* AB, *whose Time from the beginning* A *let be* AB: *and let the Horizon be* CBE, *and let a Time be given less than* AB, *to which let* BC *be noted equal in the Horizon: It is required in the said Perpendicular to find a Space equal to the same* AB *that shall be passed in the Time* BC. *Draw a Line from* A *to* C. *And because* BC *is lesse than* BA, *the Angle* BAC *shall be lesse than the Angle* BCA. *Let* CAE *be made equal to it, and the Line* AE *meet with the Horizon in the Point* E, *to which suppose* ED *a Perpendicular, cutting the Perpendicular in* D, *and let* DF *be cut equal to* BA. *I say, that the said* FD *is a part of the Perpendicular along which the Lation from the beginning of Motion in* A, *the Time* BC *given will be spent. For if in the Right-angled Triangle* AED, *a Perpendicular to the opposite Side* AD, *be drawn* EB, AE *shall be a Mean-Proportional betwixt* DA *and* AB, *and* BE *a Mean-Proportional betwixt* DB *and* BA, *or betwixt* FA *and* AB *(for* FA *is equal to* DB.) *And in regard* AB *hath been supposed to be the Time along* AB, AE, *or* EC *shall be the Time along the whole* AD, *and* EB *the Time along* AF: *Therefore the part* BC *shall be the Time along the part* FD: *Which was intended.*

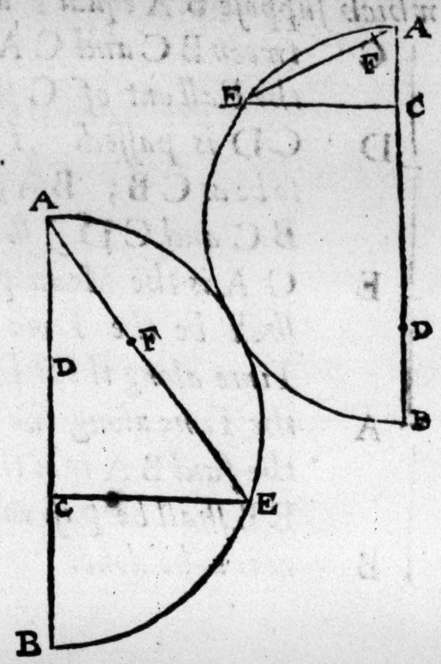


PROBL. VI. PROP. XIX.

Any Space in the Perpendicular passed from the beginning of the Motion being given, and the Time of the Fall being assigned, to find the Time in which another Space, equal to the given one, and taken in any part of the said Perpendicular, shall be afterwards past by the same Moveable.

In the Perpendicular AB *let* AC *be any Space taken from the beginning of the Motion in* A, *to which let* DB *be another equal Space taken any where at pleasure, and let the Time of the Motion along* AC *be given, and let it be* AC. *It is required to find the Time of the Motion*

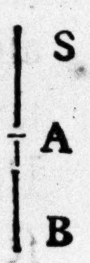
Motion along D B *after the Fall from* A. *About the whole* A B *describe a Semicircle* A E B, *and from* C *let fall* C E *a Perpendicular to* A B, *and draw a Line from* A *to* E; *which shall be greater than* E C. *Let* E F *be cut equall to* E C: *I say, that the remainder* F A *is the Time of the Motion along* D B. *For because* A E *is a Mean-proportional betwixt* B A *and and* A C, *and* A C *is the Time of the Fall along* A C; A E *shall be the Time along the Whole* A B. *And because* C E *is a Mean-proportional betwixt* D A *and* A C, (*for* D A *is equal to* B C) C E, *that is* E F *shall be the Time along* A D: *Therefore the Remainder* A F *shall be the Time along the Remainder* B B : *Which is the Proposition.*



COROLLARY.

Hence is gathered, that if the Time of any Space *ex quiete* be as the said Space, the Time thereof after another Space is added shall be the excesse of the Mean-proportional betwixt the Addition and Space taken together, and the said Space above the Mean-proportional betwixt the first Space and the Addition.

As for example, it being supposed that the Time along A B, *out of Rest in* A, *be* A B; A S *being another Space added, The Time along* A B *after* S A *shall be the excesse of the Mean-proportional betwixt* S B *and* B *A above the Mean-proportional betwixt* B *A and A* S.



PROBL VII. PROP. XX.

Any Space and a part therein after the begining of the Motion being given, to find another part towards the end that shall be past in the same Time as the first part given.

Let the Space be C B, *and let the part in it given after the begining of the Motion in* C *be* C D. *It is required to find another part towards the end* B, *which shall be past in the same Time as the*

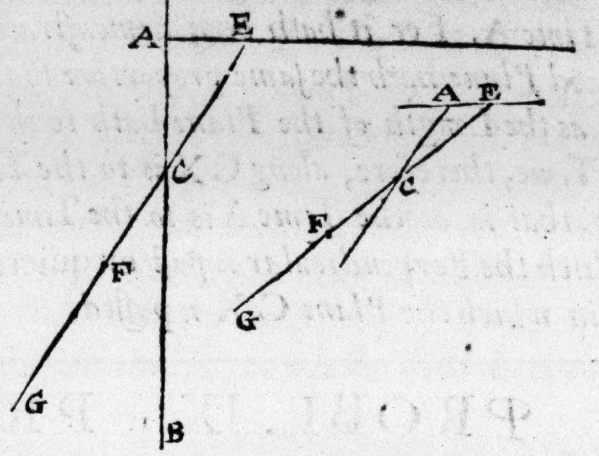
the given part CD. *Take a Mean-proportional betwixt* BC *and* CD, *to which suppose* BA *equal;* *and let* CE *be a third proportional between* BC *and* CA. *I say, that* EB *is the Space that after the Fall out of* C *shall be past in the same Time as the said* CD *is passed.* *For if we suppose the Time along* CB *to be as* CB; BA (*that is the Mean-proportional betwixt* BC *and* CD) *shall be the Time along* CD. *And because* CA *is the Mean proportional betwixt* BC *and* CE, CA *shall be the Time along* CE: *But the whole* BC *is the Time along the Whole* CB: *Therefore the part* BA *shall be the Time along the part* EB, *after the Fall out of* C: *But the said* BA *was the Time along* CD: *Therefore* CD *and* EB *shall be past in equal Times out of Rest in* C: *Which was to be done.*

THEOR. XIV. PROP. XXI.

If along the Perpendicular a Fall be made *ex quiete*, in which from the begining of the Motion a part is taken at pleasure, passed in any Time, after which an Inflex Motion followeth along any Plane however Inclined, the Space which along that Plane is passed in a Time equal to the Time of the Fall already made along the Perpendicular shall be to the Space then passed along the Perpendicular more than double, and lesse than triple.

F*Rom the Horizon* AE *let fall a Perpendicular* AB, *along which from the begining* A *let a* F*all be made, of which let a part* AC *be taken at pleasure; then out of* C *let any Plane* G *be inclined at pleasure:* *along which after the* F*all along* AC *let the Motion be continued.* *I say, the Space passed by that Motion along* CG *in a Time equall to the Time of the* F*all along* AC, *is more than double, and less than triple that same Space* AC. F*or suppose* CF *equal to* AC, *and extending out the Plane* GC *as far as the Horizon in* E, *and as* CE *is to* EF, *so let* FE *be to* EG. *If therefore we suppose the Time of the*

the Fall along A C *to be as the Line* A C ; C E *shall be the Time along* E C, *and* C F *or* C A *the Time of the Motion along* C G. *Therefore it is to be proved that the Space* C G *is more than double, and lesse than triple the said* C A. *For in regard that as* C E *is to* E F, *so is* F E *to* E G; *therefore also so is* C F *to* F G. *But* E C *is lesse than* E F: *Therefore* C F *shall be lesse than* F G, *and* G C *more than double to* F C *or* A C. *And moreover, in regard that* F E *is lesse than double to* E C, *(for* E C *is greater than* C A *or* C F *)* G F *shall also be lesse than double to* F C, *and* G C *lesse than triple to* C F *or* C A: *Which was to be demonstrated.*

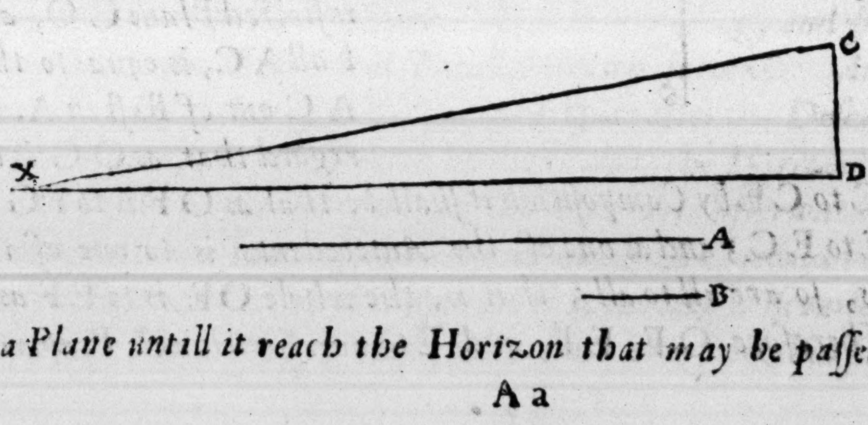


And the same may be more generally propounded: for that which hapneth in the Perpendicular and Inclined Plane, holdeth true also if after the Motion a Plane somewhat inclined it be inflected along a more inclining Plane, as is seen in the other Figure: And the Demonstration is the same.

PROBL. VIII. PROP. XXII.

Two unequall Times being given, and a Space that is past *ex quiete* along the Perpendicular in the shortest of those given Times, to inflect a Plane from the highest point of the Perpendicular unto the Horizon, along which the Moveable may descend in a Time equal to the longest of those Times given.

L*Et the unequal Times be* A *the greater, and* B *the lesser; and let the Space that is past* ex quiete *along the Perpendicular in the Time* B, *be* C D. *It is required from the Term* C *to inflect* [or



bend] *a Plane untill it reach the Horizon that may be passed in the*

 Time

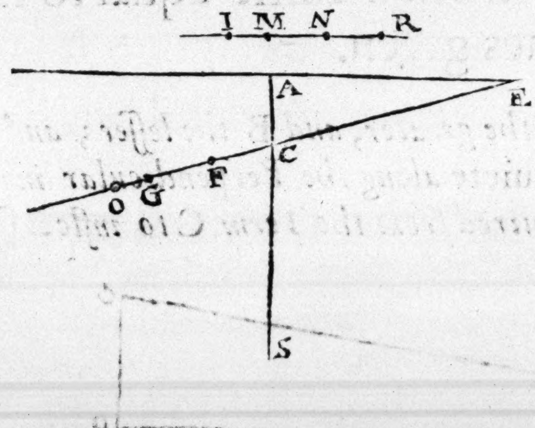
Time A. *As* B *is to* A, *so let* C D *be to another Line, to which let* C X *be equal that descendeth from* C *unto the Horizon: It is manifest that the Plane* C X *is that along which the Moveable descendeth in the Given Time* A. *For it hath been demonstrated, that the Time along the inclined Plane hath the same proportion to the Time along its * Elevation, as the Length of the Plane hath to the Length of its Elevation. The Time, therefore, along* C X *is to the Time along* C D, *as* C X *is to* C D, *that is, as the Time* A *is to the Time* B: *But the Time* B *is that in which the Perpendicular is past* ex quiete: *Therefore the Time* A *is that in which the Plane* C X *is passed.*

* Or Perpendicular.

PROBL. IX. PROP. XXIIII.

A Space past *ex quiete* along the Perpendicular in any Time being given, to inflect a Plane from the lowest term of that Space, along which, after the Fall along the Perpendicular, a Space equal to any Space given may be passed in the same Time: which neverthelesse is more than double, and lesse than triple the Space passed along the Perpendicular.

Along the Perpendicular A S, *in the Time* A C, *let the Space* A C *be past* ex quiete *in* A; *to which let* I R *be more than double, and lesse than triple. It is required from the Terme* C *to inflect a Plane, along which a Moveable after the Fall along* A C *may in the same Time* A C *passe a Space equal to the said* I R. *Let* R N, *and* N M *be equal to* A C: *And look what proportion the part* I M *hath to* M N, *the same shall the Line* A C *have to another, equal to which draw* C E *from* C *to the Horizon* A E, *which continue out towards* O, *and take* C F, F G, *and* G O, *equal to the said* R N, N M, *and* M I. *I say, that the Time along the inflected Plane* C O, *after the Fall* A C, *is equal to the Time* A C *out of Rest in* A. *For in regard that as* O G *is to* G F,



so is F C *to* C E *by Composition it shall be that as* O F *is to* F G *or* F C, *so is* F E *to* E C; *and as one of the Antecedents is to one of the Consequents, so are all to all; that is, the whole* O E *is to* E F *as* F E *to* E C: *Therefore* O E, E F, *and* E C *are Continual Proportionals:*

And

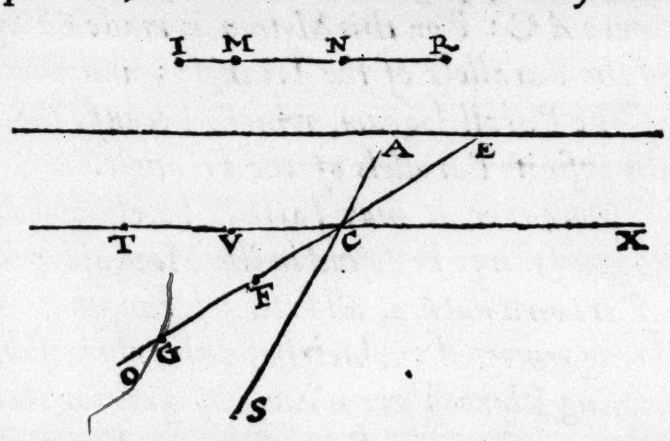
And since it was supposed that the Time along AC *is as* AC, CE *shall be the Time along* EC; *and* EF *the Time along the whole* EO; *and the part* CF *that along the part* CO: *But* CF *is equal to the said* CA: *Therefore that is done which was required: For the Time* CA *is the Time of the Fall along* AC ex quiete *in* A; *and* CF (*which is equal to* CA) *is the Time along* CO, *after the Descent along* EC, *or after the Fall along* AC: *Which was the Proposition.*

And here it is to be noted, that the same may happen if the preceding Motion be not made along the Perpendicular, but along an Inclined Plane: As in the following Figure, in which let the preceding Lation be made along the inclined Plane AS *beneath the Horizon* AE: *And the Demonstration is the very same.*

SCHOLIUM.

If one observe well, it shall be manifest, that the lesse the given Line IR wanteth of being triple to the said AC, the nearer shall the Inflected Plane, along which the second Motion is to be made, which suppose to be CO, come to the Perpendicular, along which in a Time equal to AC a Space shall be passed triple to AC.

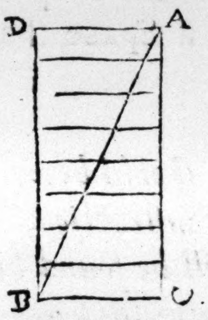
F*Or in case* IR *were very near the triple of* AC, IM *should be well-near equal to* MN: *And if, as* IM *is to* MN *by Construction, so* AC *is to* CE, *then it is evident that the said* CE *will be found but little bigger than* CA, *and, which followeth of consequence, the point* E *shall be found very near the point* A, *and* CO *to containe a very acute Angle with* CS, *and almost to concur both in one Line. And on the contrary, if the said* IR *were but a very little more than double the said* AC, IM *should be a very short Line. Hence it may happen also that* AC *may come*



to be very short in respect of CE *which shall be very long, and shall approach very near the Horizontal Parallel drawn from* C. *And from hence we may collect, that if in the present Figure after the Descent along the inclined Plane* AC, *a Reflexion be made along the Horizontal Line, as* v. gr. CT, *the Space along which the Moveable afterwards moved in a Time equal to the Time of the Descent along* AC *would be exactly double to the Space* AC. *And it appears that the like Discourse may be here applied: For it is apparent by what hath been said, that since* OE

is to EF, *as* FE *is to* EC, *that* FC *determineth the Time along* CO: *And if a part of the Horizontal Line* TC *double to* CA *be divided in two equal parts in* V, *the extension towards* X *shall be prolonged* in infinitum, *whilst it seeks to meet with the prolonged Line* AE: *And the proportion of the Infinite Line* TX *to the Infinite Line* VX, *shall be no other than the proportion of the Infinite Line* VX *to the Infinite Line* XC.

We may conclude the self-same thing another way by reassuming the same Reasoning that we used in the Demonstration of the first Proposition. For resuming the Triangle ABC, *representing to us by its Parallels to the Base* BC *the Degrees of Velocity continually encreased according to the encreases of the Time; from which, since they are infinite, like as the Points are infinite in the Line* AC, *and the Instants in any Time, shall result the Superficies of that same Triangle, if we understand the Motion to continue for such another Time, but no farther with an Accelerate, but with an Equable Motion, according to the greatest degree of Velocity acquired, which degree is represented by the Line* BC. *Of such degrees shall be made up an Aggregate like to a Parallelogram* ADBC, *which is the double of the Triangle* ABC. *Wherefore the Space which with degrees like to those shall be passed in the same Time, shall be double to the Space past with the degrees of Velocity represented by the Triangle* ABC: *But along the Horizontal Plane the Motion is Equable, for that there is no cause of Acceleration, or Retardation: Therefore it may be concluded that the Space* CD, *passed in a Time equall to the Time* AC *is double to the Space* AC: *For this Motion is made* ex quiete *Accelerate according to the Parallels of the Triangle; and that according to the Parallels of the Parallelogram, which, because they are infinite, are double to the infinite Parallels of the Triangle.*

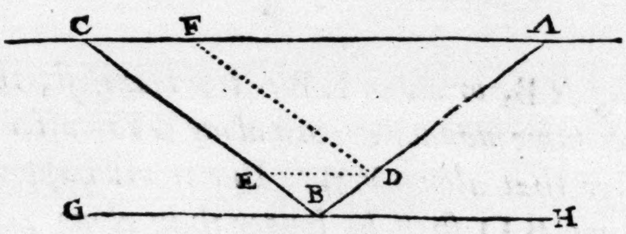


Moreover it may farther be observed, that what ever degree of swiftness is to be found in the Moveable, is indelibly impressed upon it of its own nature, all external causes of Acceleration or Retardation being removed; which hapneth only in Horizontal Planes: for in declining Planes there is cause of greater Acceleration, and in the rising Planes of greater Retardation. From whence in like manner it followeth that the Motion along the Horizontal Plane is also Perpetual: for if it be Equable, it can neither be weakned nor retarded, nor much lesse destroyed. Farthermore, the degree of Celerity acquired by the Moveable in a Natural Descent, being of its own Nature Indelible and Perpetual, it is worthy consideration, that if after the Descent along a declining Plane a Reflexion be made along another Plane that is rising, in this latter there is cause of Retardation, for in these kind of Planes

the

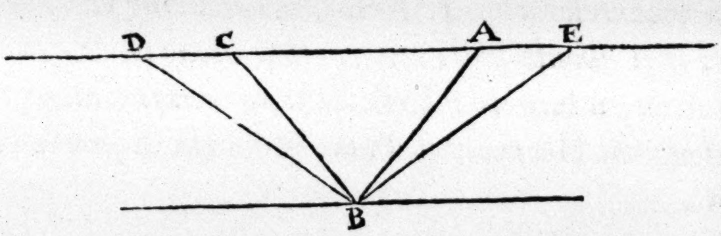
the said Moveable doth naturally descend; whereupon there results a mixture of certain contrary Affections, to wit, that degree of Celerity acquired in the precedent Descent, which would of it self carry the Moveable uniformly in infinitum, *and of Natural Propension to the Motion of Descent according to that same proportion of Acceleration wherewith it alwaies moveth. So that it will be but reasonable, if, enquiring what accidents happen when the Moveable after the Descent along any inclined Plane is Reflected along some rising Plane, we take that greatest degree acquired in the Descent to keep it self perpetually the same in the Ascending Plane; But that there is superadded to it in the Ascent the Natural Inclination downwards, that is the Motion from Rest Accelerate according to the received proportion: And lest this should, perchance, be somewhat intricate to be understood, it shall be more clearly explained by a Scheme.*

Let the Descent therefore be supposed to be made along the Declining Plane AB, *from which let the Reflex Motion be continued along another Rising Plane* BC: *And in the first place let the Planes be equal, and elevated at equal Angles to the Horizon* GH. *Now it is manifest, that the Moveable* ex quiete *in* A *descending along* AB *acquireth degrees of Velocity according to the increase of its Time, and that the degree in* B *is the greatest of those acquired and by Nature immutably impressed, I mean the Causes of new Acceleration or Retardation being removed: of Acceleration, I say, if it should passe any farther along the extended Plane; and of Retardation, whilst the Reflection is making along the Acclivity* BC: *But along the Horizontal Plane* GH *the Equable Motion according to the degree of Velocity acquired from* A *unto* B *would extend* in infinitum. *And such a Velocity would that be which in a Time equal to the Time of the*



Descent along AB *would passe a Space in double the Horizon to the said* AB. *Now let us suppose the same Moveable to be Equably moved with the same degree of Swiftnesse along the Plane* BC, *in such sort that also in this Time equal to the Time of the Descent along* AB *a Space may be passed along* BC *extended double to the said* AB. *And let us understand that as soon as it beginneth to ascend there naturally befalleth the same that hapneth to it from* A *along the Plane* AB, *to wit, a certain Descent* ex quiete *according to those degrees of Acceleration, by vertue of which, as it befalleth in* AB, *it may descend as much in the same Time along the Reflected Plane as it doth along* AB: *It is manifest, that by this same Mixture of the Equable Motion of Ascent, and the Accelerate of Descent the Moveable may be carried up to the Term* C *along the Plane* BC *according to those degrees of Velocity, which shall be equal.*

equal. And that two points at pleasure D *and* E *being taken, equally remote from the Angle* B, *the Transition along* D B *is made in a Time equal to the Time of the Reflection along* B E, *we may collect from hence: Draw* D F, *which shall be Parallel to* B C; *for it is manifest that the Descent along* A D *is reflected along* D F: *And if after* D *the Moveable passe along the Horizontal Plane* D E, *the* Impetus *in* E *shall be the same as the* Impetus *in* D: *Therefore it will ascend from* E *to* C: *And therefore the degree of Velocity in* D *is equal to the degree in* E. *From these things, therefore, we may rationally affirm, that, if a descent be made along any inclined Plane, after which a Reflection may follow along an elevated Plane, the Moveable may by the conceived* Impetus *ascend untill it attain the same height, or Elevation from the Horizon. As if a Descent be made along* A B, *the Moveable would passe along the Reflected Plane* B C, *untill it arrive at the Horizon* A C D; *and that not only when the Inclinations of the Planes are equal, but also when they are unequal, as is the Plane* B D: *For it was first supposed, that the degrees of Velocity are equal, which are acquired upon Planes unequally inclined, so long as the Elevation of those Planes above the Horizon was the same: But, if there being the same Inclination of the Planes* E B *and* B D, *the Descent along* E B *sufficeth to drive the Moveable along the Plane* B D *as far as* D, *seeing this Impulse is made by the* Impetus *of Velocity in the point* B; *and if the* Impetus *be the same in* B, *whether the Moveable descend along* A B, *or along* E B: *It is manifest, that the Moveable shall be in the same manner driven along* B D, *after the Descent along* A B, *and after that along* E B: *But it will happen that the Time of the Ascent along* B D *shall be longer than along* B C, *like as the Descent along* E B *is made in a longer time than along* A B: *But the* Proportion *of those Times was before demonstrated to be the same as the Lengths of those Planes. Now it follows, that we seek the proportion of the Spaces past in equal Times along Planes, whose Inclinations are different, but their Elevations the same; that is, which are comprehended between the same Horizontal Parallels. And this hapneth according to the following Proposition.*

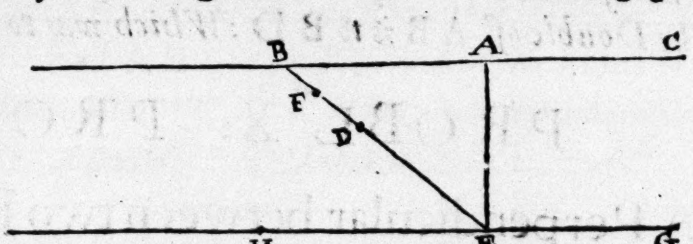


THEOR.

THEOR. XV. PROP. XXIV.

There being given between the same Horizontal Parallels a Perpendicular and a Plane elevated from its lowest term, the Space that a Moveable after the Fall along the Perpendicular passeth along the Elevated Plane in a Time equal to the Time of the Fall, is greater than that Perpendicular, but lesse than double the same.

Between the same Horizontal Parallels B C *and* H G *let there be the Perpendicular* A E; *and let the Elevated Plane be* E B, *along which after the Fall along the Perpendicular* A E *out of the Term* E *let a Reflexion be made towards* B. *I say, that the Space, along which the Moveable ascendeth in a Time equal to the Time of the Descent* A E, *is greater than* A E, *but lesse than double the same* A E. *Let* E D *be equal to* A E, *and as* E B *is to* B D, *so let* D B *be to* B F. *It shall be proved, first that the point* F *is the Term at which the Moveable with a Reflex Motion along* E B *arriveth in a Time equal to the Time* A E: *And then, that* E F *is greater than* E A, *but lesse than double the same. If we suppose the Time of the Descent along* A E *to be as* A E, *the Time of the Descent along* B E, *or Ascent along* E B *shall be as the same Line* B E: *And* D B *being a Mean-Proportional betwixt* E B *and* B F, *and* B E *being the Time of Descent along the whole* B E, B D *shall be the Time of the Descent along* B F, *and the Remaining part* D E *the Time of the Descent along the Remaining part* F E: *But the Time along* F E *ex quiete in* B, *and the Time of the Ascent along* E F *is the same, since that the Degree of Velocity in* E *was acquired along the Descent* B E, *or* A E: *Therefore the same Time* D E *shall be that in which the Moveable after the Fall out of* A *along* A E, *with a Reflex Motion along* E B *shall reach to the Mark* F: *But it hath been supposed that* E D *is equal to the said* A E: *Which was first to be proved. And because that as the whole* E B *is to the whole* B D, *so is the part taken away* D B *to the part taken away* B F, *therefore, as the whole* E B *is to the whole* B D, *so shall the Remainder* E D *be to* D F: *But* E B *is greater than* B D: *Therefore* E D *is greater than* D F, *and* E F *lesse than double to* D E *or* A E: *Which was to be proved.*



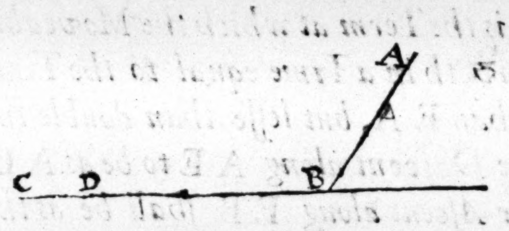
And

And the same also hapneth if the precedent Motion be not made along the Perpendicular, but along an Inclined Plane; and the Demonstration is the same, provided that the Reflex Plane be lesse rising, that is, longer than the declining Plane.

THEOR. XVI. PROP. XXV.

If after the Descent along any Inclined Plane a Motion follow along the Plane of the Horizon, the Time of the Descent along the Inclined Plane shall be to the Time of the Motion along any Horizontal Line; as the double Length of the Inclined Plane is to the Line taken in the Horizon.

Let the Horizontal Line be C B, *the inclined Plane* A B, *and after the Descent along* A B *let a Motion follow along the Horizon, in which take any Space* B D. *I say, that the Time of the Descent along* A B *to the Time of the Motion along* B D *is as the double of* A B *to* B D. *For* B C *being supposed the double of* A B, *it is manifest by what hath already been demonstrated that the Time of the Descent along* A B *is equal to the Time of the Motion along* B C: *But the Time of the Motion along* B C *is to the Time of the Motion along* B D, *as the Line* C B *is to the Line* B D: *Therefore the Time of the Motion along* A B *is the Time along* B D, *as the Double of* A B *is to* B D: *Which was to be proved.*

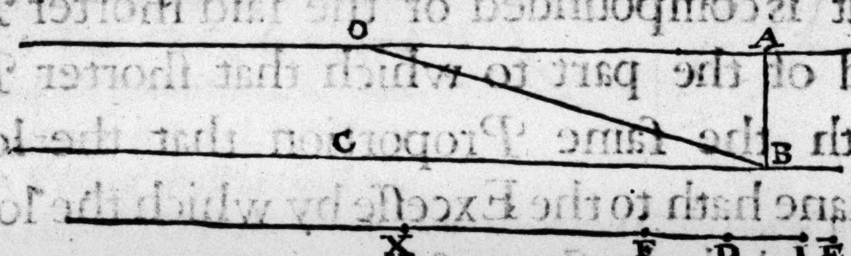


PROBL. X. PROP. XXVI.

A Perpendicular between two Horizontal Parallel Lines, as also a Space greater than the said Perpendicular, but lesse than double the same, being given, to raise a Plane between the said Parallels from the lowest Term of the Perpendicular, along which the Moveable may with a Reflex Motion after the Fall along the Perpendicular passe a Space equal to the Space given, and in a Time equal to the Time of the Fall along the Perpendicular.

Let

Let AB *be a Perpendicular between the Horizontal Parallels* AO *and* BC; *and let* FE *be greater than* BA, *but lesse than double the same. It is required between the said Parallels from the point* B *to raise a Plane, along which the Moveable after the Fall from* A *to* B *may with a Reflex Motion in a Time equal to the Time of the Fall along* AB *passe a Space ascending equal to the said* EF. *Suppose* ED *equal to* AB, *the Remaining Part* DF *shall be lesse, for that the whole* EF *is lesse than double to* AB: *Let* DI *be equal to* DF, *and as* EI *is to* ID, *so let* DF *be to another Space* FX, *and out of* B *let the Right-*

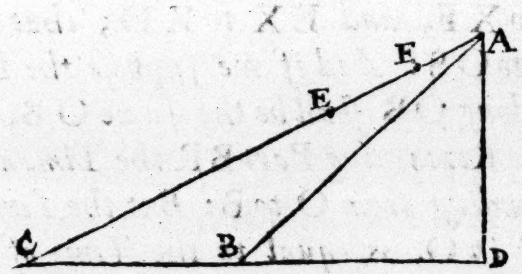


Line BO *be reflected, equal to* EX. *I say, that the Plane along* BO *is that along which after the Fall* AB *a Moveable in a Time equal to the Time of the Fall along* AB *passeth ascending a Space equal to the given Space* EF. *Suppose* BR *and* RS *equal to the said* ED *and* DF. *And because that as* EI *is to* ID, *so is* DF *to* FX; *therefore, by Composition, as* ED *is to* DI, *so shall* DX *be to* XF; *that is, as* ED *is to* DF, *so shall* DX *be to* XF, *and* EX *to* XD; *that is, as* BO *is to* OR, *so shall* RO *be to* OS: *And if we suppose the Time along* AB *to be* AB, *the Time along* OB *shall be the same* OB, *and* RO *the Time along* OS, *and the Remaining Part* BR *the Time along the Remaining Part* SB, *descending from* O *to* B: *But the Time of the Descent along* SB *from Rest in* O, *is equal to the Time of the Ascent from* B *to* S *after the Fall* AB: *Therefore* BO *is the Plane elevated from* B, *along which after the Fall along* AB *the Space* BS *equal to the given Space* EF *is passed in the Time* BR *or* BA: *Which was required to be done.*

THEOR. XVII. PROP. XXVII.

If a Moveable descend along unequal Planes, whose Elevation is the same, the Space that shall be past along the lower part of the longest in a Time equal to that in which the whole shorter Plane is passed, is equal to the Space that is compounded of the said shorter Plane and of the part to which that shorter Plane hath the same Proportion that the longer Plane hath to the Excesse by which the longest exceedeth the shortest.

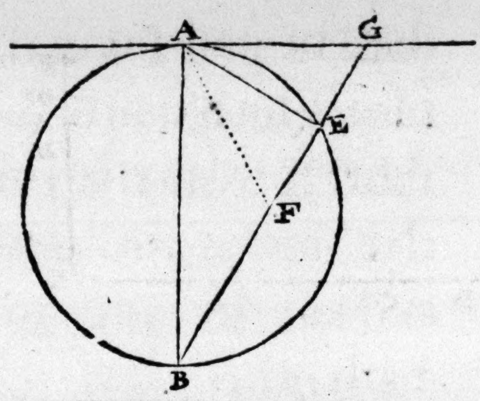
Let AC *be the longer Plane, and* AB *the shorter, whose Elevation* AD *is the same; and in the lower part of* AC *take the Space* CE, *equal to the said* AB; *and as* CA *is to* AE, *(that is to the excesse of the Plane* CA *above* AB*) so let* CE *be to* EF. *I say, that the Space* FC *is that which is past after the Descent out of* A *in a Time equal to the Time of the Descent along* AB. *For the whole* CA, *being to the whole* AE, *as the part taken away* CE *is to the part taken away* EF, *therefore the remaining part* EA *shall be to the remaining part* AF, *as the whole* CA *is to the whole* AE*: Therefore the three Spaces* CA, AE, *and* AF *are three Continual proportionals. And if the Time along* AB *be supposed to be as* AB, *the Time along* AC *shall be as* AC, *and the Time along* AF *shall be as* AE, *and along the remaining part* FC *shall be as* EC*: But* EC *is equal to the said* AB*: Therefore the Proposition is manifest.*



THEOR. XVIII. PROP. XXVIII.

Let the Horizontal Line AG *be Tangent to a Circle, and from the point of Contact let* AB *be the Diameter, and* AEB *two Chords at pleasure: We are to assign the proportion of the Time of the Fall along* AB *to the Time of the Descent along both the Chords* AEB. *Let* BE *be continued out till it meet the Tangent in* G, *and let*

let the Angle B A E *be cut in two equal parts, and draw* A F. *I say, that the Time along* A B *is to the Time along* A E B, *as* A E *is to* A E F. *For in regard the Angle* F A B *is equal to the Angle* F A E, *and the Angle* E A G *to the Angle* A B F, *the whole Angle* G A F *shall be equal to the two Angles* F A B, *and* A B F; *to which also the Angle* G F A *is equal: Therefore the Line* G F *is equal to* G A. *And because the Rectangle* B G E *is equal to the Square of* G A, *it shall likewise be equal to the Square of* G F, *and the three Lines* B G, G F, *and* G E *shall be proportionals. And if we suppose* A E *to be the Time along* A E, G E *shall be the Time along* G E, *and* G F *the Time along the whole* G B, *and* E F *the Time along* E B, *after the Descent out of* G, *or out of* A, *along* A E: *The Time, therefore, along* A E, *or along* A B *shall be to the Time along* A E B, *as* A E *is to* A E F: *Which was to be determined.*



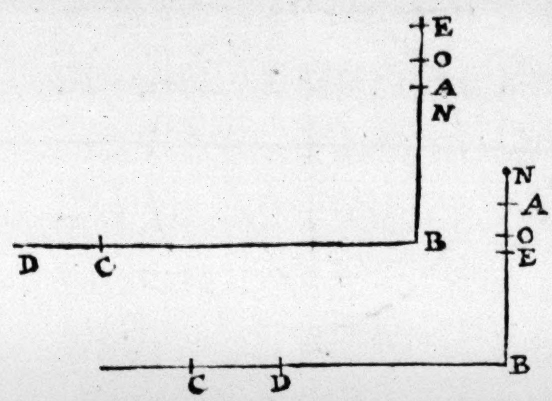
More briefly thus. Let G F *be cut equal to* G A: *It is manifest that* G F *is the Mean-proportional between* B G, *and* G E. *The rest as before.*

PROBL. XI. PROP. XXIX.

Any Horizontal Space being given upon the end of which a Perpendicular is erected, in which a part is taken equal to half of the Space given in the Horizontal a Moveable falling from that height, and turned along the Horizon, shall passe the Horizontal Space together with the Perpendicular in a shorter Time than any other Space of the Perpendicular with the same Horizontal Space.

L*Et there be an Horizontal Space in which let any Space be given* B C, *and on* B *let there be a Perpendicular erected, in which let* B A *be the half of the foresaid* B C. *I say, that the Time in which a Moveable let fall out of* A *passeth both the Spaces* A B *and* B C *is the shortest of all Times in which the said Space* B C *with a part of the Perpendicular, whether greater or lesser than the part* A B, *shall be passed. Let a greater be taken, as in the first Figure, or lesser, as in the*

second, which let be E B. *It is to be proved that the Time in which the Spaces* E B *and* B C *are passed is longer than the Time in which* A B *and* B C *are passed. Let the Time along* A B *be as* A B; *the same shall be the Time of the Motion along the Horizontal Space* B C; *because* B C *is double to* A B, *and the Time along both the Spaces* A B C *shall be double of* O B A. *Let* B O *be a Mean-proportional between* E B *and* B A. B O *shall be the Time of the Fall along* E B. *Again, let the Horizontal Space* B D *be double to the said* E E: *It is manifest that the Time of it after the Fall* E B *is the same* B O. *As* D B *is to* B C, *or as* E B *is to* B A, *so let* O B *be to* B N: *and in regard the Motion along the Horizontal Plane is Equable, and* O B *being the Time along* B D *after the Fall out of* E, *therefore* N B *shall be the Time along* B C *after the Fall from the same Altitude* E. *Hence it is manifest, that* O B, *together with* B N *is the Time along* E B C; *and because the double of* B A *is the Time along* A B C; *it remains to be proved, that* O B, *together with* B N *is more than double* B A. *Now because* O B *is a Mean between* E B *and* B A, *the proportion of* E B *to* B A *is double the proportion of* O B *to* B A: *and, in regard that* E B *is to* B A, *as* O B *is to* B N, *the proportion of* O B *to* B N *shall also be double the proportion of* O B *to* B A: *But that proportion of* O B *to* B N *is compounded of the proportions of* O B *to* B A, *and of* A B *to* B N: *therefore the proportion of* A B *to* B N *is the same with that of* O B *to* B A. *Therefore* B O, B A, *and* B N *are three continual Proportionals, and* O B, *together with* B N, *are greater than double* B A: *Whereupon the Proposition is manifest.*

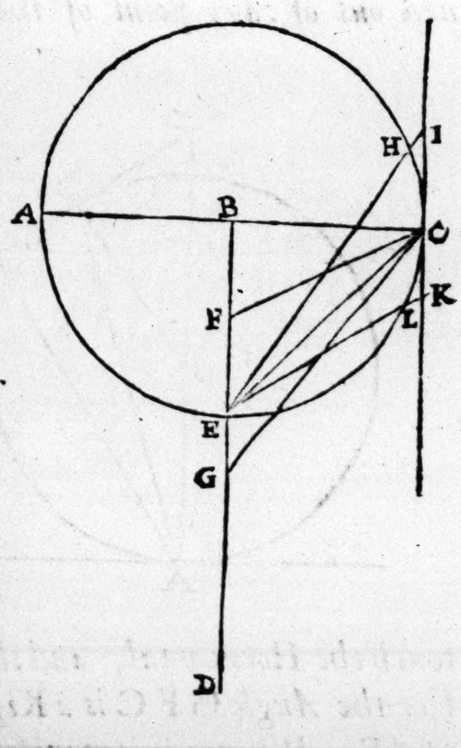


THEOR.

THEOR. XIX. PROP. XXX.

If a Perpendicular be let fall from any point of the Horizontal Line, and out of another point in the ſame Horizontal Line a Plane be drawn forth untill it meet the Perpendicular, along which a Moveable deſcendeth in the ſhorteſt time unto the ſaid Perpendicular, this Plane ſhall be that which cutteth off a part equall to the diſtance of the aſſigned point from the end of the Perpendicular.

Let the Perpendicular BD *be let fall from the point* B *of the Horizontal Line* AC, *in which let there be any point* C, *and in the Perpendicular let the Diſtance* BE *be ſuppoſed equal to the Diſtance* BC, *and draw* CE. *I ſay, that of all Planes inclined out of the point* C *till they meet the Perpendicular* CE *is that, along which in the ſhorteſt of all Times the Deſcent is made unto the Perpendicular. For let the Planes* CF *and* CG *be inclined above and below, and draw* IK *a Tangent unto the Semidiameter* BC *of the deſcribed Circle in* C, *which ſhall be equidiſtant from the Perpendicular; and unto the ſaid* CF *let* EK *be Parallel cutting the Circumference of the Circle in* L: *It is manifeſt that the Time of the Deſcent along* LE *is equal to the Time of the Deſcent along* CE: *But the Time along* KE *is longer than along* LE: *Therefore the Time along* KE *is longer than that along* CE: *But the Time along* KE *is equal to the Time along* CF, *they being equal, and drawn according to the ſame Inclination: Likewiſe ſince* CG, *and* IE *are equal, and inclined according to the ſame Inclination, the Times of the Motions along them ſhall be equal: But* HE *being ſhorter than* IE, *the Time along it is alſo ſhorter than* IE: *Therefore the Time alſo along* CE, (*which is equal to the Time along* HE) *ſhall be ſhorter than the Time along* IE: *The Propoſition, therefore, is manifeſt.*

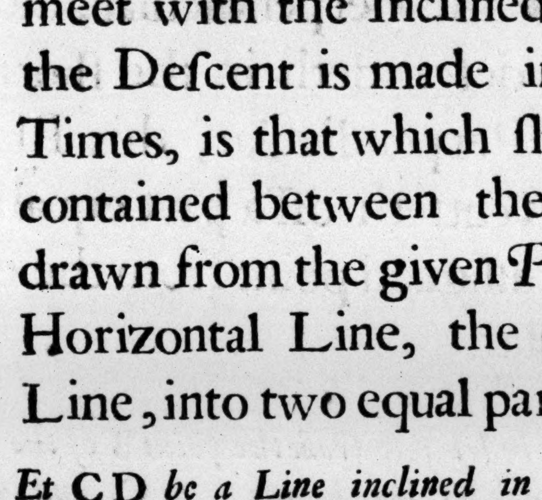


THEOR.

THEOR. XX. PROP. XXXI.

If a Right-Line shall be in any manner inclined upon the Horizontal Line, the Plane produced from a given point in the Horizon untill it meet with the Inclined Plane, along which the Descent is made in the shortest of all Times, is that which shall divide the Angle contained between the two Perpendiculars drawn from the given Point, the one unto the Horizontal Line, the other to the Inclined Line, into two equal parts.

Let CD be a Line inclined in any manner upon the Horizontal Line AB, and let any point A be given in the Horizon, and from it let AC be drawn Perpendicular to AB, and AE Perpendicular to CD, and let the Line FA divide the Angle CAE into two equal parts. I say, that of all Planes inclined out of any point of the Line CD to the point A that same produced along FA is it along which the Descent is made in the shortest of all Times. Let FG be drawn Parallel to AE; the alternate Angles GFA and FAE shall be equal: But EAF is equal to that other FAG: Therefore of the Triangle the Sides FG and GA shall be equal. If therefore about the Center G, at the distance GA, a Circle be described it shall passe by F, and shall touch the Horizontal, and the Inclined Lines in the points A and F: For the Angle GFC is a Right Angle, and likewise GF is equidistant to AE: Whence it is manifest that all Lines produced from the point A unto the inclined Plane do extend beyond the Circumference, and, which followeth of consequence, that the Motions along the same do take up more Time than along FA. Which was to be demonstrated.

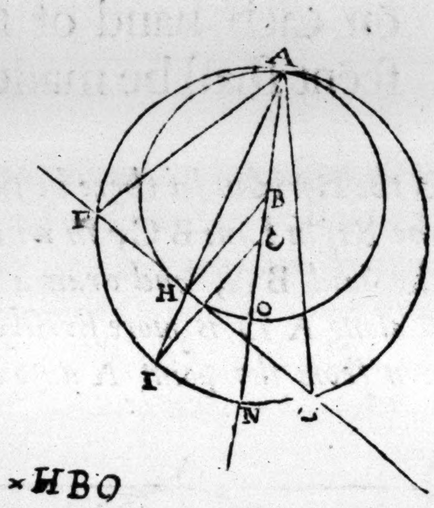


LEMMA

LEMMA.

If two Circles touch one another within, the innermost of which toucheth some Right Line, and the exteriour one cutteth it, three Lines produced from the Contact of the Circles unto three points of the Tangent Right-Line, that is, to the Contact of the interiour Circle, and to the Sections of the exteriour shall contain equall Angles in the Contact of the Circles.

Let two Circles touch one another in the point A, *of which let the Centers be* B, *that of the lesser, and* C *that of the greater; and let the interiour Circle touch any Line* F G *in the point* H, *and let the greater cut it in the points* F *and* G, *and connect the three Lines* A F, A H, *and* A G. *I say, that the Angles by them contained* F A H *and* G A H *are equal. Produce* A H *untill it meeteth the Circumference in* I, *and from the Centers draw* B H *and* C I, *and thorow the said Centers let* B C *be drawn, which continued forth shall meet with the Contact* A, *and with the Circumferences of the Circles in* O *and* N. *And because the Angles* I C N *and*



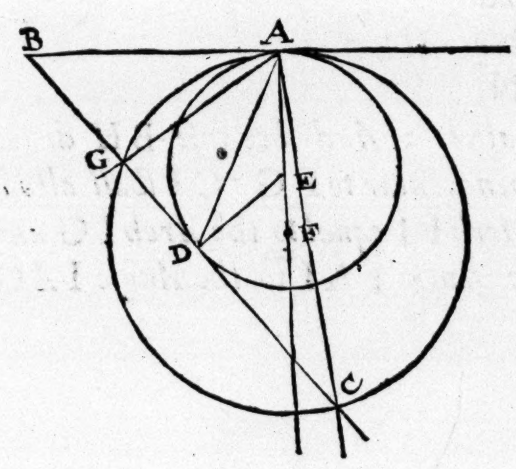
* ~~HOB~~ *are equal, for as much as either of them is double to the Angle* I A N, * HBO *the Lines* B H *and* C I *shall be Parallels: And because* B H *drawn from the Center to the Contact is Perpendicular to* F G; C I *shall also be Perpendicular to the same, and the Arch* F I *equal to the Arch* I G, *and, which followeth of consequence, the Angle* F A I = *to the Angle* I A G: *Which was to be demonstrated.*

THEOR.

THEOR. XXI. PROP. XXXII.

If two points be taken in the Horizon, and any Line ſhould be inclined from one of them towards the other, out of which a Right-Line is drawn unto the Inclined Line, cutting off a part thereof equal to that which is included between the points of the Horizon, the Deſcent along this laſt drawn ſhall be ſooner performed, than along any other Right Lines produced from the ſame point unto the ſaid Inclined Line. And along other Lines which are on each hand of this by equal Angles a Deſcent ſhall be made in equal Times.

IN the Horizon let there be two points A *and* B, *and from* B *incline the Right Line* B C, *in which from the Term* B *take* B D *equal to the ſaid* B A, *and draw a Line from* A *to* D. *I ſay, that the Deſcent along* A D *is more ſwiftly made, than along any other whatſoever drawn from the point* A *unto the inclined Line* B C. *For out of the points* A *and* D *unto* B A *and* B D *draw the Perpendiculars* A E *and* D E, *interſecting one another in* E: *and foraſmuch as in the equicrural Triangle* A B D *the Angles* B A D *and* B D A *are equal, the remainders to the Right-Angles* D A E *and* E D A *ſhall be equal. Therefore a Circle deſcribed about the Center* E *at the diſtance* A E *ſhall alſo paſſe by* D; *and the Lines* B A *and* B D *will touch it in the points* A *and* D. *And ſince* A *is the end of the Perpendicular* A E, *the Deſcent along* A D *ſhall be ſooner performed, than along any other produced from the ſame Term* A *unto the Line* B C *beyond the Circumference of the Circle: Which was firſt to be proved.*



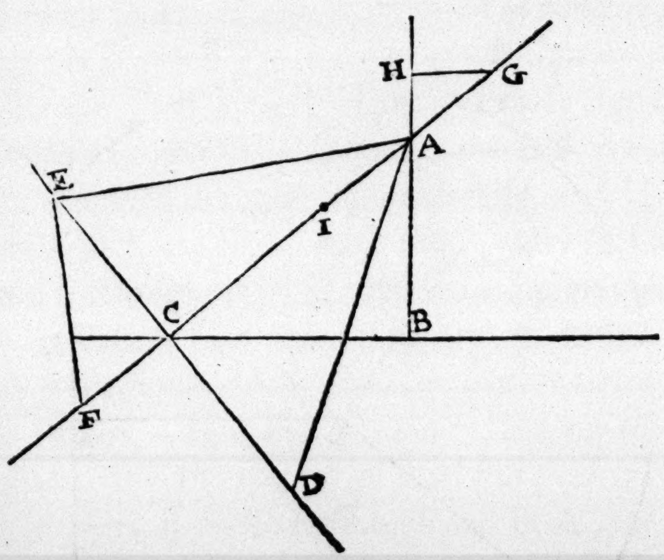
But if in the Perpendicular A E *being prolonged any Center be taken as* F, *and at the diſtance* F A *the Circle* A G C *be deſcribed cutting the Tangent Line in the points* G *and* C; *drawing* A G *and* A C *they ſhall make equal Angles with the middle Line* A D *by what hath been afore demon-*

demonstrated, and the Motions thorow them shall be performed in equal Times seeing that they terminate in A unto the Circumference of the Circle A G O *from the highest point of it A.*

PROBL. XII. PROP. XXXIII.

A Perpendicular and Plane inclined to it being given, whose height is one and the same, as also the highest term, to find a point in the Perpendicular above the common term, out of which if a Moveable be demitted that shall afterwards turn along the inclined Plane, the said Plane may be past in the same Time in which the Perpendicular *ex quiete* would be passed.

Let the Perpendicular and inclined Plane, whose Altitude is the same, be A B *and A* C. *It is required in the Perpendicular* B *A, continued out from the point A to find a Point out of which a Moveable descending may passe the Space A* C *in the same Time in which it will passe the said Perpendicular A* B *out of Rest in A. Draw* D C E *at Right-Angles to A* C, *and let* C D *be cut equal to A* B, *and draw a Line from A to* D : *The Angle A* D C *shall be greater than the Angles* C *A* D : (*for* C *A is greater than A*B *or* C D :) *Let the Angle* D *A* E *be equal to the Angle A* D E ; *and to A* E *let* E F *an inclined Plane be Perpendicular, and let both being prolonged meet in* F, *and unto both A* I *and A* G *suppose* C F *to be equal, and by* G *draw* G H *equidistant to the Horizon. I say, that* H *is the point which is sought. For supposing the Time of the Fall along the Perpendicular A* B *to be A* B, *the Time along*



A C ex quiete *in A shall be the same A* C. *And because in the Right-angled Triangle A* E F, *from the Right Angle* E *unto the Base A* F, E C *is a Perpendicular, A* E *shall be a Mean-Proportional betwixt* F *A and A* C, *and* C E *a Mean betwixt A* C *and* C F, *that is, betwixt* C *A and A* I : *and forasmuch as the Time of A* C *out of A is A* C, *A* E

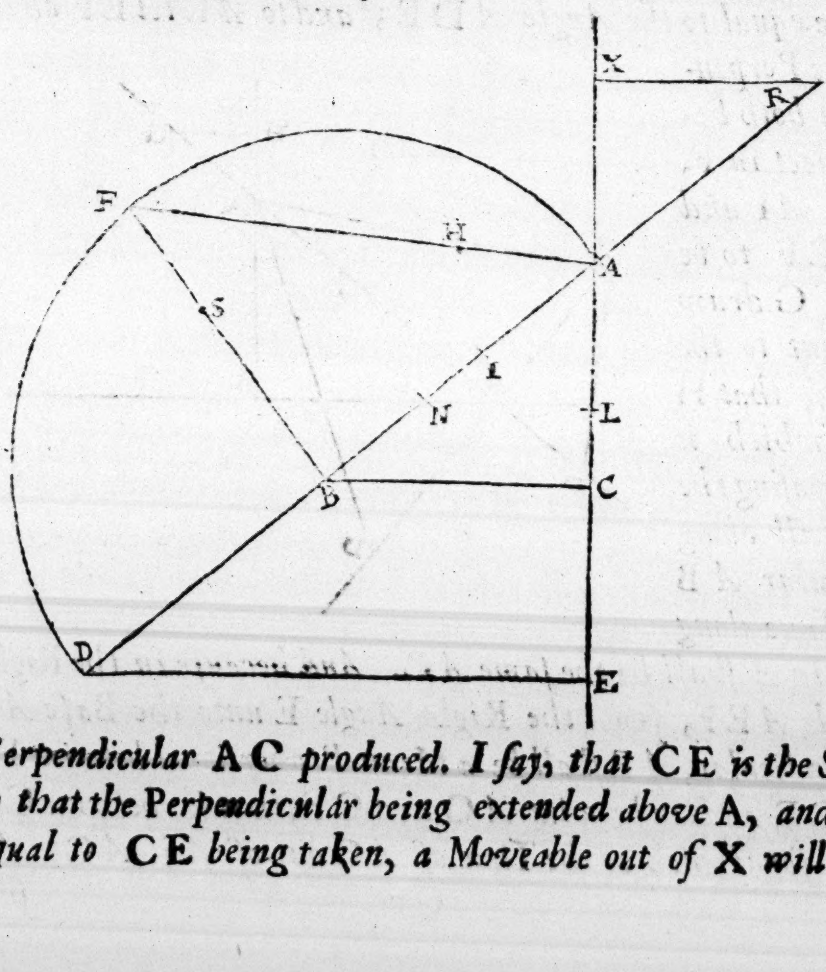
shall

shall be the Time of the whole AF, *and* EC *the Time of* AI: *And because in the Equicrural Triangle* AED *the Side* AE *is equal to the Side* ED, ED *shall be the Time along* AF, *and* EC *is the Time along* AI: *Therefore* CD, *that is* AB *shall be the Time along* AF ex quiete *in* A; *which is the same as if we said, that* AB *is the Time along* AC *out of* G, *or out of* H: *Which was to be done.*

PROBL. XIII. PROP. XXXIV.

An inclined Plane and Perpendicular whose sublime term is the same being given, to find a more sublime point in the Perpendicular prolonged out of which a Moveable falling, and being turned along the inclined Plane, may passe them both in the same Time, as it doth the sole inclined Plane *ex quiete* in its superiour Term.

L*Et the inclined Plane and Perpendicular be* AB *and* AC, *whose Term* A *is the same. It is required in the Perpendicular prolonged from* A *to find a sublime point, out of which the Moveable descending, and being turned along the Plane* AB, *may passe the assigned part of the Perpendicular and the Plane* AB *in the same Time, as it would the sole Plane* AB *out of Rest in* A.



Let the Horizontal Line be BC, *and let* AN *be cut equal to* AC; *and as* AB *is to* BN, *so let* AL *be to* LC: *and unto* AL *let* AI *be equal, and unto* AC *and* BI *let* CE *be a third proportional, marked in the Perpendicular* AC *produced. I say, that* CE *is the Space acquired; so that the Perpendicular being extended above* A, *and the part* AX *equal to* CE *being taken, a Moveable out of* X *will passe both the*

Spaces

Spaces X A B *in the same Time as it would the sole Space* A B *out of* A. *Draw the Horizontal Line* X R *Parallel to* B C, *with which let* B A *being prolonged meet in* R, *and then* A B *being continued out unto* D *draw* E D *Parallel to* C B, *and upon* A D *describe a Semicircle, and from* B, *and Perpendicular to* D A, *erect* B F *till it meet with the Circumference. It is manifest that* F B *is a Mean-proportional betwixt* A B *and* B D, *and that the Line drawn from* F *to* A *is a Mean-proportional betwixt* D A *and* A B. *Suppose* B S *equal to* B I, *and* F H *equal to* F B: *And because, as* A B *is to* B D, *so is* A C *to* C E, *and because* B F *is a Mean-proportional betwixt* A B *and* B D, *and because* B I *is a Mean-proportional betwixt* A C *and* C E; *therefore as* B A *is to* A C, *so is* F B *to* B S. *And because as* B A *is to* A C, *or* A N, *so is* F B *to* B S, *therefore, by Conversion of the proportion,* B F *is to* F S, *as* A B *is to* B N, *that is,* A L *to* L C; *therefore the Rectangle under* F B *and* C L, *is equal to the Rectangle under* A L, *and* S F: *But this Rectangle* A L, *and* S F, *is the excesse of the Rectangle under* A L *and* F B, *or* A I *and* B F, *over and above the Triangle* A I *and* B S, *or* A I B; *and the Rectangle* F B *and* L C *is the excesse of the Rectangle* A C *and* B F *over and above the Rectangle* A L *and* B F: *But the Rectangle* A C *and* B F *is equal to the Rectangle* A B I; (*for as* B A *is to* A C, *so is* F B *to* B I:) *The excesse, therefore, of the Rectangle* A B I *above the Rectangle* A I *and* B F, *or* A I *and* F H, *is equal to the excesse of the Rectangle* A I *and* F H *above the Rectangle* A I B: *Therefore twice the Rectangle* A I *and* F H *is equal to the two Rectangles* A B I *and* A I B; *that is twice* A I B *with the Square of* B I. *Let the Square* A I *be common to both, and twice the Rectangle* A I B *with the two Squares* A I, *and* I B, (*that is, the Square* A B) *shall be equal to twice the Rectangle* A I *and* F H, *with the Square* A I: *Again, taking in commonly the Square* B F; *the two Squares* A B *and* B F, *that is the sole Square* A F *shall be equal to twice the Rectangle* A I *and* F H, *with the two Squares* A I *and* F B, *that is* A I *and* F H: *But the same Square* A F *is equal to twice the Rectangle* A H F, *with the two Squares* A H *and* H F: *Therefore twice the Rectangle* A I *and* F H, *with the Squares* A I *and* F H, *are equal to twice the Rectangle* A H F, *with the Squares* A H *and* H F: *And, the Common Square* H F *being taken away, twice the Rectangle* A I *and* F H, *with the Square* A I, *shall be equal to twice the Rectangle* A H F, *with the Square* A H. *And because that in all the Rectangles* F H *is the Common Side, the Line* A H *shall be equal to* A I: *For if it should be greater or lesser, then the Rectangles* F H A *and the Square* H A *would also be greater or lesser than the Rectangles* F H *and* I A, *and the Square* I A: *Contrary to what hath been demonstrated.*

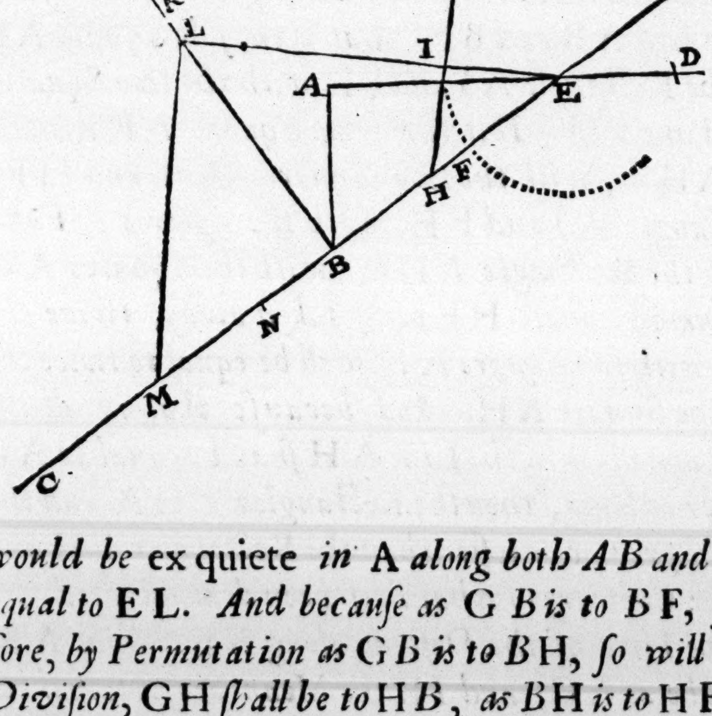
Now if we suppose the Time of the Descent along A B *to be as* A B, *the Time along* A C *shall be as* A C, *and* I B *the Mean-proportional betwixt* A C *and* C E *shall be the Time along* C E, *or along* X A *from Rest in* X: *And because betwixt* D A *and* A B, *or* R B *and* B A *the*

Mean-proportional is A F, *and between* A B *and* B D, *that is,* R A *and* A B *the Mean is* B F, *to which* F H *is equal; Therefore,* ex prædemonstratis, *the excesse* A H *shall be the Time along* A B ex quiete *in* R, *or after the Fall out of* X; *since the Time along the said* A B ex quiete *in* A, *shall be* A B. *Therefore the Time along* X A *is* I B; *and along* A B *after* R A, *or after* X A, *is* A I: *Therefore the Time along* X A B *shall be as* A B, *namely the self-same with the Time along the sole* A B ex quiete *in* A. *Which was the Proposition.*

PROBL. XIV. PROP. XXXV.

An Inflected Line unto a given Perpendicular being assigned, to take part in the Inflected Line, along which alone *ex quiete* a Motion may be made in the same Time, as it would be along the same together with the Perpendicular.

L*Et the Perpendicular be* A B, *and a Line inflected to it* B C. *It is required in* B C *to take a part, along which alone out of Rest a Motion may be made in the same Time as it would along the same together with the Perpendicular* A B. *Draw the Horizon* A D, *with which let the Inclined Line* C B *prolonged meet in* E; *and suppose* B F *equal to* B A, *and on the Center* E *at the distance* E F *describe the Circle* F I G; *and continue out* F E *unto the Circumference in* G; *and as* G B *is to* B F, *so let* B H *be to* H F; *and let* H I *touch the Circle in* I. *Then out of* B *erect* B K *Perpendicular to* F C, *with which let the Line* E I L *meet in* L; *and last of all let fall* L M *Perpendicular to* E L, *meeting* B C *in* M. *I say, that along the Line* B M *from Rest in* B *a Motion may be made in the same Time, as it would be* ex quiete *in* A *along both* A B *and* B M. *Let* E N *be made equal to* E L. *And because as* G B *is to* B F, *so is* B H *to* H F; *therefore, by Permutation as* G B *is to* B H, *so will* B F *be to* F H; *and, by Division,* G H *shall be to* H B, *as* B H *is to* H F: *Wherefore the Rectangle* G H F *shall be equal to the Square* H B: *But the said Rectangle is also equal to the Square* H I: *Therefore* B H *is equal to the same* H I.

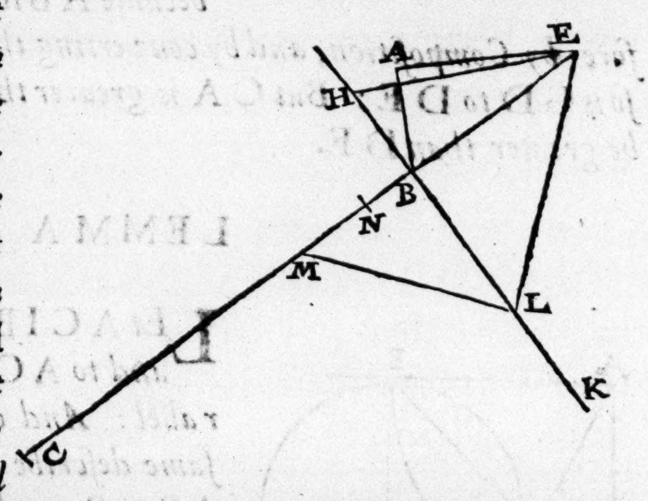


And

And because in the Quadrilateral Figure I L B H *the Sides* H B *and* H I *are equal, and the Angles* B *and* I *Right Angles, the Side* B L *shall likewise be equal to the Side* L I *: But* E I *is equal to* E F *: Therefore the whole Line* L E, *or* N E *is equal to the two Lines* L B *and* E F *: Let the Common Line* E F *be taken away, and the remainder* F N *shall be equal to* L B *: And* F B *was supposed equal to* B A *: Therefore* L B *shall be equal to the two Lines* A B *and* B N. *Again, if we suppose the Time along* A B *to be the said* A B, *the Time along* E B *shall be equal to* E B *; and the Time along the whole* E M *shall be* E N, *namely, the Mean-proportional betwixt* M E *and* E B *: Therefore the Time of the Descent of the remaining part* B M *after* E B, *or after* A B, *shall be the said* B N *: But it hath been supposed, that the Time along* A B *is* A B *: Therefore the Time of the Fall along both* A B *and* B M *is* A B N *: And because the Time along* E B ex quiete *in* E *is* E B, *the Time along* B M ex quiete *in* B *shall be the Mean-proportional between* B E *and* B M *; and this is* B L *: The Time, therefore, along both* A B M ex quiete *in* A *is* A B N *: And the Time along* B M *only* ex quiete *in* B *is* B L *: But it was proved that* B L *is equal to the two* A B *and* B N *: Therefore the Proposition is manifest.*

Otherwise with more expedition.

Let B C *be the Inclined Plane, and* B A *the Perpendicular. Continue out* C B *to* E, *and unto* E C *erect a Perpendicular at* B, *which being prolonged suppose* B H *equal to the excesse of* B E *above* B A *; and to the Angle* B H E *let the Angle* H E L *be equal; and let* E L *continued out meet with* B K *in* L *; and from* L *erect the Perpendicular* L M *unto* E L *meeting* B C *in* M. *I say, that* B M *is the Space acquired in the Plane* B C. *For because the Angle* M L E *is a Right-Angle, therefore* B L *shall be a Mean-proportional betwixt* M B *and* B E *; and* L E *a Mean proportional betwixt* M E *and* E B *; to which* E L *let* E N *be cut equal: And the three Lines* N E, E L, *and*

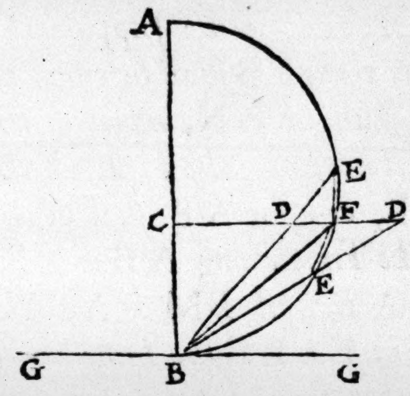


L H *shall be equal; and* H B *shall be the excesse of* N E *above* B L *: But the said* H B *is also the excesse of* N E *above* N B *and* B A *: Therefore the two Lines* N B *and* B A *are equal to* B L. *And if we suppose* E B *to be the Time along* E B, B L *shall be the Time along* B M ex quiete *in* B *; and* B N *shall be the Time of the same* B M *after* E B *or after* A B *; and* A B *shall be the Time along* A B*: Therefore the Times along* A B M, *namely,* A B N, *are equal to the Times along the sole Line* B M ex quiete *in* B *: Which was intended.*

LEMMA

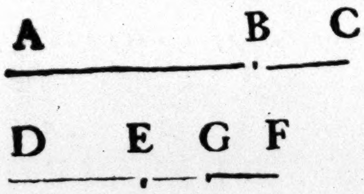
LEMMA I.

LEt D C *be Perpendicular to the Diameter* B A; *and from the Term* B *continue forth* B E D *at pleasure, and draw a Line from* F *to* B. *I say, that* F B *is a Mean-proportional betwixt* D B *and* B E. *Draw a Line from* E *to* F, *and by* B *draw the Tangent* B G; *which shall be Parallel to the former* C D: *Wherefore the Angle* D B G *shall be equal to the Angle* F D B, *like as the same* G B D *is equal also to the Angle* E F B *in the altern Portion or Segment: Therefore the Triangles* F B D *and* F E B *are alike: And, as* B D *is to* B F, *so is* F B *to* B E.



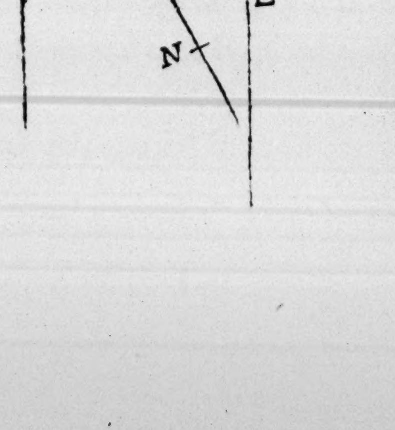
LEMMA II.

LEt the Line A C *be greater than* D F; *and let* A B *have greater proportion to* B C, *than* D E *hath to* E F. *I say, that* A B *is greater than* D E. *For because* A B *hath to* B C *greater proportion than* D E *hath to* D F, *therefore look what proportion* A B *hath to* B C, *the same shall* D E *have to a Line lesser than* E F; *let it have it to* E G: *And because* A B *to* B C, *is as* D E, *to* E G, *therefore, by Composition, and by converting the Proportion, as* C A *is to* A B, *so is* G D *to* D E: *But* C A *is greater than* G D: *Therefore* B A *shall be greater than* D E.



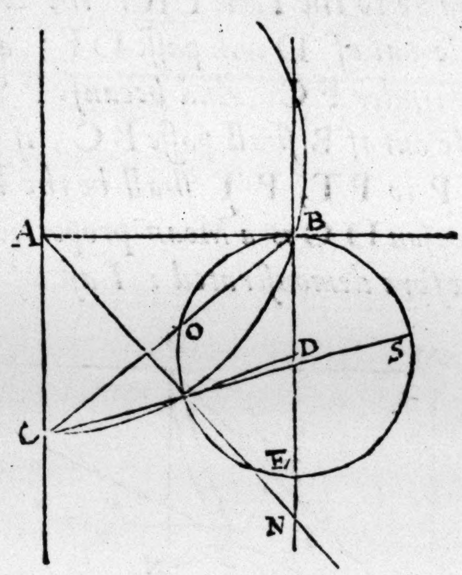
LEMMA III.

LEt A C I B *be the Quadrant of a Circle: and to* A C *let* B E *be drawn from* B *Parallel: And out of any Center taken in the same describe the Circle* B O E S, *touching* A B *in* B, *and cutting the Circumference of the Quadrant in* I; *and draw a Line from* C *to* B, *and another from* C *to* I *continued out to* S. *I say, that the Line* C I *is alwaies lesse than* C O. *Draw a Line from* A *to* I; *which toucheth the Circle* B O E. *And if* D I *be drawn it shall be equal to* D B: *And because* D B *toucheth the Quadrant, the said* D I *shall likewise touch it; and shall be Perpendicular*



pendicular to the Diameter A I: *Wherefore also* A I *toucheth the Circle* BOE *in* I. *And, because the Angle* AIC *is greater than the Angle* ABC, *as insisting on a larger Periphery: Therefore the Angle* SIN *shall be also greater than the same* ABC: *Therefore the Portion* IES *is greater than the Portion* BO; *and the Line* CS, *nearer to the Center, greater than* CB: *Therefore also* CO *is greater than* CI; *for that* SC *is to* CB, *as* OC *is to* CI.

And the same also would happen to be greater, if (as in the other Figure) the Quadrant BIC *were lesser: For the Perpendicular* DB *will cut the Circle* CIB: *Wherefore* DI *also is equal to the said* DB; *and the Angle* DIA *shall be Obtuse, and therefore* AIN *will also cut* BIN: *And because the Angle* ABC *is lesse than the Angle* AIC, *which is equal to* SIN; *and this now is lesse than that which would be made at the Contact in* I *by the Line* SI: *Therefore the Portion* SEI *is much greater than the Portion* BO: *Wherefore, &c. Which was to be demonstrated.*

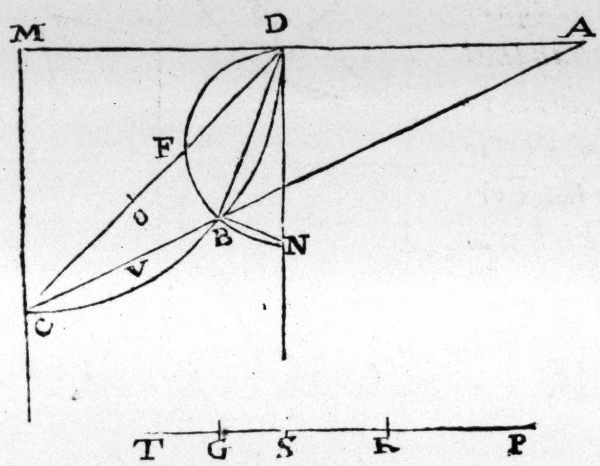


THEOR. XXII. PROP. XXXVI.

If from the lowest point of a Circle erect unto the Horizon a Plane should be elevated subtending a Circumference not greater than a Quadrant, from whose Terms two other Planes are Inflected to any point of the Circumference, the Descent along both the Inflected Planes would be performed in a shorter Time than along the former elevated Plane alone, or than along but one of the other two, namely, along the lower.

LEt CBD *be the Circumference not greater than a Quadrant of a Circle erect unto the Horizon on the lower point* C, *in which let* CD *be an elevated Plane; and let two Planes be inflected from the Terms* D *and* C *to any point in the Circumference taken at pleasure, as* B. *I say, that the Time of the Descent along both those Planes* DBC *is shorter than the Time of the Descent along the sole Plane* DC, *or along the other only* BC ex quiete *in* B. *Let the Horizontal Line* MDA *be*

be drawn by D, *with which let* C B *prolonged meet in* A; *and let fall the Perpendiculars* D N *and* M C *to* M D, *and* B N *to* B D; *and about the Right-angled Triangle* D B N *describe the Semicircle* D F B N, *cutting* D C *in* F; *and let* D O *be a Mean-proportional betwixt* C D *and* D F; *and* A V *a Mean-proportional betwixt* C A *and* A B: *And let* P S *be the time in which the whole* D C, *or* B C, *shall be passed; (for it is manifest that they shall be both past in the same Time;) And look what proportion* C D *hath to* D O, *the same shall the Time* S P *have to the Time* P R: *the Time* P R *shall be that in which a Moveable out of* D *will passe* D F; *and* R S *that in which it shall passe the remainder* F C. *And because* P S *is also the Time in which the Moveable out of* B *shall passe* B C; *if it be supposed that as* B C *is to* C D, *so is* S P *to* P T, P T *shall be the Time of the Descent out of* A *to* C: *by reason* D C *is a Mean-proportional betwixt* A C *and* C B, *by what was before demonstrated: Last of all, as* C A *is to* A V, *so let* T P *be to* P G: P G *shall be the Time, in which the Moveable out of* A *descendeth to* B. *And because of the Circle* D F N *the Diameter erect to the Horizon is* D N, *the Lines* D F *and* D B *shall be passed in equal Times. So that if it should be demonstrated that the Moveable would sooner passe* B C *after the Descent* D B, *than* F C *after the Lation* D F; *we should have our intent. But the Moveable will with the same Celerity of Time passe* B C *coming out of* D *along* D B, *as if it came out of* A *along* A B: *for that in both the Descents* D B *and* A B *it acquireth equal Moments of Velocity: Therefore it shall rest to be demonstrated that the Time is shorter in which* B C *is passed after* A B, *than that in which* F C *is past after* D F. *But it hath been demonstrated, that the Time in which* B C *is passed after* A B *is* G T; *and the Time of* F C *after* D F *is* R S. *It is to be proved therefore, that* R S *is greater than* G T: *Which is thus done. Because as* S P *is to* P R, *so is* C D *to* D O, *therefore, by Conversion of proportion, and by Inversion, as* R S *is to* S P, *so is* O C *to* C D: *and as* S P *is to* P T, *so is* D C *to* C A: *And, because as* T P *is to* P G, *so is* C A *to* A V: *Therefore also, by Conversion of the proportion, as* P T *is to* T G, *so is* A C *to* C V: *therefore,* ex equali, *as* R S *is to* G T, *so is* O C *to* C V. *But* O C *is greater than* C V, *as shall anon be demonstrated: Therefore the Time* R S *is greater than the Time* G T: *Which it was required to demonstrate. And because* C F *is greater than* C B, *and* F D *lesse than* B A, *therefore* C D *shall have greater proportion to* D F *than* C A *to* A B: *And as* C D *is to* D F, *so is the Square*



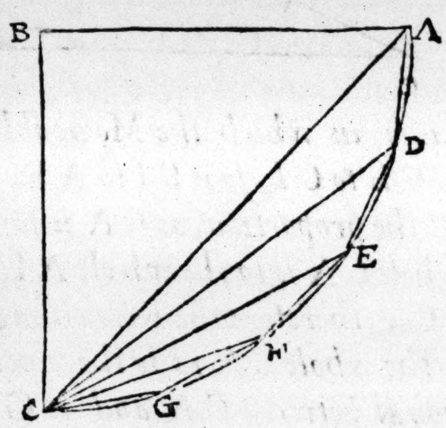
C O *to*

CO to the Square OF; forasmuch as CD, DO, and OF are Proportionals: And as CA is to AB, so is the Square CV to the Square VB: Therefore CO hath greater proportion to OF, than CV to VB: Therefore, by the foregoing Lemma, CO is greater than CV. It is manifest moreover, that the Time along DC is to the Time along DBC, as DOC is to DO together with CV.

SCHOLIUM.

From these things that have been demonstrated may evidently be gathered, that the swiftest of all Motions betwixt Term and Term is not made along the shortest Line, that is by the Right, but along a portion of a Circle.

FOR in the Quadrat BAEC, whose Side BC is erect to the Horizon, let the Arch AC be divided into any number of equal parts, AD, DE, EF, FG, GC; and let Right-lines be drawn from C to the Points A, D, E, F, G, H; and also by Lines joyn AD, DE, EF, FG. and GC. It is manifest, that the Motion along the two Lines ADC is sooner performed than along the sole Line AC, or DC out of Rest in D: But out of Rest in A, DC is sooner past than the two ADC: But along the two DEC out of Rest in A the Descent is likewise sooner made than along the sole CD: Therefore the Descent along the three Lines ADEC shall be performed sooner than along the two ADC. And in like manner the Descent along ADE preceding, the Motion is more speedily consummated along the two EFC than along the sole FC: Therfore along the four ADEFC the Motion is quicklier accomplished than along the three ADEC: And so, in the last place, along the two FGC after the precedent Descent along ADEF the Motion will be sooner consummated than along the sole FC: Therefore along the five ADEFGC the Descent shall be effected in a yet shorter Time than along the four ADEFC: Whereupon the nearer by inscribed Poligons we approach the Circumference, the sooner will the Motion be performed between the two assigned points AC.

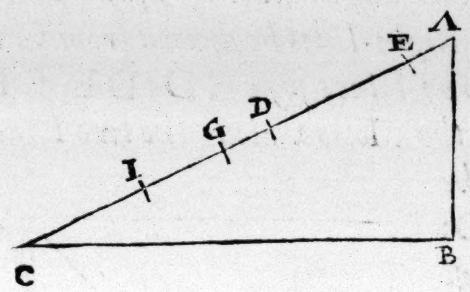


And that which is explained in a Quadrant, holdeth true likewise in a Circumference lesse than the Quadrant: and the Ratiocination is the same.

PROBL. XV. PROP. XXXVII.

A Perpendicular and Inclined Plane of the same Elevation being given, to find a part in the Inclined Plane that is equal to the Perpendicular, and passed in the same Time as the said Perpendicular.

LET A B *be the Perpendicular, and* A C *the Inclined Plane. It is required in the Inclined to find a part equal to the Perpendicular* A B, *that after Rest in* A *may be passed in a Time equal to the Time in which the Perpendicular is passed. Let* A D *be equal to* A B, *and cut the Remainder* B C *in two equal parts in* I; *and as* A C *is to* C I, *so let* C I *be to another Line* A E; *to which let* D G *be equal: It is manifest that* E G *is equal to* A D *and to* A B. *I say moreover, that this same* E G *is the same that is passed by the Moveable coming out of Rest in* A *in a Time equal to the Time in which the Moveable falleth along* A B. *For because that as* A C *is to* C I, *so is* C I *to* A E, *or* I D *to* D G; *Therefore by Conversion of the proportion, as* C A *is to* A I, *so is* D I *to* I G. *And because as the whole* C A *is to the whole* A I, *so is the part taken away* C I *to the part* I G; *therefore the Remaining part* I A *shall be to the Remainder* A G, *as the whole* C A *is to the whole* A I: *Therefore* A I *is a Mean-proportional betwixt* C A *and* A G; *and* C I *a Mean-proportional betwixt* C A *and* A E: *If therefore we suppose the Time along* A B *to be as* A B; A C *shall be the Time along* A C, *and* C I *or* I D *the Time along* A E: *And because* A I *is a Mean-proportional betwixt* C A *and* A G; *and* C A *is the Time along the whole* A C: *Therefore* A I *shall be the Time along* A G; *and the Remainder* I C *that along the Remainder* G C: *But* D I *was the Time along* A E: *Therefore* D I *and* I C *are the Times along both the Spaces* A E *and* C G: *Therefore the Remainder* D A *shall be the Time along* E G, *to wit, equal to the Time along* A B. *Which was to be done.*



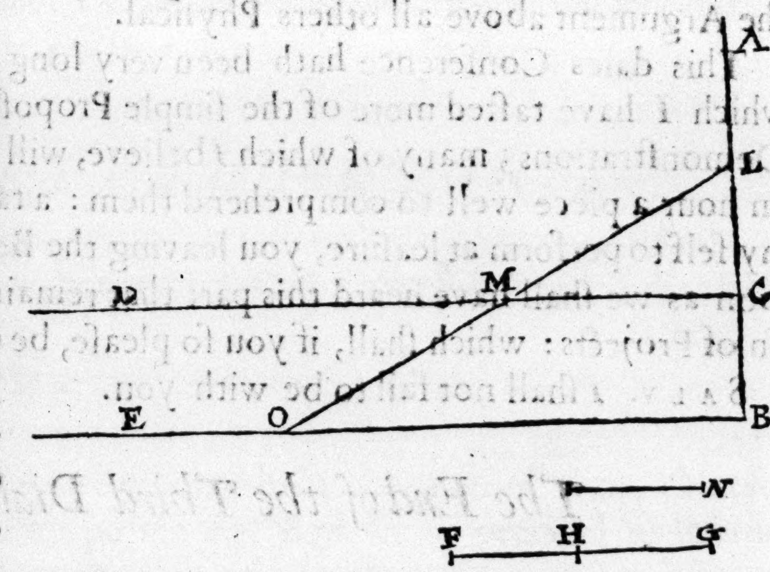
COROLLARIE.

Hence it is manifest, that the Space required is an intermedial between the upper and lower parts that are past in equal Times.

PROBL. XVI. PROP. XXXVIII.

Two Horizontal Planes cut by the Perpendicular being given, to find a sublime point in the Perpendicular, out of which Moveables falling and being reflected along the Horizontal Planes may in Times equal to the Times of the Descents along the said Horizontal Planes, namely, along the upper and along the lower, passe Spaces that have to each other any given proportion of the lesser to the greater.

LET *the Planes* C D *and* B E *be intersected by the Perpendicular* A C B, *and let the given proportion of the lesse to the greater be* N *to* F G. *It is required in the Perpendicular* A B *to find a point on high, out of which a Moveable falling, and reflected along* C D *may in a Time equal to the Time of its Fall, passe a Space, that shall have unto the Space passed by the other Moveable coming out of the same sublime point in a Time equal to the Time of its Fall with a Reflex Motion along the Plane* B E *the same proportion as the given Line* N *hath to* F G. *Let* G H *be made equal to the said* N; *and as* F H *is to* H G, *so let* B C *be to* C L. *I say,* L *is the sublime point required. For taking* C M *double to* C L, *draw* L M *meeting the Plane* B E *in* O; B O *shall be double to* B L: *And because, as* F H *is to* H G, *so is* B C *to* C L; *therefore, by Composition and Inversion, as* H G, *that is,* N *is to* G F, *so is* C L *to* L B, *that is,* C M *to* B O: *But because* C M *is double to* L C; *let the Space* C M *be that which by the Moveable coming from* L *after the Fall* L C *is passed along the Plane* C D; *and by the same reason* B O *is that which is passed after the Fall* L B *in a Time equal to the Time of the Fall along* L B; *forasmuch as* B O *is double to* B L: *Therefore the Proposition is manifest.*



SAGR. Really me thinks that we may justly grant our *Academian* what he without arrogance assumed to himself in the begining of this his Treatise of shewing us a *New Science* about *a very old Subject*. And to see with what Facility and Perspicuity he deduceth from one sole Principle the Demonstrations of so many Propositions, maketh me not a little to wonder how this business escaped unhandled by *Archimedes*, *Apollonius*, *Euclid*, and so many other Illustrious Mathematicians and Phylosophers: especially since there are found many great Volumns of *Motion*.

SALV. There is extant a small Fragment of *Euclid* touching *Motion*, but there are no marks to be seen therein of any steps that he took towards the discovery of the Proportion of *Acceleration*, and of its Varieties along different Inclinations. So that indeed one may say, that never till now was the door opened to a new Contemplation fraught with infinite and admirable Conclusions, which in times to come may busie other Wits.

SAGR. I verily believe, that as those few Passions (I will say for example) of the Circle demonstrated by *Euclid* in the third of his *Elements* are an introduction to innumerable others more abstruce, so those produced and demonstrated in this short Tractate, when they shall come to the hands of other Speculative Wits, shall be a manuduction unto infinite others more admirable: and it is to be believed that thus it will happen by reason of the Nobility of the Argument above all others Physical.

This daies Conference hath been very long and laborious; in which I have tasted more of the simple Propositions than of their Demonstrations; many of which, I believe, will cost me more than an hour a piece well to comprehend them: a task that I reserve to my self to perform at leasure, you leaving the Book in my hands so soon as we shall have heard this part that remains about the Motion of Projects: which shall, if you so please, be to morrow.

SALV. I shall not fail to be with you.

The End of the Third Dialogue.

GALILEUS, HIS DIALOGUES OF MOTION.

The Fourth Dialogue.

INTERLOCUTORS,

SALVIATUS, SAGREDUS, and SIMPLICIUS.

SALVIATUS.

Simplicius likewise cometh in the nick of time, therefore without interposing any *Rest* let us proceed to *Motion*; and see here the *Text* of our *Author*.

OF THE MOTION OF PROJECTS.

WHat accidents belong to Equable Motion, *as also to the* Naturally Accelerate *along all whatever Inclinations of Planes, we have considered above. In this Contemplation which we are now entering upon, I will attempt to declare, and with solid Demonstrations*

to establish some of the principal Symptomes, and those worthy of knowledge, which befall a Moveable whilst it is moved with a Motion compounded of a twofold Lation, to wit, of the Equable and Naturally-Accelerate; and this is that Motion, which we call the Motion of Projects: whose Generation I constitute to be in this manner.

I fancy in my mind a certain Moveable projected or thrown along an Horizontal Plane, all impediment secluded: Now it is manifest by what we have elsewhere spoken at large, that that Motion will be Equable and Perpetual along the said Plane, if the Plane be extended in infinitum*: but if we suppose it terminate, and placed on high, the Moveable, which I conceive to be endued with Gravity, being come to the end of the Plane, proceeding forward, it addeth to the Equable and Indelible first Lation that propension downwards which it receiveth from its Gravity, and from thence a certain Motion doth result compounded of the Equable Horizontal, and of the Descending naturally-Accellerate Lations: which I call* Projection. *Some of whose Accidents we will demonstrate; the first of which shall be this.*

THEOR. I. PROP. I.

A Project, when it is moved with a Motion compounded of the Horizontal Equable, and of the Naturally-Accelerate downwards, shall describe a Semiparabolical Line in its Lation.

SAGR. It is requisite, *Salviatus*, in favour of my self, and, as I believe, also of *Simplicius*, here to make a pause; for I am not so far gone in Geometry as to have studied *Apollonius*, save only so far as to know that he treateth of these Parabola's, and of the other Conick Sections, without the knowledge of which, and of their Passions, I do not think that one can understand the Demonstrations of other Propositions depending on them. And because already in the very first Proposition it is proposed by the Author to prove the Line described by the Project to be Parabolical, I imagine to my self, that being to treat of none but such Lines, it is absolutely necessary to have a perfect knowledge, if not of all the Passions of those Figures that are demonstrated by *Apollonius*, at least of those that are necessary for the Science in hand.

SALV. You undervalue your self very much, to make strange of those Notions, which but even now you admitted as very well understood: I told you heretofore, that in the Treatise of Resistances we had need of the knowledge of certain Propositions of *Apollonius*,

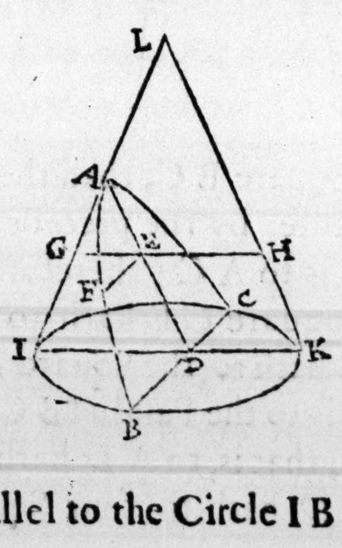
Apollonius, at which you made no scruple.

SAGR. It may be either that I knew them by chance, or that I might for once guesse at, and take for granted so much as served my turn in that Tractate: but here where I imagine that we are to hear all the Demonstrations that concern those Lines, it is not convenient, as we say, to swallow things whole, losing our time and pains.

SIMP. But as to what concerns me, although *Sagredus* were, as I believe he is, well provided for his occasions, the very first Terms already are new to me: for though our Philosophers have handled this Argument of the Motion of Projects, I do not remember that they have confined themselves to define what the Lines are which they describe, save only in general that they are alwaies Curved Lines, except it be in Projections Perpendicularly upwards. Therefore in case that little Geometry that I have learnt from *Euclid* since the Time that we have had other Conferences, be not sufficient to render me capable of the Notions requisite for the understanding of the following Demonstrations, I must content my self with bare Propositions believed, but not understood.

SALV. But I will have you to know them by help of the Author of this Book himself, who when he heretofore granted me a sight of this his Work, because I also at that time was not perfect in the Books of *Apollonius*, took the pains to demonstrate to me two most principal Passions of the Parabola without any other Precognition, of which two, and no more, we shall stand in need in the present Treatise; which are both likewise proved by *Apollonius*, but after many others, which it would take up a long time to look over, and I am desirous that we may much shorten the Journey, taking the first immediately from the pure and simple generation of the said Parabola, and from this also immediately shall be deduced the Demonstration of the second. Coming therefore to the first;

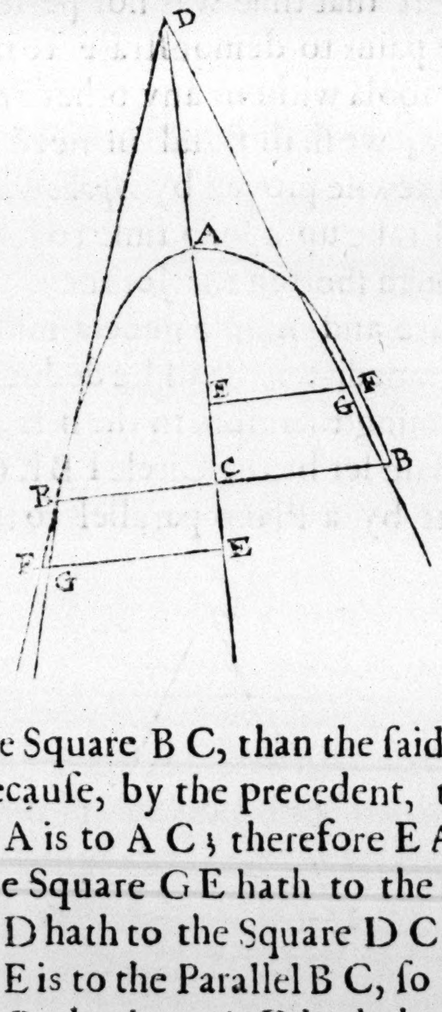
Describe the Right Cone, whose Base let be the Circle I B K C, and Vertex the point L, in which, cut by a Plane parallel to the Side L K, ariseth the Section B A C called a Parabola; and let its Base B C cut the Diameter I K of the Circle I B K C at Right-Angles; and let the Axis of the Parabola A D be Parallel to the side L K; and taking any point F in the Line B F A, draw the Right-Line F E parallel to B D. I say, that the Square of B D hath to the Square of F E the same proportion that the Axis D A hath to the part A E. Let a Plane parallel to the Circle I B K C



be

be supposed to passe by the Point E, which shall make in the Cone a Circular Section, whose Diameter is G E H. And because upon the Diameter I K of the Circle I B K, B D is a Perpendicular, the Square of B D shall be equal to the Rectangle made by the parts I D and D K: And likewise in the upper Circle which is understood to passe by the points G F H, the Square of the Line F E is equal to the Rectangle of the parts G E H: Therefore the Square of B D hath the same proportion to the Square of F E, that the Rectangle I D K hath to the Rectangle G E H. And because the Line E D is Parallel to H K, E H shall be equal to D K, which also are Parallels: And therefore the Rectangle I D K shall have the same proportion to the Rectangle G E H, as I D hath to G E; that is, that D A hath to A E: Therefore the Rectangle I D K to the Rectangle G E H, that is, the Square B D to the Square F E, hath the same proportion that the Axis D A hath to the part A E: Which was to be demonstrated.

The other Proposition, likewise necessary to the present Tract, we will thus make out. Let us describe the Parabola, of which let the Axis C A be prolonged out unto D; and taking any point B, let the Line B C be supposed to be continued out by the same Parallel unto the Base of the said Parabola; and let D A be supposed equal to the part of the Axis C A. I say, that the Right-Line drawn by the points D and B, falleth not within the Parabola, but without, so as that it only toucheth the same in the said point B: For, if it be possible for it to fall within, it cutteth it above, or being prolonged, it cutteth it below. And in that Line let any point G be taken, by which passeth the Right Line F G E. And because the Square F E is greater than the Square G E, the said Square F E shall have greater proportion to the Square B C, than the said Square G E hath to the said B C. And because, by the precedent, the Square F E is to the Square B C as E A is to A C; therefore E A hath greater proportion to A C, than the Square G E hath to the Square B C; that is, than the Square E D hath to the Square D C: (because in the Triangle D G E as G E is to the Parallel B C, so is E *D* to *D* C:) But the Line E A to A C, that is, to A *D* hath the same proportion that four Rectangles E A *D* hath to four Squares of A *D*, that is, to the Square C *D*, (which



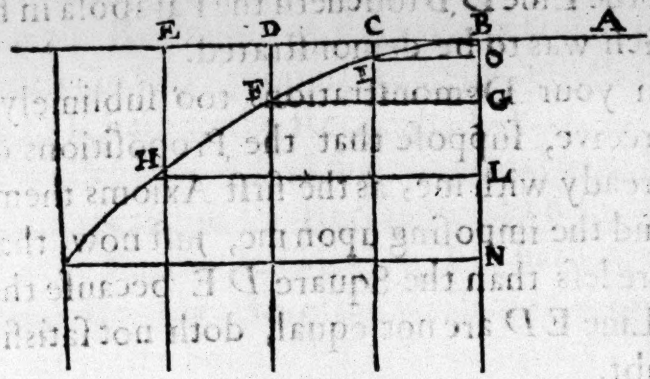
(which is equal to four Squares of A D:) Therefore four Rectangles E A D shall have greater proportion to the Square C D, than the Square E D hath to the Square D C: Therefore four Rectangles E A D shall be greater than the Square E D: which is false, for they are lesse; because the parts E A and A D of the Line E D are not equal: Therefore the Line D B toucheth the Parabola in B, and doth not cut it: Which was to be demonstrated.

SIMP. You proceed in your Demonstrations too sublimely, and still, as far as I can perceive, suppose that the Propositions of *Euclid* are as familiar and ready with me, as the first Axioms themselves, which is not so. And the imposing upon me, just now, that four Rectangles E A *D* are less than the Square *D* E because the parts E A and A *D* of the Line E *D* are not equal, doth not satisfie me, but leaveth me in doubt.

SALV. The truth is, all the Mathematicians that are not vulgar suppose, that the Reader hath ready by heart the Elements of *Euclid*: And here to supply your want, it shall suffice to remember you of a Proposition in the second Book, in which it is demonstrated that when a Line is cut into equal parts, and into unequal, the Rectangle of the unequal parts is less than the Rectangle of the equal, (that is, than the Square of the half) by so much as is the Square of the Line comprized between the Sections. Whence it is manifest, that the Square of the whole, which containeth four Squares of the Half, is greater than four Rectangles of the unequal parts. Now it is necessary that we bear in mind these two Propositions which have been demonstrated, taken from the Conick Elements, for the better understanding the things that follow in the present Treatise: for of these two, and no more, the Author makes use. Now we may reassume the Text to see in what manner he doth demonstrate his first Proposition, in which he intendeth to prove unto us, That the Line described by the Grave Moveable, when it descends with a Motion compounded of the Equable Horizontal, and of the Natural Descending is a Semiparabola.

Suppose the Horizontal Line or Plane A B *placed on high; upon* [or along] *which let the Moveable passe with an Equable Motion out of* A *unto* B: *and the support of the Plane failing in* B *let there be derived upon the Moveable from its own Gravity a Motion naturally downwards according to the Perpendicular* B N. *Let the Line* B E *be supposed applyed unto the Plane* A B *right out, as if it were the Efflux or measure of the Time, on which at pleasure note any equal parts of Time,* B C, C D, D E: *And out of the points* B C D E *suppose Perpendicular Lines to be let fall equidistant or parallel to* B N: *In the first of which take any part* C I, *whose quadruple take in the following one* D F, *nonuple* E H, *and so in the rest that follow according to the propor-*

tion of the Squares of C B, D B, E B, *or, if you will, in the doubled proportion of the Lines. And if unto the Moveable moved beyond* B *towards* C *with the Equable Lation we suppose the Perpendicular Descent to be superadded according to the quantity* C I, *in the Time* B C *it shall be found constituted in the Term* I. *And proceeding farther, in the Time* D B, *namely, in the double of* B C, *the Space of the Descent downwards shall be quadruple to the first Space* C I: *For it hath been demonstrated in the first Tractate, that the Spaces passed by Grave Bodies with a Motion Naturally Accelerate are in duplicate proportion of their Times. And it likewise followeth, that the Space* E H *passed in the Time* B E, *shall be as* G. *So that it is manifestly proved, that the Spaces* E H, D F, C I, *are to one another as the Squares of the Lines* E B, D B, C B. *Now from the points* I, F, *and* H *draw the Right Lines* I O, F G, H L, *Parallel to the said* E B; *and each of the Lines* H L, F G, *and* I O *shall be equal to each of the other Lines* E B, D B, *and* C B; *as also each of those* B O, B G, *and* B L, *shall be equal to each of those* C I, D F, *and* E H: *And the Square* H L *shall be to the Square* F G, *as the Line* L B *to* B G: *And the Square* F G *shall be to the Square* I O, *as* G B *to* B O: *Therefore the Points* I, F, *and* H *are in one and the same Parabolical Line. And in like manner it shall be demonstrated, any equal particles of Time of whatsoever Magnitude being taken, that the place of the Moveable whose Motion is compounded of the like Lations, is in the same Times to be found in the same Parabolick Line: Therefore the Proposition is manifest.*



SALV. This Conclusion is gathered from the Conversion of the first of those two Propositions that went before, for the Parabola being, for example, described by the points B H, if either of the two F or I were not in the described Parabolick Line, it would be within, or without; and by consequence the Line F G would be either greater or lesser than that which should determine in the Parabolick Line; Wherefore the Square of H L would have, not to the Square of F G, but to another greater or lesser, the same proportion that the Line L B hath to B G, but it hath the same proportion to the Square of F G: Therefore the point F is in the Parabolick Line: And so all the rest, &c.

SAGR. It cannot be denied but that the Discourse is new, ingenious and concludent, arguing *ex suppositione*, that is, supposing that the Transverse Motion doth continue alwaies Equable, and

that the Natural *Deorsum* do likewise keep its tenour of continually Accelerating according to a proportion double to the Times; and that those Motions and their Velocities in mingling be not altered, disturbed, and impeded, so that finally the Line of the Project do not in the continuation of the Motion degenerate into another kind; a thing which seemeth to me to be impossible. For, in regard that the Axis of our Parabola, according to which we suppose the Natural Motion of Graves to be made, being Perpendicular to the Horizon, doth terminate in the Center of the Earth; and in regard that the Parabolical Line doth successively enlarge from its Axis, no Project would ever come to terminate in the Center, or if it should come thitherwards, as it seemeth necessary that it must, the Line of the Project should describe another most different from that of the Parabola.

SIMP. I add to these difficulties several others; one of which is that we suppose, that the Horizontal Plane which hath neither acclivity or declivity is a Right Line; as if that such a Line were in all its parts equidistant from the Center, which is not true: for departing from its middle it goeth towards the extreams, alwaies more and more receding from the Center, and therefore alwaies ascending: which of consequence rendereth it Impossible that its Motion should be perpetual, or that it should for any time continue Equable, and necessitates it to grow continually more and more weak. Moreover, it is, in my Opinion, impossible to avoid the Impediment of the *Medium*, but that it will take away the Equability of the Transverse Motion, and the Rule of the Acceleration in falling Grave Bodies. By all which difficulties it is rendred very improbable that the things demonstrated with such inconstant Suppositions should afterwards hold true in the practical Experiments.

SALV. All the Objections and Difficulties alledged are so well grounded, that I esteem it impossible to remove them; and for my own part I admit them all, as also I believe the Author himself would do. And I grant that the Conclusions thus demonstrated in Abstract, do alter and prove false, and that so egregiously, in Concrete, that neither is the Transverse Motion Equable, nor is the Acceleration of the Natural in the proportion supposed, nor is the Line of the Project Parabolical, *&c.* But yet on the contrary, I desire that you would not scruple to grant to this our Author that which other famous Men have supposed, although false. And the single Authority of *Archimedes* may satisfie every one: who in his Mechanicks, and in the first Quadrature of the Parabola, taketh it as a true Principle, that the Beam of the Ballance or Stilliard is a Right Line in all its points equidistant from the Common Center of Grave Bodies, and that the Scale-ropes, to which the Weights are hanged, are parallel to one another. Which

Liberty of his hath been excused by some, for that in our practices the Instruments we use, and the Distances which we take are so small in comparison of our great remoteness from the Center of the Terrestrial Globe, that we may very well take a Minute of a degree of the great Circle as if it were a Right Line, and two Perpendiculars that should hang at its extreams as if they were Parallels. For if we were in practical Operations to keep account of such like Minutes, we should begin to reprove the Architects, who with the Plumb Line suppose that they raise very high Towers between Lines equidistant. And I here add, that we may say that *Archimedes*, and others suppose in their Contemplations that they were constituted remote at an infinite distance from the Center; in which case their Assumptions were not false: And that therefore they did conclude by Absolute Demonstration. Again, if we will practice the demonstrated Conclusions in terminate Distances, by supposing an immense Distance, we ought to defalk from the truth demonstrated that which our Distance from the Center doth import, not being really infinite, but yet such as that it may be termed Immense in comparison of the Artifices that we make use of, the greatest of which will be the Ranges of Projects, and amongst these that only of Canon shot; which though it be great, yet shall it not exceed four of those Miles of which we are remote from the Center well-nigh so many thousands: and these coming to determine in the Surface of the Terrestrial Globe may very well only insensibly alter that Parabolick Figure, which we grant would be extreamly transformed in going to determine in the Center. In the next place as to the perturbation proceeding from the Impediment of the *Medium*, this is more considerable, and, by reason of its so great multiplicity of Varieties, incapable of being brought under any certain Rules, and reduced to a Science: for if we should propose to consideration no more but the Impediment which the Air procureth to the Motions considered by us, this alone shall be found to disturb all, and that infinite waies, according as we infinite waies vary the Figures, Gravities, and Velocities of the Moveables. For as to the Velocity, according as this shall be greater, the greater shall the opposition be that the Air makes against them, which shall yet more impede the said Moveable according as they are less Grave: so that although the descending Grave Body ought to go Accelerating in a duplicate proportion to the Duration of its Motion, yet neverthelesse, albeit the Moveable were very Grave, in coming from very great heights, the Impediment of the Air shall be so great, as that it will take from it all power of farther encreasing its Velocity, and will reduce it to an Uniform and Equable Motion: And this Adequation shall be so much the sooner obtained, and in so much lesser heights, by how much the Moveable

shall

ſhall be leſs Grave. That Motion alſo which along the Horizontal Plane, all other Obſtacles being removed, ought to be Equable and perpetual, ſhall come to be altered, and in the end arreſted by the Impediment of the Air: and here likewiſe ſo much the ſooner, by how much the Moveable ſhall be Lighter. Of which Accidents of Gravity, of Velocity, and alſo of Figure, as being varied ſeveral waies, there can no fixed Science be given. And therefore that we may be able Scientifically to treat of this Matter it is requiſite that we abſtract from them; and, having found and demonſtrated the Concluſions abſtracted from the Impediments, that we make uſe of them in practice with thoſe Limitations that Experience ſhall from time to time ſhew us. And yet nevertheleſs the benefit ſhall not be ſmall, becauſe ſuch Matters, and their Figures ſhall be made choice of as are leſs ſubject to the Impediments of the *Medium*; ſuch are the very Grave, the Rotund: and the Spaces, and the Velocities for the moſt part will not be ſo great, but that their exorbitances may with eaſie * Allowance be reduced to a certainty. * Tarra. Yea more, in Projects practicable by us, that are of Grave Matters, and of Round Figure, and alſo that are of Matters leſſe Grave, and of Cylindrical Figure, as Arrows, ſhot from Slings or Bows, the variation of their Motion from the exact Parabolical Figure ſhall be altogether inſenſible. Nay, (and I will aſſume to my ſelf a little more freedom) that in * Inſtruments that are practicable by * Artifizii. us, their ſmalneſs rendreth the extern and accidental Impediments, of which that of the *Medium* is moſt conſiderable, to be but of very ſmall note, I am able by two experiments to make manifeſt. I will conſider the Motions made thorow the Air, for ſuch are thoſe chiefly of which we ſpeak: againſt which the ſaid Air in two manners exerciſeth its power. The one is by more impeding the Moveables leſs Grave, than thoſe very Grave. The other is in more oppoſing the greater than the leſs Velocity of the ſame Moveable. As to the firſt; Experience ſhewing us that two Balls of equal bigneſs, but in weight one ten or twelve times more Grave than the other, as, for example, one of Lead and another of Oak would be, deſcending from an height of 150, or 200 Yards, arrive to the Earth with Velocity very little different, it aſſureth us that the Impediment or Retardment of the Air in both is very ſmall: for if the Ball of Lead departing from on high in the ſame Moment with that of Wood, were but little retarded, and this much, the Lead at its coming to the ground ſhould leave the Wood a very conſiderable Space behind, ſince it is ten times more Grave; which nevertheleſs doth not happen: nay, its Anticipation ſhall not be ſo much as the hundredth part of the whole height. And between a Ball of Lead, and another of Stone which weighs a third part, or half ſo much as it, the difference of the Times of their coming to the

the ground would be hardly observable. Now because the *Impetus* that a Ball of Lead acquireth in falling from an height of 200 Yards (which is so much that continuing it in an Equable Motion it would in a like Time run 400 Yards) is very considerable in comparison of the Velocity that we confer with Bows or other Machines, upon our Projects (excepting the *Impetus's* that depend on the Fire) we may without any notable Errour conclude and account the Propositions to be absolutely true that are demonstrated without any regard had to the alteration of the *Medium*. In the next place as touching the other part, that is to shew, that the Impediment that the said Moveable receiveth from the Air whilst it moveth with great Velocity is not much greater than that which opposeth it in moving slowly, the ensuing Experiment giveth us full assurance of it. Suspend by two threads both of the same length, *v. gr.* four or five Yards, two equal Balls of Lead : and having fastned the said threads on high, let both the Balls be removed from the state of Perpendicularity ; but let the one be removed 80. or more degrees, and the other not above 4 or 5 : so that one of them being left at liberty descendeth, and passing beyond the Perpendicular, describeth very great Arches of 160, 150, 140, &c. degrees, diminishing them by little and little : but the other swinging freely passeth little Arches of 10, 8, 6, &c. this also diminishing them in like manner by little and little. Here I say, in the first place, that the first Ball shall pass its 180, 160, &c. degrees in as much Time as the other doth its 10, 8, &c. From whence it is manifest, that the Velocity of the first Ball shall be 16 and 18 times greater than the Velocity of the second : so that in case the greater Velocity were to be more impeded by the Air than the lesser, the Vibrations should be more * rare in the greatest Arches of 180, or 160 degrees, &c. than in the least of 10, 8, 4, and also of 2, and of 1 ; but this is contradicted by Experience : for if two Assistants shall set themselves to count the Vibrations, one the greatest, the other the least, they will find that they shall number not only tens, but hundreds also, without disagreeing one single Vibration, yea, or one sole point. And this observation joyntly assureth us of the two Propositions, namely, that the greatest and least Vibrations are all made one after another under equal Times, and that the Impediment and Retardment of the Air operates no more in the swiftest Motion, than in the slowest : contrary to that which before it seemed that we our selves also would have judged for company.

* Or fewer.

SAGR. Rather, because it cannot be denied but that the Air impedeth both those and these, since they both continually grow more languid, and at last cease, it is requisite to say that those Retardations are made with the same proportion in the one and in the

other

other Operation. And then, the being to make greater Resistance at one time than at another, from what other doth it proceed, but only from its being assailed at one time with a greater *Impetus* and Velocity, and at another time with lesser? And if this be so, then the same quantity of the Velocity of the Moveable is at once the Cause and the Measure of the quantity of the Resistance. Therefore all Motions, whether they be slow or swift, are retarded and impeded in the same proportion: a Notion in my judgment not contemptible.

Salv. We may also in this second case conclude, That the Fallacies in the Conclusions, which are demonstrated, abstracting from the extern Accidents, are in our Instruments of very small consideration, in respect of the Motions of great Velocities of which for the most part we speak, and of the Distances which are but very small in relation to the Semidiameter and great Circles of the Terrestrial Globe.

Simp. I would gladly hear the reason why you sequestrate the Projects from the *Impetus* of the Fire, that is, as I conceive from the force of the Powder, from the other Projects made by Slings, Bows, or Cross-bows, touching their not being in the same manner subject to the Acceleration and Impediment of the Air.

Salv. I am induced thereto by the excessive, and, as I may say, Supernatural Fury or Impetuousness with which those Projects are driven out: For indeed I think that the Velocity with which a Bullet is shot out of a Musket or Piece of Ordinance may without any Hyperbole be called Supernatural. For one of those Bullets descending naturally thorow the Air from some immense height, its Velocity, by reason of the Resistance of the Air will not go increasing perpetually: but that which in Cadent Bodies of small Gravity is seen to happen in no very great * Space, I mean their being reduced in the end to an Equable Motion, shall also happen after a Descent of thousands of yards, in a Ball of Iron or Lead: and this determinate and ultimate Velocity may be said to be the greatest that such a Body can obtain or acquire thorow the Air: which Velocity I account to be much lesser than that which cometh to be impressed on the same Ball by the fired Powder. And of this a very apposite Experiment may advertise us. At an height of an hundred or more yards let off a Musket charged with a Leaden Bullet perpendicularly downwards upon a Pavement of Stone; and with the same Musket shoot against such another Stone at the Distance of a yard or two, and then see which of the two Bullets is more flatted: for if that coming from on high be less * dented than the other, it shall be a sign that the Air hath impeded it, and diminished the Velocity conferred upon it by the Fire in the beginning of the Motion: and that, consequently, so great a Velocity the Air would

* Or Way.

* Or battered.

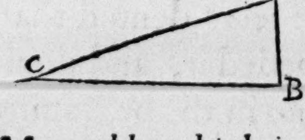
would not suffer it to gain coming from never so great an height: for in case the Velocity impressed upon it by the Fire should not exceed that which it might acquire of its self descending naturally, the battery downwards ought rather to be more valid than less. I have not made such an Experiment, but incline to think that a Musket or Cannon Bullet falling from never so great an height, will not make that percussion which it maketh in a Wall at a Distance of a few yards, that is of so few that the short perforation, or, if you will, Scissure to be made in the Air sufficeth not to obviate the excess of the supernatural impetuosity impressed on it by the Fire. This excessive *Impetus* of such like forced shots may cause some deformity in the Line of the Projection; making the beginning of the Parabola less inclined or curved than the end. But this can be but of little or no prejudice to our Author in practical Operations: amongst the which the principal is the composition of a Table for the Ranges, or Flights, which containeth the distances of the Falls of Balls shot according to all Elevations. And because these kinds of Projections are made with Mortar-Pieces, and with no great charge; in these the *Impetus* not being supernatural, the Ranges describe their Lines very exactly.

But for the present let us proceed forwards in the Treatise, where the Author desireth to lead us to the Contemplation and Investigation of the *Impetus* of the Moveable whilst it moveth with a Motion compounded of two. And first of that compounded of two Equable Motions; the one Horizontal, and the other Perpendicular.

THEOR. II. PROP. II.

If any Moveable be moved with a twofold Equable Motion, that is, Horizontal and Perpendicular, the *Impetus* or Moment of the Lation compounded of both the Motions shall be *potentia* equal to both the Moments of the first Motions.

For let any Moveable be moved Equably with a double Lation, and let the Mutations of the Perpendicular answer to the Space A B, *and let* B C *answer to the Horizontal Lation passed in the same Time. Forasmuch therefore as the Spaces* A B, *and* B C *are passed by the Equable Motion in the same Time, their Moments shall be to each other as the said* A B *and* B C. *But the Moveable which is moved according to these two Mutations shall describe*



ſcribe the Diagonal A C,*and its Moment ſhall be as* A C. *But* A C *is* potentia *equal to the ſaid* A B *and* B C *: therefore the Moment compounded of both the Moments* A B *and* B C, *is* potentia *equal to them both taken together : Which was to be demonſtrated.*

SIMP. It is neceſſary that you eaſe me of one Scruple that cometh into my mind, it ſeemeth to me that this which is now concluded oppugneth another Propoſition of the former Tractate: in which it is affirmed, That the *Impetus* of the Moveable coming from A into B is equal to that coming from A into C; and now it is concluded, that the *Impetus* in C is greater than that in B.

SALV. The Propoſitions, *Simplicius*, are both true, but very different from one another. Here the Author ſpeaks of one ſole Moveable moved with one ſole Motion, but compounded of two, both Equable; and there he ſpeaks of two Moveables moved with Motions Naturally Accelerated, one along the Perpendicular A B, and the other along the Inclined Plane A C : and moreover, the Times there are not ſuppoſed equal, but the Time along the Inclined Plane A C is greater than the Time along the Perpendicular A B: but in the Motion ſpoken of at preſent, the Motions along A B, B C and A C are underſtood to be Equable, and made in the ſame Time.

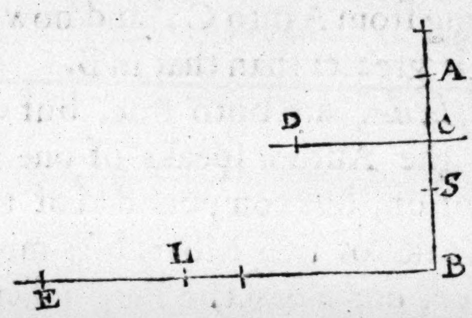
SIMP. Excuſe me, and go on, for I am ſatisfied.

SALV. The Author proceeds to ſhew us that which hapneth concerning the *Impetus* of a Moveable moved in like manner with one Motion compounded of two, that is to ſay, the one Horizontal and Equable, and the other Perpendicular but Naturally-Accelerate, of which in fine the Motion of the Project is compounded, and by which the Parabolick Line is deſcribed; in each point of which the Author endeavours to determine what the *Impetus* of the Project is; for underſtanding of which he ſheweth us the manner, or, if you will, Method of regulating and meaſuring that ſame *Impetus* upon the ſaid Line, along which the Motion of the Grave Moveable deſcending with a Natural-Accelerate Motion departing from Reſt is made, ſaying :

THEOR. III. PROP. III.

Let a Motion be made along the Line A B *out of Reſt in A, and take in ſome point* C; *and ſuppoſe the ſaid* A C *to be the Time or Meaſure of the Time of the ſaid Fall along the Space* A C, *as alſo the Meaſure of the* Impetus *or Moment in the Point* C *acquired by the Deſcent along* A C. *Now let there be taken in the ſaid Line* A B *any other Point, as ſuppoſe* B, *in which we are to determine of the* Impetus *acquired by the Moveable along the Fall* A B, *in proportion to the*

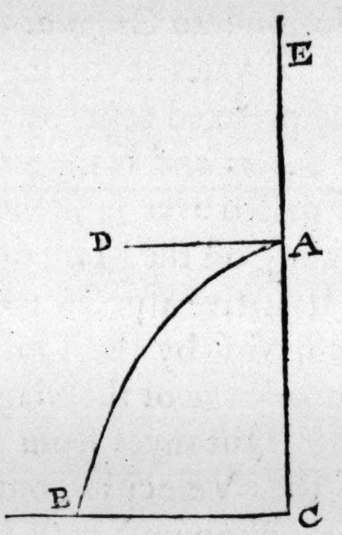
the Impetus, *which it obtaineth in* C, *whose Measure is supposed to be* A C, *Let* A S *be a Mean-proportional betwixt* B A *and* A C. *We will demonstrate that the* Impetus *in* B *is to the* Impetus *in* C, *as* S A *is to* A C. *Let the Horizontal Line* C D *be double to the said* A C; *and* B E *double to* B A. *It appeareth by what hath been demonstrated, That the Cadent along* A C *being turned along the Horizon* C D, *and according to the* Impetus *acquired in* C, *with an Equable Motion, shall pass the Space* C D *in a Time equal to that in which the said* A C *is passed with an Accelerate Motion; and likewise that* B E *is passed in the same time as* A B: *But the Time of the Descent along* A B *is* A S: *Therefore the Horizontal Line* B E *is passed in* A S. *As the Time* S A *is to the Time* A C, *so let* E B *be to* B L. *And because the Motion by* B E *is Equable, the Space* B L *shall be passed in the Time* A C *according to the Moment of Celerity in* B: *But in the same Time* A C *the Space* C D *is passed, according to the Moment of Velocity in* C: *the Moments of Velocity therefore are to one another as the Spaces which according to the same Moments are passed in the same Time: Therefore the Moment of Velocity in* C *is to the Moment of Celerity in* B, *as* D C *is to* B L. *And because as* D C *is to* B E, *so are their halfs, to wit,* C A *to* A B: *but as* E B *is to* B L, *so is* B A *to* A S: *Therefore,* ex æquali, *as* D C *is to* B L, *so is* C A *to* A S: *that is, as the Moment of Velocity in* C *is to the Moment of Velocity in* B, *so is* C A *to* A S; *that is, the Time along* C A *to the Time along* A B. *The manner of Measuring the* Impetus, *or the Moment of Velocity upon a Line along which it makes a Motion of Descent is therefore manifest; which* Impetus *is indeed supposed to encrease according to the Proportion of the Time.*



But this, before we proceed any farther, is to be premonished, that in regard we are to speak for the future of the Motion compounded of the Equable Horizontal, and of the Naturally Accelerate downwards, (for from this Mixtion results, and by it is designed the Line of the Project, that is a Parabola;) it is necessary that we define some common measure according to which we may measure the Velocity, Impetus, *or Moment of both the Motions. And seeing that of the Equable Motion the degrees of Velocity are innumerable, of which you may not take any promiscuously, but one certain one which may be be compared and conjoyned with the Degree of Velocity naturally Accelerate. I can think of no more easie way for the electing and determining of that, than by assuming another of the same kind. And that I may the better express my meaning; Let* A C *be Perpendicular to the Horizon* C B; *and* A C

to

to be the Altitude, and CB *the Amplitude of the Semiparabola A* B; *which is described by the Composition of two Lations; of which one is that of the Moveable descending along A* C *with a Motion Naturally Accelerate* ex quiete *in A; the other is the Equable Transversal Motion according to the Horizontal Line A*D. *The* Impetus *acquired in* C *along the Descent A* C *is determined by the quantity of the said height A*C; *for the* Impetus *of a* Moveable *falling from the same height is alwaies one and the same: but in the Horizontal Line one may assign not one, but innumerable Degrees of Velocities of* Equable Motions*: out of which multitude that I may single out, and as it were point with the finger to that which I make choice of, I extend or prolong the Altitude* C *A* in sublimi, *in which, as was done before, I will pitch upon A*E; *from which if I conceive in my mind a* Moveable *to fall* ex quiete *in* E, *it appeareth that its* Impetus *acquired in the Time A, is one with which I conceive the same* Moveable *being turned along A*D *to be moved; and its degree of Velocity to be that, which in the Time of the Descent along* E *A passeth a Space in the Horizon double to the said* E *A. This Præmonition I judged necessary.*



It is moreover to be advertized that the Amplitude of the Semiparabola A B shall be called by me the Horizontal Line [or Plane] C *B.*

The Altitude, to wit A C, *the Axis of the said Parabola.*

And the Line E *A, by whose Descent the Horizontal* Impetus *is determined, I call the Sublimity, or height.*

These things being declared and defined, I proceed to Demonstration.

SAGR. Stay, I pray you, for here me thinks it is convenient to adorn this Opinion of our Author with the conformity of it to the Conceit of *Plato* about the determining the different Velocities of the Equable Motions of the Revolutions of the Cœlestial Bodies; who, having perhaps had a conjecture that no Moveable could passe from Rest into any determinate degree of Velocity in which it ought afterwards to be perpetuated, unless by passing thorow all the other lesser degrees of Velocity, or, if you will, greater degrees of Tardity, which interpose between the assigned degree, and the highest degree of Tardity, that is of Rest, said that God after he had created the Moveable Cœlestial Bodies that he might assign them those Velocities wherewith they were afterwards

to be perpetually moved with an Equable Circular Motion, made them, they departing from Reſt, to move along determinate Spaces with that Natural Motion in a Right Line, according to which we ſenſibly ſee our Moveables to move from the ſtate of Reſt ſucceſſively Accelerating. And he addeth, that having made them to acquire that degree in which it pleaſed him that they ſhould afterwards be perpetually conſerved, he converted their Right or direct Motion into Circular; which only is apt to conſerve it ſelf Equable, alwaies revolving without receding from, or approaching to any prefixed term by them deſired. The Conceit is truly worthy of *Plato*; and is the more to be eſteemed in that the grounds thereof paſſed over in ſilence by him, and diſcovered by our Author by taking off the Mask or Poetick Repreſentation, do ſhew it to be in its native aſpect a true Hiſtory. And I think it very credible that we having by the Doctrine of Aſtronomy ſufficiently competent Knowledge of the Magnitudes of the Orbes of the Planets, and of their Diſtances from the Center about which they move, as alſo of their Velocities, our Author (to whom *Plato*'s Conjecture was not unknown) may ſometime for his curioſity have had ſome thought of attempting to inveſtigate whether one might aſſign a determinate Sublimity from which the Bodies of the Planets departing, as from a ſtate of Reſt, and moved for certain Spaces with a Right and Naturally Accelerate Motion, afterwards converting the Acquired Velocity into Equable Motions, they might be found to correſpond with the greatneſs of their Orbes, and with the Times of their Revolutions.

Salv. I think I do remember that he hath heretofore told me, that he had once made the Computation, and alſo that he found it exactly to anſwer the Obſervations; but that he had no mind to ſpeak of them, doubting leſt the two many Novelties by him diſcovered, which had provoked the diſpleaſure of many againſt him, might blow up new ſparks. But if any one ſhall have the like deſire he may of himſelf by the Doctrine of the preſent Tract give himſelf content. But let us purſue our buſineſs, which is to ſhew;

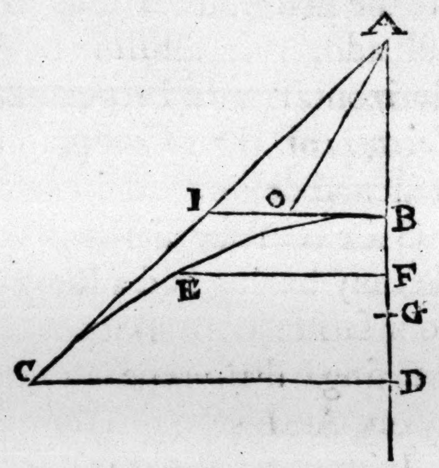
PROBL. I. PROP. IV.

How in a Parabola given, deſcribed by the Project, the *Impetus* of each ſeveral point may be determined.

Let the Semiparabola be B E C, *whoſe Amplitude is* C D *and Altitude* D B, *with which continued out on high the Tangent of the Parabola* C A *meeteth in* A; *and along the* Vertex B *let* B I *be*

an

an Horizontal Line, and parallel to C D. *And if the Amplitude* C D *be equal to the whole Altitude* D A, B I *shall be equal to* B A *and* B D. *And if the Time of the Fall along* A B, *and the Moment of Velocity acquired in* B *along the Descent* A B ex quiete *in* A *be supposed to be measured by the said* A B, *then* D C (*that is twice* B I) *shall be the Space which shall be passed by the* Impetus A B *turned along the Horizontal Line in the same Time : But in the same Time falling along* B D *out of Rest in* B, *it shall pass the Altitude* B D : *Therefore the Moveable falling out of Rest in* A *along* A B, *being converted with the* Impetus A B *along the Horizontal Parallel shall pass a Space equal to* D C. *And the Fall along* B D *supervening, it passeth the Altitude* B D, *and describes the Parabola* B C ; *whose* Impetus *in the Term* C *is compounded of the Equable Transversal whose Moment is as* A B, *and of another Moment acquired in the Fall* B D *in the Term* D *or* C ; *which Moments are Equal. If therefore we*



suppose A B *to be the Measure of one of them, as suppose of the Equable Transversal ; and* B I, *which is equal to* B D, *to be the Measure of the* Impetus *acquired in* D *or* C ; *then the Subtense* I A *shall be the quantity of the Moment compound of them both : Therefore it shall be the quantity or Measure of the whole Moment which the Project descending along the Parabola* B C *shall acquire of* Impetus *in* C. *This premised, take in the Parabola any point* E, *in which we are to determine of the* Impetus *of the Project. Draw the Horizontal Parallel* E F, *and let* B G *be a Mean-proportional between* B D *and* B F. *And forasmuch as* A B *or* B D *is supposed to be the Measure of the Time, and of the Moment of the Velocity in the Fall* B D ex quiete *in* B : B G *shall be the Time, or the Measure of the Time, and of the* Impetus *in* F, *coming out of* B. *If therefore* B O *be supposed equal to* B G, *the Diagonal drawn from* A *to* O *shall be the quantity of the* Impetus *in* E ; *for* A B *hath been supposed the determinator of the Time, and of the* Impetus *in* B, *which turned along the Horizontal Parallel doth alwaies continue the same : And* B O *determineth the* Impetus *in* F *or in* E *along the Descent* ex quiete *in* B *in the Altitude* B F : *But these two* A B *and* B O *are* potentia *equal to the Power* A O. *Therefore that is manifest which was sought.*

SAGR. The Contemplation of the Composition of these different *Impetus's*, and of the quantity of that *Impetus* which results from this mixture, is so new to me, that it leaveth my mind in no small confusion. I do not speak of the mixtion of two Motions

Equable,

Equable, though unequal to one another, made the one along the Horizontal Line, and the other along the Perpendicular, for I very well comprehend that there is made a Motion of these two *potentia* equal to both the Compounding Motions, but my confusion ariseth upon the mixing of the Equable-Horizontal and Perpendicular-Naturally-Accelerate Motion. Therefore I could wish we might together a little better consider this business.

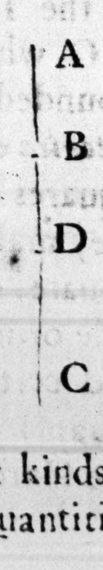
SIMP. And I stand the more in need thereof in that I am not yet so well satisfied in Mind as I should be, in the Propositions that are the first foundations of the others that follow upon them. I will add, that also in the Mixtion of the two Motions Equable Horizontal, and Perpendicular, I would better understand that *Potentia* of their Compound. Now, *Salviatus*, you see what we want and desire.

SALV. Your desire is very reasonable: and I will essay whether my having had a longer time to think thereon may facilitate your satisfaction. But you must bear with and excuse me if in discoursing I shall repeat a great part of the things hitherto delivered by our Author.

It is not possible for us to speak positively touching Motions and their Velocities or *Impetus's*, be they Equable, or be they Naturally-Accelerate, unless we first agree upon the Measure that we are to use in the commensuration of those Velocities, as also of the Time. As to the Measure of the Time, we have already that which is commonly received by all of Hours, Prime-Minutes, and Seconds, *&c.* and as for the measuring of Time we have that common Measure received by all, so it is requisite to assign another Measure for the Velocities that is commonly understood and received by every one; that is, which every where is the same. The Author, as hath been declared, adjudged the Velocity of Naturally descending Grave-Bodies to be fit for this purpose; the encreasing Velocities of which are the same in all parts of the World. So that that same degree of Velocity which (for example) a Ball of Lead of a pound acquireth in having, departing from Rest, descended Perpendicularly as much as the height of a Pike, is alwaies, and in all places the same, and therefore most commodious for explicating the quantity of the *Impetus* that is derived from the Natural Descent. Now it remains to find a way to determine likewise the Quantity of the *Impetus* in an Equable Motion in such a manner, that all those which discourse about it may form the same conceit of its greatness and Velocity; so that one may not imagine it more swift, and another less; whereupon afterwards in conjoyning and mingling this Equable Motion imagined by them with the established Accelerate Motion several men may form several Conceits of several greatnesses of *Impetus's*. To determine and represent this

Impetus,

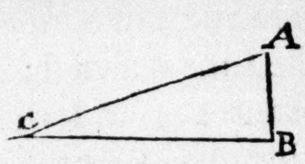
Impetus, and particular Velocity our Author hath not found any way more commodious, than the making use of the *Impetus* which the Moveable from time to time acquires in the Naturally-Accelerate Motion, any acquired Moment of which being reduced into an Equable Motion retaineth its Velocity precisely limited, and such, that in such another Time as that wherein it did Descend it passeth double the Space of the Height from whence it fell. But because this is the principal point in the business that we are upon, it is good to make it to be perfectly understood by some particular Example. Reassuming therefore the Velocity and *Impetus* acquired by the Cadent Moveable, as we said before, from the height of a Pike, of which Velocity we will make use for a Measure of other Velocities and *Impetusses* upon other occasions, and supposing, for example, that the Time of that Fall be four second Minutes of an hour, to find by this same Measure how great the *Impetus* of the Moveable would be falling from any other height greater, or lesser, we ought not from the proportion that this other height hath to the height of a Pike to argue and conclude the quantity of the *Impetus* acquired in this second height, thinking, for example, that the Moveable falling from quadruple the height hath acquired quadruple Velocity, for that it is false: for that the Velocity of the Naturally-Accelerate Motion doth not increase or decrease according to the proportion of the Spaces, but according to that of the Times, than which that of the Spaces is greater in a duplicate proportion, as was heretofore demonstrated. Therefore when in a Right Line we have assigned a part for the Measure of the Velocity, and also of the Time, and of the Space in that Time passed (for that for brevity sake all these three Magnitudes are often represented by one sole Line,) to find the quantity of the Time, and the degree of Velocity that the same Moveable would have acquired in another Distance we shall obtain the same, not immediataly by this second Distance, but by the Line which shall be a Mean-proportional betwixt the two Distances. But I will better declare my self by an Example. In the Line A C Perpendicular to the Horizon let the part A B be understood to be a Space passed by a Moveable naturally descending with an Accelerate Motion: the Time of which passage, in regard I may represent it by any Line, I will, for brevity, imagine it to be as much as the same Line A B and likewise for a Measure of the *Impetus* and Velocity acquired by that Motion, I again take the same Line A B; so that of all the Spaces that are in the progress of the Discourse to be considered the part A B may be the Measure. Having all our pleasure established under one sole Magnitude A B these three Measures of different kinds of



Quantities,

Quantities, that is to say, of Spaces, of Times, and of *Impetus's*, let it be required to determine in the assigned Space, and at the height A C, how much the Time of the Fall of the Moveable from A to C is to be, and what the *Impetus* is that shall be found to have been acquired in the said Term C, in relation to the Time and to the *Impetus* measured by A B. Both these questions shall be resolved taking A D the Mean-proportional betwixt the two Lines A C and A B; affirming the Time of the Fall along the whole Space A C to be as the Time A D is in relation to A B, assigned in the beginning for the Quantity of the Time in the Fall A B. And likewise we will say that the *Impetus*, or degree of Velocity that the Cadent Moveable shall obtain in the Term C, in relation to the *Impetus* that it had in B, is as the same Line A D is in relation to A B, being that the Velocity encreaseth with the same proportion as the Time doth: Which Conclusion although it was assumed as a *Postulatum*, yet the Author was pleased to explain the Application thereof above in the third Proposition.

This point being well understood and proved, we come to the Consideration of the *Impetus* derived from two compound Motions: whereof let one be compounded of the Horizontal and alwaies Equable, and of the Perpendicular unto the Horizon, and it also Equable: but let the other be compounded of the Horizontal likewise alwaies Equable, and of the Perpendicular Naturally-Accelerate. If both shall be Equable, it hath been seen already that the *Impetus* emerging from the composition of both is *potentia* equal to both, as for more plainness we will thus Exemplifie. Let the Moveable descending along the Perpendicular A B be supposed to have, for example, three degrees of Equable *Impetus*, but being transported along A B towards C, let the said Velocity and *Impetus* be supposed four degrees, so that in the same Time that falling it would



pass along the Perpendicular, *v. gr.* three yards, it would in the Horizontal pass four, but in that compounded of both the Velocities it cometh in the same Time from the point A unto the Term C, descending all the way along the Diagonal Line A C, which is not seven yards long, as that should be which is compounded of the two Lines A B, 3, and B C, 4, but is 5; which 5 is *potentia* equal to the two others, 3 and 4: For having found the Squares of 3 and 4, which are 9 and 16, and joyning these together, they make 25 for the Square of A C, which is equal to the two Squares of A B and B C: whereupon A C shall be as much as is the Side, or, if you will, Root of the Square 25, which is 5. For a constant and certain Rule therefore, when it is required to assign the Quantity of the *Impetus* resulting from two *Impetus's* given, the one Horizontal, and the other Perpendicular, and both Equable,

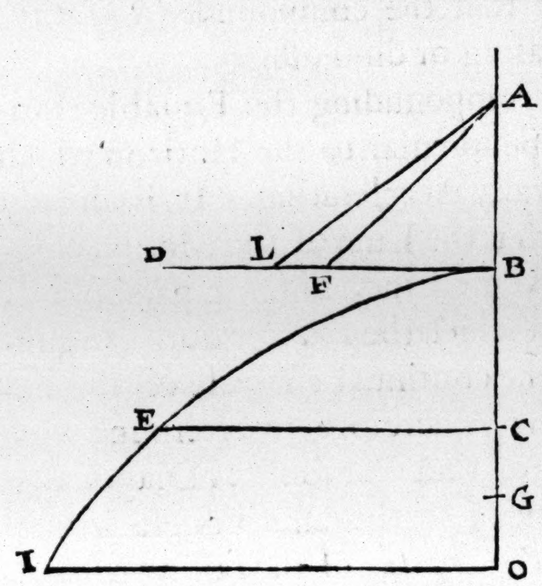
they

they are each of them to be squared, and their Squares being put together the Root of the Aggregate is to be extracted, which shall give us the quantity of the *Impetus* compounded of them both. And thus in the foregoing example, that Moveable that by vertue of the Perpendicular Motion would have percussed upon the Horizon with three degrees of Force, and with only the Horizontal Motion would have percussed in C with four degrees, percussing with both the *Impetus's* conjoyned, the blow shall be like to that of the Percutient moved with five degrees of Velocity and Force. And this same Percussion would be of the same Impetuosity in all the points of the Diagonal A C, for that the compounded *Impetus's* are alwaies the same, never encreasing or diminishing.

Let us now see what befalls in compounding the Equable Horizontal Motion with another Perpendicular to the Horizon which beginning from Rest goeth Naturally Accelerating. It is already manifest, that the Diagonal, which is the Line of the Motion compounded of these two, is not a Right Line, but Semiparabolical, as hath been demonstrated; * in which the *Impetus* doth go continually encreasing by means of the continual encrease of the Velocity of the Perpendicular Motion: Wherefore, to determine what the *Impetus* is in an assigned point of that Parabolical Diagonal, it is requisite first to assign the Quantity of the Uniform Horizontal *Impetus*, and then to find what is the *Impetus* of the falling Moveable in the point assigned: the which cannot be determined without the consideration of the Time spent from the beginning of the Composition of the two Motions: which Consideration of the Time is not required in the Composition of Equable Motions, the Velocities and *Impetus's* of which are alwaies the same: but here where there is inserted into the mixture a Motion which beginning from extream Tardity goeth encreasing in Velocity according to the continuation of the Time, it is necessary that the quantity of the Time do shew us the quantity of the degree of Velocity in the assigned point: for, as to the rest, the *Impetus* compounded of these two (as in Uniform Motions) is *potentia* equal to both the others compounding. But here again I will better explain my meaning by an example. In A C the Perpendicular to the Horizon let any part be taken A B; the which I will suppose to stand for the Measure of the Space of the Natural Motion made along the said Perpendicular, and likewise let it be the Measure of the Time, and also of the degree of Velocity, or, if you will, of the *Impetus's*. It is manifest in the first place, that if the *Impetus* of the Moveable in B *ex quiete* in A shall be turned along B D parallel to the Horizon in an Equable Motion, the quantity of its Velocity shall be such that in the Time A B it shall pass a Space double to the Space A B, which let be the Line B D. Then let B C be supposed equal to B A, and

* Or along which.

let C E be drawn parallel and equal to B D, and thus by the Points B and E we ſhall deſcribe the Parabolick Line B E I. And becauſe that in the Time A B with the *Impetus* A B the Horizontal Line B D or C E is paſſed, double to A B, and in ſuch another Time the Perpendicular B C is paſſed with an acquiſt of *Impetus* in C equal to the ſaid Horizontal Line; therefore the Moveable in ſuch another Time as A B ſhall be found to have paſſed from B to E along the Parabola B E with an *Impetus* compounded of two, each equal to the *Impetus* A B. And becauſe one of them is Horizontal, and the other Perpendicular, the *Impetus* compound of them ſhall be equal in Power to them both, that is double to one of them. So that ſuppoſing B F equal to B A; and drawing the Diagonal A F, the *Impetus* or the Percuſſion in E ſhall be greater than the Percuſſion in B of the Moveable falling from the Height A, or than the Percuſſion of the Horizontal *Impetus* along B D, according to the proportion of A F to A B. But in caſe, ſtill retaining B A for the Meaſure of the Space of the Fall from Reſt in A unto B, and for the Meaſure of the Time and of the *Impetus* of the falling Moveable acquired in B, the Altitude B O ſhould not be equal to, but greater than A B, taking B G to be a Mean-proportional betwixt the ſaid A B and B O, the ſaid B G would be the Meaſure of the Time and of the *Impetus* in O, acquired in O by the Fall from the height B O; and the Space along the Horizontal Line, which being paſſed with the *Impetus* A B in the Time A B would be double to A B, ſhall, in the whole duration of the Time B G, be ſo much the greater, by how much in proportion B G is greater than B A. Suppoſing therefore L B equal to B G, and drawing the Diagonal A L, it ſhall give us the quantity compounded of the two *Impetus's* Horizontal and Perpendicular, by which the Parabola is deſcribed; and of which the Horizontal and Equable is that acquired in B by the fall of A B, and the other is that acquired in O, or, if you will, in I by the Deſcent B O, whoſe Time, as alſo the quantity of its Moment was B G. And in this Method we ſhall inveſtigate the *Impetus* in the extream term of the Parabola, in caſe its Altitude were leſſer than the Sublimity A B, taking the Mean-proportional betwixt them both: which being ſet off upon the Horizontal Line in the place of B F, and the Diagonal drawn, as A F, we ſhall hereby have the quantity of the *Impetus* in the extream term of the Parabola.



And

And to what hath hitherto been proposed touching *Impetus's*, Blows, or if you please, Percussions of such like Projects, it is necessary to add another very necessary Consideration; and this it is: That it doth not suffice to have regard to the Velocity only of the Project for the determining rightly of the Force and Violence of the Percussion, but it is requisite likewise to examine apart the State and Condition of that which receiveth the Percussion, in the efficacy of which it hath for many respects a great share and interest. And first there is no man but knows that the thing smitten doth so much suffer violence from the Velocity of the Percutient by how much it opposeth it, and either totally or partially checketh its Motion: For if the Blow shall light upon such an one as yieldeth to the Velocity of the Percutient without any Resistance, that Blow shall be nullified: And he that runneth to hit his Enemy with his Launce, if at the overtaking of him it shall fall out that he moveth, giving back with the like Velocity, he shall make no thrust, and the Action shall be a meer touch without doing any harm.

But if the Percussion shall happen to be received upon an Object which doth not wholly yield to the Percutient, but only partially, the Percussion shall do hurt, though not with its whole *Impetus*, but only with the excess of the Velocity of the said Percutient above the Velocity of the recoile and recession of the Object percussed: so that, if *v. g.* the Percutient shall come with 10 degrees of Velocity upon the Percussed Body, which giving back in part retireth with 4 degrees, the *Impetus* and Percussion shall be as if it were of 6 degrees. And lastly, the Percussion shall be entire and perfect on the part of the Percutient when the thing percussed yieldeth not, but wholly opposeth and stoppeth the whole Motion of the Percutient; if haply there can be such a case. And I say on the part of the Percutient, for when the Body percussed moveth with a contrary Motion towards the Percutient, the Blow and Shock shall be so much the more Impetuous by how much the two Velocities united are greater than the sole Velocity of the Percutient. Moreover, you are likewise to take notice, that the more or less yielding may proceed not only from the quality of the Matter more or less hard, as if it be of Iron, of Lead, or of Wooll, *&c.* but also from the Position of the Body that receiveth the Percussion. Which Position if it shall be such as that the Motion of the Percutient happeneth to hit it at Right-Angles, the *Impetus* of the Percussion shall be the greatest: but if the Motion shall proceed obliquely, and, as we say, aslant, the Percussion shall be weaker; and that more, and more according to its greater and greater Obliquity: for an Object in that manner scituate, albeit of very solid matter, doth not damp or arrest the whole *Impetus* and Motion of the Percutient, which slanting passeth farther, continuing at least in some part to

move along the Surface of the opposed Body Resisting. When therefore we have even now determined of the greatness of the *Impetus* of the Project in the end of the Par. bolicall Line, it ought to be understood to be meant of the Percussion received upon a Line at Right Angles with the same Parabolick Line, or with the Line that is Tangent to the Parabola in the foresaid point: for although that same Motion be compounded of an Horizontal and a Perpendicular Motion, the *Impetus* is not at the greatest either upon the Horizontal Plane, or upon that erect to the Horizon, being received upon them both obliquely.

SAGR. Your speaking of these Blows, and these Percussions hath brought into my mind a Problem, or, if you will, Question in the Mechanicks, the solution whereof I could never find in any Author, nor any thing that doth diminish my admiration, or so much as in the least afford my judgment satisfaction. And my doubt and wonder lyeth in my not being able to comprehend whence that Immense Force and Violence should proceed, and on what Principle it should depend, which we see to consist in Percussion, in that with the simple stroke of an Hammer, that doth not weigh above eight or ten pounds, we see such Resistances to be overcome as would not yield to the weight of a Grave Body that without Percussion hath an *Impetus* only by pressing and bearing upon it, albeit the weight of this be many hundreds of pounds more. I would likewise find out a way to measure the Force of this Percussion, which I do not think to be infinite, but rather hold that it hath its Term in which it may be compared, and in the end Regulated with other Forces of pressing Gravities, either of Leavers, or of Screws, or of other Mechanick Instruments, of whose multiplication of Force I am thorowly satisfied.

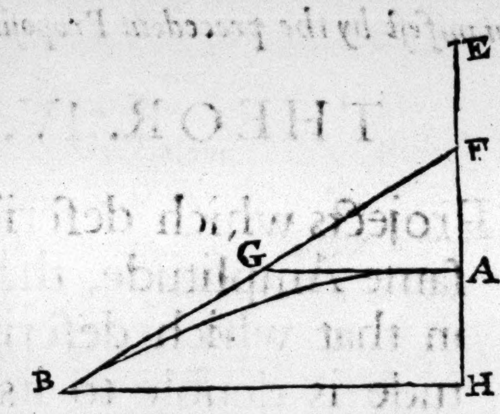
SALV. You are not alone in the admirableness of the effect, and the obscurity of the cause of so stupendious an Accident. I ruminated a long time upon it in vain, my stupifaction still encreasing; till in the end meeting with our *Academian*, I received from him a double satisfaction: first in hearing that he also had been a long time at the same loss; and next in understanding that after he had at times spent many thousands of hours in studying and contemplating thereon, he had light upon certain Notions far from our first conceptions, and therefore new, and for their Novelty to be admired. And because that I already see that your Curiosity would gladly hear those Conceits which are Remote from common Conjecture, I shall not stay for your entreaty, but I give you my word that so soon as we shall have finished the Reading of this Treatise of Projects, I will set before you all those Fancies, or, I might say, Extravagancies that are yet left in my memory of the Discourses of the Academick. In the mean time let us prosecute the Propositions of our Author.

PROBL.

PROBL. II. PROP. V.

In the Axis of a given Parabola prolonged to find a sublime point out of which the Moveable falling shall describe the said Parabola.

LET *the Parabola be* A B, *its Amplitude* H B, *and its prolonged Axis* H E; *in which a Sublimity is to be found, out of which the Moveable falling, and converting the* Impetus *conceived in* A *along the Horizontal Line, describeth the Parabola* A B. *Draw the Horizontal Line* A G, *which shall be Parallel to* B H, *and supposing* A F *equal to* A H *draw the Right Line* F B, *which toucheth the Parabola in* B, *and cutteth the Horizontal Line* A G *in* G; *and unto* F A *and* A G *let* A E *be a third Proportional. I say, that* E *is the sublime Point required, out of which the Moveable falling* ex quiete *in* E, *and the* Impetus *conceived in* A *being converted along the Horizontal Line overtaking the* Impetus *of the Descent in* H ex quiete *in* A, *describeth the Parabola* A B. *For if we suppose* E A *to be the Measure of the Time of the Fall from* E *to* A, *and of the* Impetus *acquired in* A, A G *(that is a Mean-proportional between* E A *and* A F*) shall be the Time and the* Impetus *coming from* F *to* A, *or from* A *to* H. *And because the Moveable coming out of* E *in the Time* E A *with the* Impetus *acquired in* A *passeth in the Horizontal Lation with an Equable Motion the double of* E A; *Therefore likewise moving with the same* Impetus *it shall in the Time* A G *pass the double of* G A, *to wit, the Mean-proportional* B H *(for the Spaces passed with the same Equable Motion are to one another as the Times of the said Motions:) And along the Perpendicular* A H *shall be passed with a Motion* ex quiete *in the same Time* G A: *Therefore the Amplitude* H B, *and Altitude* A H *are passed by the Moveable in the same Time: Therefore the Parabola* A B *shall be described by the Descent of the Project coming from the Sublimity* E: *Which was required.*



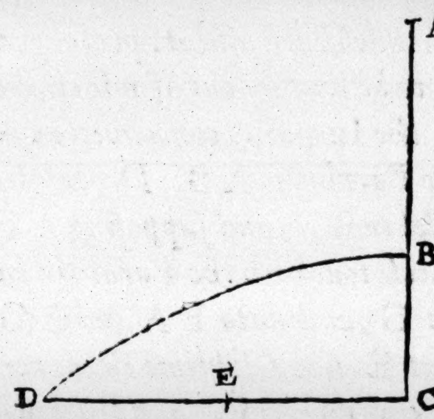
COROLLARY.

Hence it appeareth that the half of the *Base* or Amplitude of the Semiparabola (which is the fourth part of the Amplitude of the whole Parabola) is a Mean-proportional betwixt its Altitude and the Sublimity out of which the Moveable falling describeth it.

PROBL.

PROBL. III. PROP. VI.

The Sublimity and Altitude of a Semiparabola being given to find its Amplitude.



LEt A C *be perpendicular to the Horizontal Line* D C, *in which let the Altitude* C B *and the Sublimity* B *A be given: It is required in the Horizontal Line* D C *to find the Amplitude of the Semiparabola that is described out of the Sublimity* B *A with the Altitude* B C. *Take a Mean-proportional between* C B *and* B *A, to which let* C D *be double, I say, that* C D *is the Amplitude required. The which is manifest by the precedent Proposition.*

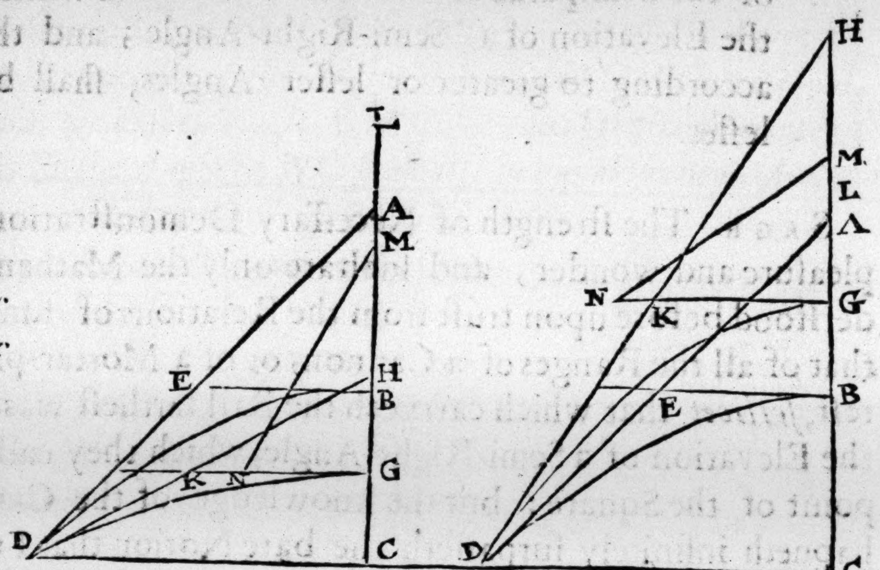
THEOR. IV. PROP. VII.

In Projects which describe Semiparabola's of the same Amplitude, there is less *Impetus* required in that which describeth that whose Amplitude is double to its Altitude, than in any other.

FOR *let the Semiparabola be* B D, *whose Amplitude* C D *is double to its Altitude* C B; *and in its Axis extended on high let* B *A be supposed equal to the Altitude* B C; *and draw a Line from A to* D *which toucheth the Semiparabola in* D, *and shall cut the Horizontal Line* B E *in* E; *and* B E *shall be equal to* B C *or to* B *A: It is manifest that it is described by the Project whose Equable Horizontal* Impetus *is such as is that gained in* B *of a thing falling from Rest in A, and the* Impetus *of the Natural Motion downwards, such as is that of a thing coming to* C ex quiete *in* B. *Whence it is manifest, that the* Impetus *compounded of them, and that striketh in the Term* D *is as the Diagonal A* E, *that is* potentia *equal to them both. Now let there be another Semiparabola* G D, *whose Amplitude is the same* C D, *and the Altitude* C G *less, or greater than the Altitude* B C, *and let* H D *touch the same, cutting the Horizontal Line drawn by* G *in the point* K; *and as* H G *is to* G K, *so let* K G *be to* G L: *by what hath been demonstrated* G L *shall be the Altitude from which the Project falling describeth the*

Parabola

Parabola G D. *Let* G M *be a Mean-proportional betwixt* A B *and* G L; G M *shall be the Time, and the Moment or* Impetus *in* G *of the Project falling from* L, (*for it hath been supposed that* A B *is the Measure of the Time and* Impetus.) *Again, let* G N *be a Mean-proportional betwixt* B C *and* C G: *this* G N *shall be the Measure of the Time and the* Impetus *of the Project falling from* G *to* C. *If therefore a Line be drawn from* M *to* N *it shall be the the Measure of the* Impetus *of the Project along the Parabola* B D, *striking in the term* D. *Which* Impetus, *I say, is greater than the* Impetus *of the Project along the Parabola* B D, *whose quantity was* A E. *For because* G N *is supposed the Mean-proportional betwixt* B C *and* C G, *and* B C *is equal to* B E, *that is to* H G; (*for they are each of them subduple to* D C:) *Therefore as* C G *is to* G N, *so shall* N G *be to* G K: *and, as* C G *or* H G *is to* G K, *so shall the Square* N G *be to the Square of* G K: *But as* H G *is to* G K, *so was* K G *supposed to be to* G L: *Therefore as* N G *is to the Square* G K, *so is* K G *to* G L: *But as* K G *is to* G L, *so is the Square* K G *unto the Square* G M, (*for* G M *is the Mean between* K G *and* G L:) *Therefore the three Squares* N G, K G, *and* G M *are continual proportionals: And the two extream ones* N G *and* G M *taken together, that is the Square* M N, *is greater than double the Square* K G, *to which the Square* A E *is double: Therefore the Square* M N *is greater than the Square* A E: *and the Line* M N *greater than the Line* A E: *Which was to be demonstrated.*



CORROLLARY I.

Hence it appeareth, that on the contrary, in the Project out of D along the Semiparabola D B, less *Impetus* is required than along any other according to the greater or lesser Elevation of the Semiparabola B D, which is according to the Tangent A D, containing half a Right-Angle upon the Horizon.

COROLLARY

COROLLARRY II.

And that being so, it followeth, that if Projections be made with the same *Impetus* out of the Term D, according to several Elevations, that shall be the greatest Projection or Amplitude of the Semiparabola or whole Parabola which followeth at the Elevation of a * Semi-Right-Angle; and the rest, made according to greater or lesser Angles, shall be greater or lesser.

* Or, at the Elevation of 45 degrees.

SAGR. The strength of Necessary Demonstrations are full of pleasure and wonder; and such are only the Mathematical. I understood before upon trust from the Relations of sundry Gunners, that of all the Ranges of a Cannon, or of a Mortar-piece, the greatest, *scilicet* that which carryeth the Ball farthest was that made at the Elevation of a Semi-Right-Angle, which they call, of the Sixth point of the Square: but the knowledge of the Cause whence it hapneth infinitely surpasseth the bare Notion that I received upon their attestation, and also from many repeated Experiments.

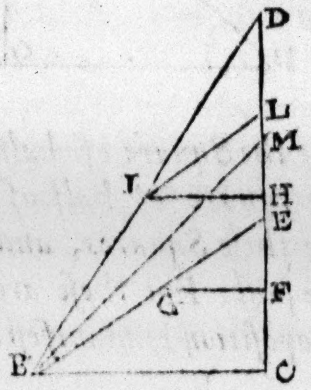
SALV. You say very right: and the knowledge of one single Effect acquired by its Causes openeth the Intellect to understand and ascertain our selves of other effects, without need of repairing unto Experiments, just as it hapneth in the present Case; in which having found by demonstrative Discourse the certainty of this, That the greatest of all Ranges is that of the Elevation of a Semi-Right-Angle, the Author demonstrates unto us that which possibly hath not been observed by Experience: and that is, that of the other Ranges those are equal to one another whose Elevations exceed or fall short by equal Angles of the Semi-right: so that the Balls shot from the Horizon, one according to the Elevation of seven Points, and the other of 5, shall light upon the Horizon at equal Distances: and so the Ranges of 8 and of 4 points, of 9 and of 3, &c. shall be equal. Now hear the Demonstration of it.

THEOR. V. PROP. VIII.

The Amplitudes of Parabola's described by Projects expulsed with the same *Impetus* according to the Elevations by Angles equidistant above, and beneath from the * Semi-right, are equal to each other.

* Or Angle of 45.

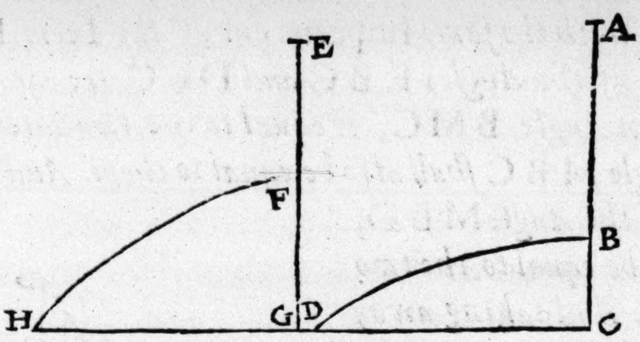
OF the Triangle M C B, *about the Right-Angle* C, *let the Horizontal Line* B C *and the Perpendicular* C M *be equal; for so the Angle* M B C *shall be Semi-right; and prolonging* C M *to* D, *let there be constituted in* B *two equal Angles above and below the Diagonal* M B, *viz.* M B E, *and* M B D. *It is to be demonstrated that the Amplitudes of the Parabola's described by the Projects being emitted* [or shot off] *with the same* Impetus *out of the Term* B, *according to the Elevations of the Angles* E B C *and* D B C, *are equal. For in regard that the extern Angle* B M C, *is equal to the two intern* M D B *and* M B D, *the Angle* M B C *shall also be equal to them. And if we suppose* M B E *instead of the Angle* M B D, *the said Angle* M B C *shall be equal to the two Angles* M B E *and* B D C*: And taking away the common Angle* M B E, *the remaining Angle* B D C *shall be equal to the remaining Angle* E B C*: Therefore the Triangles* D C B *and* B C E *are alike. Let the Right Lines* D C *and* E C *be divided in the midst in* H *and* F*; and draw* H I *and* F G *parallel to the Horizontal Line* C B*; and as* D H *is to* H I, *so let* I H *be to* H L*: the Triangle* I H L *shall be like to the Triangle* I H D, *like to which also is* E G F. *And seeing that* I H *and* G F *are equal (to wit, halves of the same* B C*:) Therefore* F E, *that is* F C, *shall be equal to* H L*: And, adding the common Line* F H, C H *shall be equal to* F L. *If therefore we understand the Semiparabola to be described along by* H *and* B, *whose Altitude shall be* H C, *and Sublimity* H L, *its Amplitude shall be* C B, *which is double to* H I, *that is, the Mean betwixt* D H, *or* C H, *and* H L*: And* D B *shall be a Tangent to it, the Lines* C H *and* H D *being equal. And if, again, we conceive the Parabola to be described along by* F *and* B *from the Sublimity* F L, *with the Altitude* F C, *betwixt which the Mean-proportional is* F G, *whose double is the Horizontal Line* C B*:* C B, *as before, shall be its Amplitude; and* E B *a Tangent to it, since* E F *and* F C *are equal: But the Angles* D B C *and* E B C (scilicet, *their Elevations) shall be equidistant from the Semi-Right Angle: Therefore the Proposition is demonstrated.*



THEOR. VI. PROP. IX.

The Amplitudes of Parabola's, whose Altitudes and Sublimities answer to each other *è contrario*, are equall.

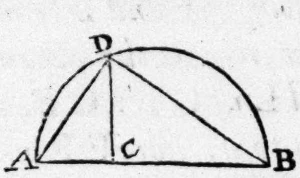
Let the Altitude G F *of the Parabola* F H *have the same proportion to the Altitude* C B *of the Parabola* B D, *as the Sublimity* B A *hath to the Sublimity* F E. *I say, that the Amplitude* H G *is equal to the Amplitude* D C. *For since the first* G F *hath the same proportion to the second* C B, *as the third* B A *hath to the fourth* F E; *Therefore, the Rectangle* G F E *of the first and fourth, shall be equal to the Rectangle* C B A *of the second and third: Therefore the Squares that are equal to these Rectangles shall be equal to one another: But the Square of half of* G H *is equal to the Rectangle* G F E; *and the Square of half of* C D *is equal to the Rectangle* C B A: *Therefore these Squares, and their Sides, and the doubles of their Sides shall be equal: But these are the Amplitudes* G H *and* C D: *Therefore the Proposition is manifest.*



LEMMA *pro sequenti.*

If a Right Line be cut according to any proportion, the Squares of the Mean-proportionals between the whole and the two parts are equal to the Square of the whole.

Let A B *be cut according to any proportion in* C. *I say, that the Squares of the Mean-proportional Lines between the whole* A B *and the parts* A C *and* C B, *being taken together are equal to the Square of the whole* A B. *And this appeareth, a Semicircle being described upon the whole Line* B A, *and from* C *a Perpendicular being erected* C D, *and Lines being drawn from* D *to* A, *and from* D *to* B. *For* D A *is the Mean-proportional betwixt* A B *and* A C; *and* D B *is the Mean-proportional between* A B *and* B C: *And the Squares of the Lines* D A *and* D B *taken together are equal to the Square of the whole Line* A B, *the Angle* A D B *in the Semicircle being a Right-Angle: Therefore the Proposition is manifest.*

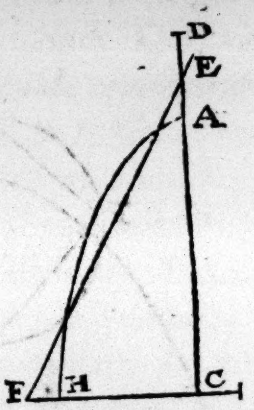


THEOR.

THEOR. VII. PROP. X.

The *Impetus* or Moment of any Semiparabola is equal to the Moment of any Moveable falling naturally along the Perpendicular to the Horizon that is equal to the Line compounded of the Sublimity and of the Altitude of the Semiparabola.

Let the Semiparabola be A B, *its Sublimity* D A, *and Altitude* A C, *of which the Perpendicular* D C *is compounded. I say, that the* Impetus *of the Semiparabola in* B *is equal to the Moment of the Moveable Naturally falling from* D *to* C. *Suppose* D C *it self to be the Measure of the Time and of the* Impetus; *and take a Mean-proportional betwixt* C D *and* D A, *to which let* C F *be equal; and withal let* C E *be a Mean-proportional between* D C *and* C A: *Now* C F *shall be the Measure of the Time and of the Moment of the Moveable falling along* D A *out of Rest in* D; *and* C E *shall be the Time and Moment of the Moveable falling along* A C, *out of Rest in* A, *and the Moment of the Diagonal* E F *shall be that compounded of both the others,* scil. *that of the Semiparabola in* B. *And because* D C *is cut according to any proportion in* A, *and because* C F *and* C E *are Mean-Proportionals between* C D *and the parts* D A *and* A C; *the Squares of them taken together shall be equal to the Square of the whole; by the Lemma aforegoing: But the Squares of them are also equal to the Square of* E F: *Therefore* D F *is equal also to the Line* DC: *Whence it is manifest that the Moments along* D C, *and along the Semiparabola* A B, *are equal in* C *and* B: *Which was required.*



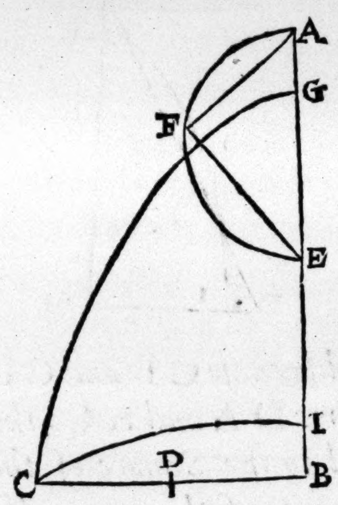
COROLLARY.

Hence it is manifest, that of all Parabola's whose Altitudes and Sublimities being joyned together are equal, the *Impetus's* are also equal.

PROBL. IV. PROP. XI.

The *Impetus* and Amplitude of a Semiparabola being given, to find its Altitude, and consequently its Sublimity.

LEt the Impetus *given be defined by the Perpendicular to the Horizon* A B; *and let the Amplitude along the Horizontal Line be* B C. *It is required to find the Altitude and Sublimity of the Parabola whose* Impetus *is* A B, *and Amplitude* B C. *It is manifest, from what hath been already demonstrated, that half the Amplitude* B C *will be a Mean-proportional betwixt the Altitude and the Sublimity of the said Semiparabola, whose* Impetus, *by the precedent Proposition, is the same with the* Impetus *of the Moveable falling from Rest in* A *along the whole Perpendicular* A B: *Wherefore* B A *is so to be cut that the Rectangle contained by its parts may be equal to the Square of half of* B C, *which let be* B D. *Hence it appeareth to be necessary that* D B *do not exceed the half of* B A; *for of Rectangles contained by the parts the greatest is when the whole Line is cut into two equal parts. Therefore let* B A *be divided into two equal parts in* E. *And if* B D *be equal to* B E *the work is done; and the Altitude of the Semiparabola shall be* B E, *and its Sublimity* E A: *(and see here by the way that the Amplitude of the Parabola of a Semi-right Elevation, as was demonstrated above, is the greatest of all those described with the same* Impetus.*)*



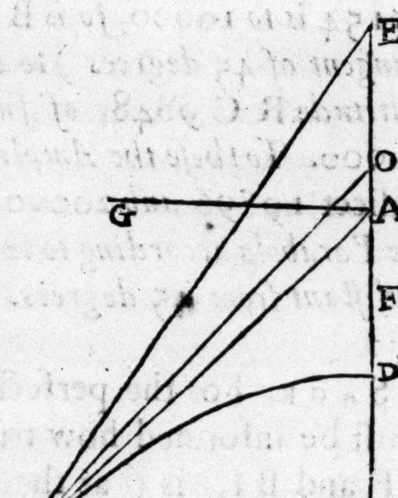
But let B D *be less than the half of* B A, *which is so to be cut that the Rectangle under the parts may be equal to the Square* B D. *Upon* E A *describe a Semicircle, upon which out of* A *set off* A F *equal to* B D, *and draw a Line from* F *to* E, *to which cut a part equal* E G. *Now the Rectangle* B G A, *together with the Square* E G, *shall be equal to the Square* E A; *to which the two Squares* A F *and* F E *are also equal: Therefore the equal Squares* G E *and* F E *being substracted, there remaineth the Rectangle* B G A *equal to the Square* A F, *scilicet, to* B D; *and the Line* B D *is a Mean-proportional betwixt* B G *and* G A. *Whence it appeareth, that of the Semiparabola whose Amplitude is* B C, *and* Impetus A B, *the Altitude is* B G, *and the Sublimity* G A. *And if we set off* B I *below equal to* G A, *this shall be the Altitude, and* I A *the Sublimity of the Semiparabola* I C. *From what hath been already demonstrated we are able,*

PROBL.

PROBL. V. PROP. XII.

To collect by Calculation of the Amplitudes of all Semiparabola's that are described by Projects expulsed with the same *Impetus*, and to make Tables thereof.

It is obvious, from the things demonstrated, that Parabola's are described by Projects of the same Impetus *then, when their Sublimities together with their Altitudes do make up equal Perpendiculars upon the Horizon. These Perpendiculars therefore are to be comprehended between the same Horizontal Parallels. Therefore let the Horizontal Line* C B *be supposed equal to the Perpendicular* B A, *and draw the Diagonal from* A *to* C. *The Angle* A C B *shall be Semiright, or 45 Degrees. And the Perpendicular* B A *being divided into two equal parts in* D, *the Semiparabola* D C *shall be that which is described from the Sublimity* A D *together with the Altitude* D B: *and its* Impetus *in* C *shall be as great as that of the Moveable coming out of Rest in* A *along the Perpendicular* A B, *is in* B. *And if* A G *be drawn parallel to* B C, *the united Altitudes and Sublimities of all other remaining Semiparabola's whose future* Impetus's *are the same with those now mentioned must be bounded by the Space between the Parallels* A G *and* B C. *Farthermore, it having been but now demonstrated, that the Amplitudes of the Semiparabola's whose Tangents are equidistant either above or below from the Semi right Elevation are equal, the Calculations that we frame for the greater Elevations will likewise serve for the lesser. We choose moreover a number of ten thousand parts for the greatest Amplitude of the Projection of the Semiparabola made at the Elevation of 45 degrees: so much therefore the Line* B A, *and the Amplitude of the Semiparabola* B C, *are to be supposed. And we make choice of the number* 10000, *because we in our Calculation use the Table of Tangents, in which this number agreeth with the Tangent of 45 degrees. Now, to come to the business, let* C E *be drawn, containing the Angle* E C B *greater (Acute nevertheless,) than the Angle* A C B; *and let the Semiparabola be described which is touched by the Line* E C, *and whose Sublimity united with its Altitude is equal to* B A. *In the Table of Tangents take the said* B E *for the Tangent at the*



given

given Angle B C E, *which divide into two equal parts at* F. *Then find a third Proportional to* B F *and* B C, (*or to the half of* B C,) *which shall of necessity be greater than* F A; *therefore let it be* F O: *Of the Semiparabola, therefore, inscribed in the Triangle* E C B, *according to the Tangent* C E, *whose Amplitude is* C B, *the Altitude* B F, *and the Sublimity* F O *is found: But the whole Line* B O *riseth above the Parallels* A G *and* C B, *whereas our work was to bound it between them: For so both it and the Semiparabola* D C *shall be described by the Projects out of* C *expelled with the same* Impetus. *Therefore we are to seek another like to this, (for innumerable greater and smaller, like to one another, may be described within the Angle* B C E) *to whose united Sublimity and Altitude* B A *shall be equal. Therefore as* O B *is to* B A, *so let the Amplitude* B C *be to* C R: *and* C R *shall be found,* scilicet *the Amplitude of the Semiparabola according to the Elevation of the Angle* B C E, *whose conjoyned Sublimity and Altitude is equal to the Space contained between the Parallels* G A *and* C B: *Which was required. The work, therefore, shall be after this manner.*

Take the Tangent of the given Angle B C E, *to the half of which add the third Proportional of it, and half of* B C, *which let be* F O: *Then as* O B *is to* B A, *so let* B C *be to another, which let be* C R, *to wit, the Amplitude sought. Let us give an Example.*

Let the Angle E C B *be* 50 *degrees, its Tangent shall be* 11918, *whose half, to wit,* B F, *is* 5959, *and the half of* B C *is* 5000, *the third proportional of these halves is* 4195, *which added to the said* B F *maketh* 10154: *for the said* B O. *Again, as* O B *is to* B A, *that is as* 10154 *is to* 10000, *so is* B E, *that is* 10000 (*for each of them is the Tangent of* 45 *degrees*) *to another: and that shall give us the required Altitude* R C 9848, *of such as* B C (*the greatest Amplitude*) *is* 10000. *To these the Amplitudes of the whole Parabola's are double,* scilicet 19696 *anb* 20000. *And so much likewise is the Amplitude of the Parabola according to the Elevation of* 40 *degrees, since it is equally distant from* 45 *degrees.*

SAGR. For the perfect understanding of this Demonstration I must be informed how true it is, that the Third Proportional to B F and B I, is (as the Author saith) necessarily greater than F A.

SALV. That inference, as I conceive, may be deduced thus. The Square of the Mean of three proportional Lines is equal to the Rectangle of the other two: whence the Square of B I, or of B D equal to it, ought to be equal to the Rectangle of the first F B multiplied into the third to be found: which third is of necessity to be greater than F A, because the Rectangle of B F multiplied into F A is less than the Square B D: and the Defect is as much as the Square of D F, as *Euclid* demonstrates in a Proposition of his

Second

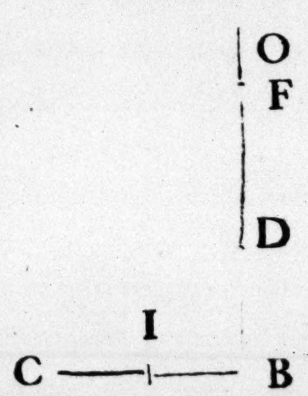
Second Book. You must also know, that the point F which divideth the Tangent E B in the middle, will many other times fall above the point A, and once also in the said A: In which cases it is evident of it self, that the third proportional to the half of the Tangent, and to B I (which giveth the Sublimity) is all above A. But the Author hath taken a Case in which it was not manifest that the said third Proportional is alwaies greater than F A: and which therefore being set off above the point F passeth beyond the Parallel A G. Now let us proceed.

It will not be unprofitable if by help of this Table we compose another, shewing the Altitudes of the same Semiparabola's of Projects of the same Impetus. *And the Construction of it is in this manner.*

PROBL. VI. PROP. XIII.

From the given Amplitudes of Semiparabola's in the following Table set down, keeping the common *Impetus* with which every one of them is described, to compute the Altitudes of each several Semiparabola.

LEt *the Amplitude given be* B C, *and of the* Impetus, *which is supposed to be alwaies the same, let the Measure be* O B, *to wit, the Aggregate of the Altitude and Sublimity. The said Altitude is required to be found and distinguished. Which shall then be done when* B O *is so divided as that the Rectangle contained under its parts is equal to the Square of half the Amplitude* B C. *Let that same division fall in* F; *and let both* O B *and* B C *be cut in the midst at* D *and* I. *The Square* I B, *therefore, is equal to the Rectangle* B F O: *And the Square* D O *is equal to the same Rectangle together with the Square* F D. *If therefore from the Square* D O *we deduct the Square* B I, *which is equal to the Rectangle* B F O, *there shall remain the Square* F D; *to whose Side* D F, B D *being added it shall give the desired Altitude Altitude* B F. *And it is thus compounded* ex datis. *From half of the Square* B O *known substract the Square* B I *also known, of the remainder take the Square Root, to which add* D B *known; and you shall have the Altitude sought* B F. *For example. The Altitude of the Parabola described at the Elevation of* 55 *degrees is to be found. The Amplitude, by the following Table is* 9396, *its half is* 4698, *the Square of that is* 22071204, *this*



this substracted from the Square of the half BO, which is alwaies the same, to wit, 2500000, the remainder is 2928796, whose Square Root is 1710 very near, this added to the half of BO, to wit, 5000, gives 67101, and so much is the Altitude BF. It will not be unprofitable, to give the Third Table, containing the Altitudes and Sublimities of Semiparabola's, whose Amplitude shall be alwaies the same.

SAGR. This I would very gladly see since by it I may come to know the Difference of the *Impetus's*, and of the Forces that are required for carrying the Project to the same Distance with Ranges which are called at Random: which Difference I believe is very great according to the different Elevations [*or Mountures*:] so that if, for example, one would at the Elevation of 3 or 4 degrees, or of 87 or 88 make the Ball to fall where it did, being shot at the Elevation of *gr*. 45. (where, as hath been shewn, the least *Impetus* is required) I believe that it would require a very much greater Force.

SALV. You are in the right: and you will find that to do the full execution in all the Elevations it is requisite to make great Progressions towards an infinite *Impetus*. Now let us see the Construction of the Table.

The

The Amplitudes of the Semiparabola's, described with the same *Impetus*.

Gr.		Gr.
45	10000	
46	9994	44
47	9976	43
48	9945	42
49	9902	41
50	9848	40
51	9782	39
52	9704	38
53	9612	37
54	9511	36
55	9396	35
56	9272	34
57	9136	33
58	8989	32
59	8829	31
60	8659	30
61	8481	29
62	8290	28
63	8090	27
64	7880	26
65	7660	25
66	7431	24
67	7191	23
68	6944	22
69	6692	21
70	6428	20
71	6157	19
72	5878	18
73	5592	17
74	5300	16
75	5000	15
76	4694	14
77	4383	13
78	4067	12
79	3746	11
80	3420	10
81	3090	9
82	2756	8
83	2419	7
84	2079	6
85	1736	5
86	1391	4
87	1044	3
88	698	2
89	349	1

Degrees of Elevation.

The Altitudes of the Semiparabola's, whose *Impetus* is the same.

Gr.		Gr.	
1	3	46	5173
2	13	47	5346
3	28	48	5523
4	50	49	5698
5	76	50	5868
6	108	51	6038
7	150	52	6207
8	194	53	6379
9	245	54	6546
10	302	55	6710
17	365	56	6873
12	432	57	7033
13	506	58	7190
14	585	59	7348
15	670	60	7502
16	760	61	7649
17	855	62	7796
18	955	63	7939
19	1060	64	8078
20	1170	65	8214
21	1285	66	8346
22	1402	67	8474
23	1527	68	8597
24	1685	69	8715
25	1786	70	8830
26	1922	71	8940
27	2061	72	9045
28	2204	73	9144
29	2351	74	9240
30	2499	75	9330
31	2653	76	9415
32	2810	77	9493
33	2967	78	9567
34	3128	79	9636
35	3289	80	9698
36	3456	81	9755
37	3621	82	9806
38	3793	83	9851
39	3962	84	9890
40	4132	85	9924
41	4302	86	9951
42	4477	87	9972
43	4654	88	9987
44	4827	89	9998
45	5000	90	10000

Degrees of Elevation.

A Table containing the Altitudes and Sublimities of the Semiparabola's, whose Amplitudes are the same, that is to say, of 10000 parts, calculated to each Deg. of Elevation.

Gr.	Altit.	Sublim.	Gr.	Altit.	Sublim.
1	87	286533	46	5177	4828
2	175	142450	47	5363	4662
3	262	95802	48	5553	4502
4	349	71531	49	5752	4345
5	437	57142	50	5959	4196
6	525	47573	51	6174	4048
7	614	40716	52	6399	3906
8	702	35587	53	6635	3765
9	792	31565	54	6882	3632
10	881	28367	55	7141	3500
11	972	25720	56	7413	3372
12	1063	23518	57	7699	3247
13	1154	21701	58	8002	3123
14	1246	20056	59	8332	3004
11	1339	18663	60	8600	2887
16	1434	17405	61	9020	2771
17	1529	16355	62	9403	2658
18	1624	15389	63	9813	2547
19	1722	14522	64	10251	2438
20	1820	13736	65	10722	2331
21	1919	13024	66	11220	2226
22	2020	12376	67	11779	2122
23	2123	11778	68	12375	2020
24	2226	11230	69	13025	1919
25	2332	10722	70	13237	1819
26	2439	10253	71	14521	1721
27	2547	9814	72	15388	1624
28	2658	9404	73	16354	1528
29	2772	9020	74	17437	1433
30	2887	8659	75	18660	1339
31	3008	8336	76	20054	1246
32	3124	8001	77	21657	1154
33	3247	7699	78	23523	1062
34	3373	7413	79	25723	972
35	3501	7141	80	28356	881
36	3633	6882	81	31560	792
37	3768	6635	82	35577	702
38	3906	6395	83	40222	613
39	4049	6174	84	47572	525
40	4196	5959	85	57150	437
41	4346	5752	86	71503	349
42	4502	5553	87	95405	262
43	4662	5362	88	143181	174
44	4828	5177	89	286499	87
45	5000	5000	90	Infinite	

PROBL. VII. PROP. XIV.

To find the Altitudes and Sublimities of Semiparabola's whose Amplitudes shall be equal for each degree of Elevation.

THis we shall easily do. For supposing the Amplitude of the Semiparabola to be of 10000 *parts, the half of the Tangent of each degree of Elevation shews the Altitude. As for example, of the Semiparabola whose Elevation is* 30 *degrees, and Amplitude, as is supposed,* 10000 *parts, the Altitude shall be* 2887, *for so much, very near, is the half of the Tangent. And having found the Altitude the Sublimity is to be known in this manner. Forasmuch as it hath been demonstrated that the half of the Amplitude of a Semiparabola is the Mean-proportional betwixt the Altitude and Sublimity, and the Altitude being already found, and the half of the Amplitude being alwaies the same, to wit,* 5000 *parts, if we shall divide the Square thereof by the Altitude found, the desired Sublimity shall come forth. As in the Example: The Altitude found was* 2887; *The Square of the* 5000 *parts is* 25000000; *which being divided by* 2887, *giveth* 8659, *very near, for the Sublimity sought.*

SALV. Now here we see, in the first place, that the Conjecture is very true which was mentioned afore, that in different Elevations the farther one goeth from the middlemost, whether it be in the Higher, or in the Lower, so much greater *Impetus* and Violence is required to carry the Project to the same Distance. For the *Impetus* lying in the mixture of the two Motions, Equable, Horizontal, and Perpendicular Naturally-Accelerate, of which *Impetus* the Aggregate of the Altitude and Sublimity is the Measure, we do see in the propounded Table that that same Aggregate is least in the Elevation of *gr.* 45, in which the Altitude and Sublimity are equal, *scilicet* each 5000, and their Aggregate 10000. But if we should look on any greater Elevation, as, for example, of *gr.* 50, we should find the Altitude to be 5959, and the Sublimity 4196, which added together make 10155. And so much also we should find the *Impetus* of *gr.* 40 to be, this and that Elevation being equally remote from the middlemost. Where we are to note, in the second place, that it is true, That equal *Impetus's* are sought by two, and two in the Elevations equidistant from the middlemost, with this pretty variation over and above that the Altitudes and the Sublimities of the * superiour Elevations answer alternally to the Sublimities and Altitudes of the Inferiour: so that whereas in the example

* i.e. Those above 45 deg.

example proposed, in the Elevation of *gr.* 50. the Altitude is 5959 and the Sublimity 4196, in the Elevation of *gr.* 40. it falls out on the contrary that the Altitude is 4196, and the Sublimity 5959: And the same happens in all others without any difference; save only that for the avoyding of tediousness in Calculations we have kept no account of some fractions, which in so great sums are of no value, but may without any prejudice be omitted.

SAGR. I am observing that of the two *Impetus's* Horizontal and Perpendicular in Projections, the more Sublime they are, they need so much the less of the Horizontal, and the more of the Perpendicular. Moreover in those of small Elevation, great must be the Force of the Horizontal *Impetus*, which is to carry the Project in a little Altitude. But although I comprehend very well that in the Total Elevation of *gr.* 90, all the force in the world sufficeth not to drive the Project one single Inch from the Perpendicular, but that it must of necessity fall in the same place whence it was expelled; yet dare I not with the like certainty affirm that likewise in the nullity of Elevation, that is in the Horizontal Line, the Project cannot by any Force less than infinite, be driven to any distance: So, as that, for example, a Culverin it self should not be able to carry a Ball of Iron Horizontally, or, as they say, at Point blank, that is at no point, which is when it hath no Elevation. I say, in this case I stand in some doubt; and that I do not resolutely deny the thing, the reason depends on another Accident which seems no less strange, and yet I have a very necessary Demonstration for it. And the Accident is this, the Impossibility of distending a Rope, so, as that it may be stretched right out, and parallel to the Horizon, but that it alwaies swayes and bendeth, nor is there any Force that can stretch it otherwise.

SALV. So then, *Sagredus*, your wonder ceaseth in this case of the Rope because you have the Demonstration of it. But if we shall well consider the matter, it may be we shall find some correspondence between the Accident of the Project and this of the Rope. The Curvity of the Line of the Horizontal Projection seemeth to be derived from two Forces, of which one, (which is that of the Projicient) driveth it Horizontally, and the other, (which is the Gravity of the Project) draweth it downwards Perpendicularly. Now so in the stretching of the Rope, there are the Forces of those that pull it Horizontally, and there is also the weight of the Rope it self, which naturally inclineth it downwards. These two effects are very much alike in the generation of them. And if you allow the weight of the Rope so much strength and power as to be able to oppose and overcome any whatever Immense Force, that would distend it right out, why will you deny the like to the weight of the Bullet? But besides, I shall tell you, and at once procure your

wonder, and delight, that the Rope thus tentered, and ſtretcht little or much, doth ſhape it ſelf into Lines that come very near to Parabolical, and the reſemblance is ſo great, that if you draw a Parabolical Line upon a plain Superficies that is erect unto the Horizon, and holding it reverſed, that is with the Vertex downwards and with the Baſe Parallel to the Horizon, you cauſe a Chain to be held pendent, and ſuſtained at the extreams of the Baſe of the Deſcribed Parabola, you ſhall ſee the ſaid Chain, as you ſlaken it more or leſs, to incurvate and apply it ſelf to the ſame Parabola, and this ſame Application ſhall be ſo much the more exact, when the deſcribed Parabola is leſs curved, that is more diſtended: So that in Parabola's deſcribed with Elevations under *gr.* 45, the Chain anſwereth the Parabola almoſt to an hair.

SAGR. It ſeems then that with ſuch a Chain wrought into ſmall Links one might in an inſtant trace out many Parabolick Lines upon a plain Superficies.

SALV. One might, and that alſo with no ſmall commodity, as I ſhall tell you anon.

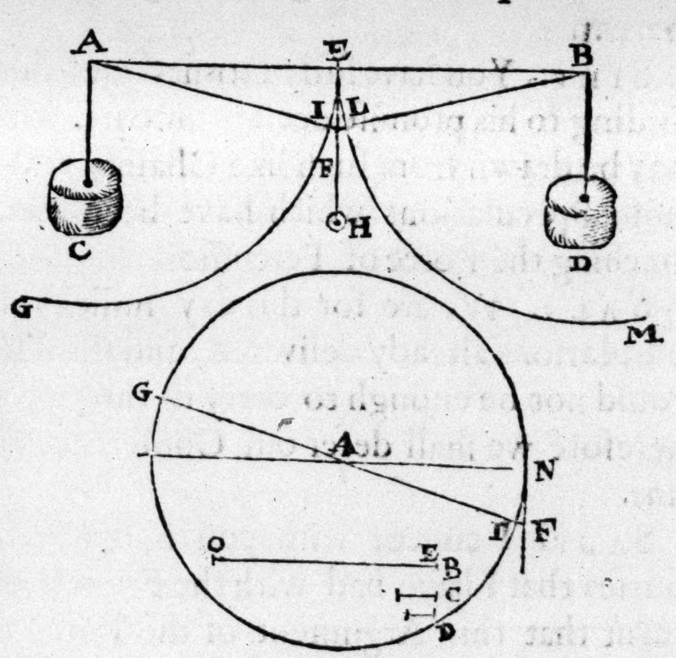
SIMP. But before you paſs any farther, I alſo would gladly be aſcertained at leaſt in that Propoſition of which you ſay there is a very neceſſary Demonſtration, I mean that of the Impoſſibility of diſtending a Rope, by any whatever immenſe Force, right out and equidiſtant from the Horizon.

SAGR. I will ſee if I remember the Demonſtration, for underſtanding of which it is neceſſary, *Simplicius*, that you ſuppoſe for true, that which in all Mechanick Inſtruments is confirmed, not only by Experience, but alſo by Demonſtration: and this it is, That the Velocity of the Mover, though its Force be very ſmall, may overcome the Reſiſtance, though very great, of a Reſiſter, which muſt be moved ſlowly when ever the Velocity of the Mover hath greater proportion to the Tardity of the Reſiſter, than the Reſiſtance of that which is to be moved hath to the Force of the Mover.

SIMP. This I know very well, and it is demonſtrated by *Ariſtotle* in his Mechanical Queſtions, and is manifeſtly ſeen in the Leaver and in the Stiliard, in which the Roman which weigheth not above 4 pounds, will lift up a weight of 400 in caſe the diſtance of the ſaid Roman from the Center on which the Beam turneth be more than an hundred times greater than the diſtance of that point at which the great weight hangeth from the ſame Center: and this cometh to paſs becauſe in the deſcent which the Roman maketh paſſeth a Space above an hundred times greater than the Space which the great weight mounteth in the ſame Time: Which is all one as to ſay, that the little Roman moveth with a Velocity above an hundred times greater than the Velocity of the great Weight.

SAGR.

SAGR. You argue very well, and make no ſcruple at all of granting, that be the Force of the Mover never ſo ſmall it ſhall ſuperate any what-ever great Reſiſtance at all times when that ſhall more exceed in Velocity than this doth in Force and Gravity. Now come we to the caſe of the Rope. And drawing a ſmall Scheme be pleaſed to underſtand for once that this Line A B, reſting upon the two fixed and ſtanding points A and B, to have hanging at its ends, as you ſee, two immenſe Weights C and D, which drawing it with great Force make it to ſtand directly diſtended, it being a ſimple Line without any gravity. And here I proceed, and tell you, that if at the midſt of that which is the point E, you ſhould hang any never ſo little a Weight, as is this H, the Line A B would yield, and inclining towards the point F, and by conſequence lengthening, will conſtrain the two great Weights C and D to aſcend upwards: which I demonſtrate to you in this manner: About the two points A and B as Centers I deſcribe two Quadrants E F G, and E L M, and in regard that the two Semidiameters A I and B L are equal to the two Semidiameters A E and E B, the exceſſes F I and F L ſhall be the quantity of the prolongations of the parts A F and F B, above A E and E B; and of conſequence ſhall determine the Aſcents of the Weights C and D, in caſe that the Weight H had had a power to deſcend to F: which might then be in caſe the Line E F, which is the quantity of the Deſcent of the ſaid Weight H, had greater proportion to the Line F I which determineth the Aſcent of the two Weights C & D, than the ponderoſity of both thoſe Weights hath to the ponderoſity of the Weight H.



But this will neceſſarily happen, be the ponderoſity of the Weights C and D never ſo great, and that of H never ſo ſmall; for the exceſs of the Weights C and D above the Weight H is not ſo great, but that the exceſs of the Tangent E F above the part of the Secant F I may bear a greater proportion. Which we will prove thus: Let there be a Circle whoſe Diameter is G A I; and look what proportion the ponderoſity of the Weights C and D have to the ponderoſity of H, let the Line B O have the ſame proportion to another, which let be C, than which let D be leſſer: So that B O ſhall

ſhall have greater proportion to D, than to C. Unto O B and D take a third proportional B E; and as O E is to E B, ſo let the Diameter G I (prolonging it) be to I F: and from the Term F draw the Tangent F N. And becauſe it hath been preſuppoſed, that as O E is to E B, ſo is G I to I F: therefore, by Compoſition, as O B is to B E, ſo is G F to F I: But betwixt O B and B E the Mean-proportional is D; and betwixt G F and F I the Mean-proportional is N F: Therefore N F hath the ſame proportion to F I that O B hath to D: which proportion is greater than that of the Weights C and D to the Weight H. Therefore, the Deſcent or Velocity of the Weight H having greater proportion to the Aſcent or Velocity of the Weights C and D, than the ponderoſity of the ſaid Weights C and D hath to the ponderoſity of the Weight H: It is manifeſt, that the Weight H ſhall deſcend, that is, that the Line A B ſhall depart from Horizontal Rectitude. And that which befalleth the right Line A B deprived of Gravity in caſe any ſmall Weight H cometh to be hanged at the ſame in E, happens alſo to the ſaid Rope A B, ſuppoſed to be of ponderous Matter, without the addition of any other Grave Body; for that the Weight of the Matter it ſelf compounding the ſaid Rope A B is ſuſpended thereat.

SIMP. You have fully ſatisfied me; therefore *Salviatus* may according to his promiſe declare unto us, what the Commodity is that may be drawn from ſuch like Chains, and after that relate unto us thoſe Speculations which have been made by our *Accademian* touching the Force of Percuſſion.

SALV. We are for this day ſufficiently employed in the Contemplations already delivered, and the Time, which is pretty late, would not be enough to carry us through the matters you mention; therefore we ſhall defer our Conference till ſome more convenient time.

SAGR. I concur with you in opinion, for that by ſundry diſcourſes that I have had with the Friends of our *Academick* I have learnt that this Argument of the Force of Percuſſion is very obſcure, nor hath hitherto any one that hath treated thereof penetrated its intricacies, full of darkneſs, and altogether remote from mans firſt imaginations: and amongſt the Concluſions that I have heard of, one runs in my mind that is very extravagant and odde, namely, That the Force of Percuſſion is Interminate, if not Infinite. We will therefore attend the leaſure of *Salviatus*. But for the preſent, tell me what things are thoſe which are written at the end of the Treatiſe of Projects?

SALV. Theſe are certain Propoſitions touching the Center of Gravity of Solids, which our *Academick* found out in his youth, conceiving that what * *Frederico Comandino* had writ touching the ſame

* *Fredericus Comandinus.*

same was not altogether without Imperfection. He therefore thought that with these Propositions, which here you see written; he might supply that which is wanting in the Book of *Comandine*; and he applyed himself to the same at the Instance of the most Illustrious Lord Marquess *Guid' Ubaldo dal Monte*, the most excellent Mathematician of his Time, as his several Printed Works do speak him; and gave a Copy thereof to that Noble Lord with thoughts to have pursued the same Argument in other Solids not mentioned by *Comandine*: But he chanced after some Time to meet with the * Book of *Signore Luca Valerio*, a most famous Geometrician, and saw that he resolveth all these matters without omission of any thing, he proceeded no farther, although his Agressions were by methods very different from these of *Signore Valerio*. * *De.*

SAGR. It would be a favour, therefore, if, for this time, which interposeth between this and our next Meeting, you would please to leave the Book in my hands: for I shall all the while be reading and studying the Propositions that are consequently therein writ.

SALV. I shall very willingly obey your Command; and hope that you will take pleasure in these Propositions.

APPEN-

AN

APPENDIX,

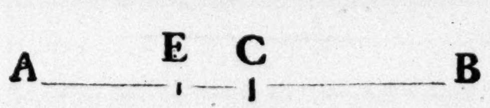
In which is contained certain THEOREMS and their DEMONSTRATIONS: Formerly written by the same Author, touching the *CENTER* of *GRAVITY*, of SOLIDS.

POSTVLATVM.

WE presuppose equall Weights to be alike disposed in severall Ballances, if the Center of Gravity of some of those Compounds shall divide the Ballance according to some proportion, and the Ballance shall also divide their Center of Gravity according to the same proportion.

LEMMA.

Let the line AB be cut in two equall parts in C, whose half AC let be divided in E, so that as BE is to EA, so may AE be to EC. I say that BE is double to EA. For as BE is to EA, so is EA to EC: therefore by Composition and by Permutation of Proportion, as BA is to AC, so is AE to EC: But as AE is to EC, that is, BA to AC, so is BE to EA: Wherefore BE is double to EA.



This supposed, we will Demonstrate, That,

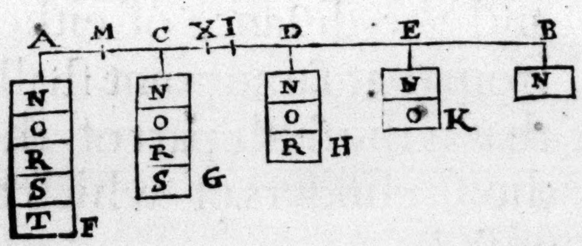
PROP.

PROPOSITION.

If certain Magnitudes at any Rate equally exceeding one another, and whose excess is equal to the least of them, be so disposed in the Balance, as that they hang at equal distances, to divide the Center of Gravity of the whole Balance so, that the part towards the lesser Magnitudes be double to the remainder.

*IN the * Ballance* AB, *therefore, let there be suspended at equal distances any number of Magnitudes, as hath been said,* F, G, H, K, N; *of which let the least be* N, *and let the points of the Suspensions be* A, C, D, E, B, *and let the Center of Gravity of all the Magnitudes so disposed be* X. *It is to be proved that the part of the Ballance* BX *towards the lesser Magnitudes is double to the remaining part* XA. * C

Let the Ballance be divided in two equal parts in D, *for it must either fall in some point of the Suspensions, or else in the middle point between two of the points of the Suspensions: and let the remaining distances of the Suspensions which fall between* A *and* D, *be all divided into halves by the Points* M *and* I; *and let all the Magnitudes be divided into parts equal to* N: *Now the parts of* F *shall be so many in number, as those Magnitudes be which are suspended at the Ballance, and the parts of* G *one fewer, and so of the rest.*



Let the parts of F *therefore be* N, O, R, S, T, *and let those of* G *be* N, O, R, S, *those of* H *also* N, O, R, *then let those of* K *be* N, O: *and all the Magnitudes in which are* N *shall be equal to* F; *and all the Magnitudes in which are* O *shall be equal to* G; *and all the Magnitudes in which are* R *shall be equal to* H; *and those in which* S *shall be equal to* K; *and the Magnitude* T *is equal to* N. *Because therefore all the Magnitudes in which are* N *are equal to one another, they shall equiponderate in the point* D, *which divideth the Ballance into two equal parts; and for the same cause all the Magnitudes in which are* O *do equiponderate in* I; *and those in which are* R *in* C; *and in which are* S *in* M *do equiponderate; and* T *is suspended in* A. *Therefore in the Ballance* AD *at the equal distances* D, I, C, M, A, *there are Magnitudes suspended exceeding one another equally, and whose excess is equal to the least: and the greatest, which is compounded of all the* NN *hangeth at* D, *the least*

least which is T *hangeth at* A; *and the rest are ordinately disposed. And again there is another Ballance* AB *in which other Magnitudes equal in number and Magnitude to the former are disposed in the same order. Wherefore the Ballances* AB *and* AD *are divided by the Center of all the Magnitudes according to the same proportion: But the Center of Gravity of the aforesaid Magnitudes is* X: *Wherefore* X *divideth the Ballances* BA *and* AD *according to the same proportion; so that as* BX *is to* XA, *so is* XA *to* XD: *Wherefore* BX *is double to* XA, *by the Lemma aforegoing: Which was to be proved.*

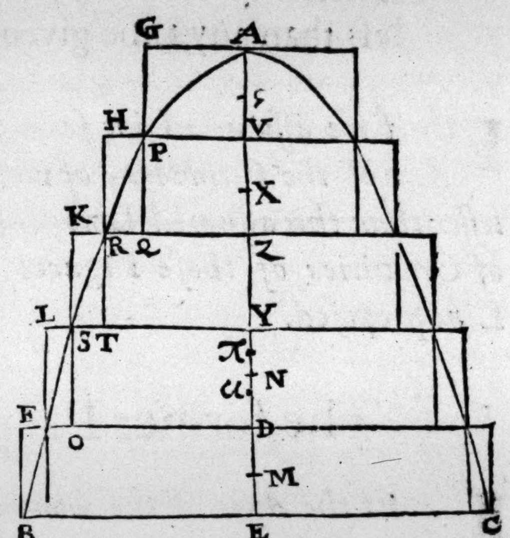
PROPOSITION.

If in a Parabolical Conoid Figure be described, and another circumscribed by Cylinders of equal Altitude; and the Axis of the said Conoid be divided in such proportion that the part towards the Vertex be double to that towards the Base; the Center of Gravity of the inscribed Figure of the Base portion shall be nearest to the said point of division; and the Center of Gravity of the circumscribed from the Base of the Conoid shall be more remote: and the distance of either of those Centers from that same point shall be equal to the Line that is the sixth part of the Altitude of one of the Cylinders of which the Figures are composed.

T*Ake therefore a Parabolical Conoid, and the Figures that have been mentioned: let one of them be inscribed, the other circumscribed; and let the Axis of the Conoid, which let be* AE, *be divided in* N, *in such proportion as that* AN *be double to* NE. *It is to be proved that the Center of Gravity of the inscribed Figure is in the Line* NE, *but the Center of the circumscribed in the Line* AN. *Let the Plane of the Figures so disposed be cut through the Axis, and let the Section be that of the Parabola* BAC: *and let the Section of the cutting Plane, and of the Base of the Conoid be the Line* BC; *and let the Sections of the Cylinders be the Rectangular Figures; as appeareth in the description. First, therefore, the Cylinder of the inscribed whose Axis is* DE, *hath the same proportion to the Cylinder whose Axis is* DY, *as the Quadrate* ID *hath to the Quadrate* SY; *that is, as* DA *hath to* AY: *and the Cylinder whose Axis is* DY *is* potentia

to

to the Cylinder YZ *as* SY *to* RZ; *that is, as* YA *to* AZ: *and, by the same reason, the Cylinder whose Axis is* ZY *is to that whose Axis is* ZV, *as* ZA *is to* AV. *The said Cylinders, therefore, are to one another as the Lines* DA, AY; ZA, AV: *But these are equally exceeding to one another, and the excess is equal to the least, so that* AZ *is double to* AV; *and* AY *is triple the same; and* DA *Quadruple. Those Cylinders, therefore, are certain Magnitudes in order equally exceeding one another, whose excess is equal to the least of them, and is the Line* XM, *in which they are suspended at equal distances (for that each of the Cylinders hath its Center of Gravity in the midst of the Axis.) Wherefore, by what hath been above demonstrated, the Center of Gravity of the Magnitude compounded of them all divideth the Line* XM *so, that the part towards* X *is double to the rest. Divide it, therefore, and, let* X α *be double* α M: *therefore is* α *the Center of Gravity of the inscribed Figure. Divide* AV *in two equal parts in* ε: ε X *shall be double to* ME: *But* X α *is double to* α M: *Wherefore* ε E *shall be triple* E α. *But* α E *is triple* EN: *It is manifest, therefore, that* EN *is greater than* EX; *and for that cause* α, *which is the Center of Gravity of the inscribed Figure, cometh nearer to the Base of the Conoid than* N. *And because that as* AE *is to* EN, *so is the part taken away* ε E *to the part taken away* E α: *and the remaining part shall be to the remaining part, that is,* A ε *to* N α, *as* AE *to* EN. *Therefore* α N *is the third part of* A ε, *and the sixt part of* AV. *And in the same manner the Cylinders of the circumscribed Figure may be demonstrated to be equally exceeding one another, and the excess to me equal to the least; and that they have their Centers of Gravity at equal distances in the Line* ε M. *If therefore* ε M *be divided in* π, *so as that* ε π *be double to the remaining part* π M; π *shall be the Center of Gravity of the whole circumscribed Magnitude. And since* ε π *is double to* π M; *and* A ε *less than double* EM: *(for that they are equal:) the whole* AE *shall be less than triple* E π: *Wherefore* E π *shall be greater than* EN. *And, since* ε M *is triple to* M π, *and* ME *with twice* ε A *is likewise triple to* ME: *the whole* AE *with* A ε *shall be triple to* E π: *But* AE *is triple to* EN: *Wherefore the remaining part* A ε *shall be triple to the remaining part* π N. *Therefore* N π *is the sixth part of* AV. *And these are the things that were to be demonstrated.*



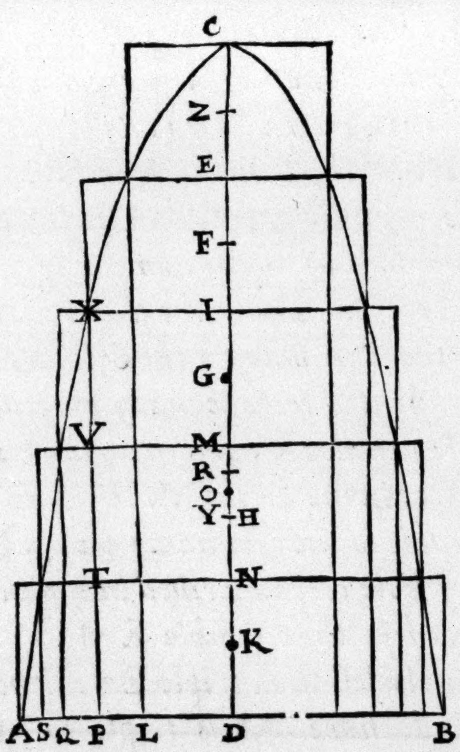
COROLLARY.

Hence it is manifeſt, that a Conoid may be inſcribed in a Parabolical Figure, and another circumſcribed, ſo, as that the Centers of their Gravities may be diſtant from the point N leſs than any Line given.

For if we aſſume a Line ſexcuple of the propoſed Line, and make the Axis of the Cylinders, of which the Figures are compounded given leſſer than this aſſumed Line, there ſhall fall Lines between the Centers of Gravities of theſe Figures and the mark N that are leſs than the Line propoſed.

The former Propoſition another way.

Let the Axis of the Conoid (which let be CD*) be divided in* O, *ſo, as that* C O *be double to* O D. *It is to be proved that the Center of Gravity of the inſcribed Figure is in the Line* O D; *and the Center of the circumſcribed in* C O. *Let the Plane of the Figures be cut through the Axis and* C, *as hath been ſaid. Becauſe therefore the Cylinders* S N, T M, V I, X E *are to one another as the Squares of the Lines* S D, T N, V M, X I; *and theſe are to one another as the Lines* N C, C M, C I, C E: *but theſe do exceed one another equally; and the exceſs is equal to the leaſt, to wit,* C E: *And the Cylinder* T M *is equal to the Cylinder* Q N; *and the Cylinder* V I *equal to* P N; *and* X E *is equal to* L N: *Therefore the Cylinders* S N, Q N, P N, *and* L N *do equally exceed one another, and the exceſs is equal to the leaſt of them, namely, to the Cylinder* L N. *But the exceſs of the Cylinder* S N, *above the Cylinder* Q N *is a Ring whoſe height is* Q T; *that is,* N D; *and its breadth* S Q. *And the exceſs of the Cylinder* Q N *above* P N, *is a Ring, whoſe breadth is* Q P. *And the exceſs of the Cylinder* P N *above* L N *is a Ring, whoſe breadth is* P L. *Wherefore the ſaid Rings* S Q, Q P, P L, *are equal to another, and to the Cylinder* L N. *Therefore the Ring* S T *equalleth the Cylinder* X E: *the Ring* Q V, *which is double to* S T, *equalleth the Cylinder* V I; *which likewiſe is double to the*



Cylinder

Cylinder XE: and for the same cause the Ring PX is equal to the Cylinder TM; and the Cylinder LE shall be equal to the Cylinder SN. In the Beam or Ballance, therefore, KF connecting the middle points of the Right-lines EI and DN, and cut into equal parts in the points H and G, are certain Magnitudes suspended, to wit the Cylinders SN, TM, VI, XE; and the Center of Gravity of the first Cylinder is K; and of the second H; of the third G; of the fourth F. And we have another Ballance MK, which is the half of the said FK, and a like number of points distributed into equal parts, to wit, MH, HN, NK, and on it other Magnitudes, equal in number and bigness to those which are on the Beam FK, and having the Centers of Gravity in the points M, H, N, and K, and disposed in the same order. For the Cylinder LE hath its Center of Gravity in M; and is equal to the Cylinder SN that hath its Center in K: And the Ring PX hath the Center H; and is equal to the Cylinder TM, whose Center is H: And the Ring QV having the Center N is equal to the VI whose Center is G: And lastly, the Ring ST having the Center K, is equal to the Cylinder XE whose Center is F. Therefore the Center of Gravity of the said Magnitudes divideth the Beam in the same proportion: But the Center of them is one, and therefore some point common to both the Beams or Ballance, which let be Y. Therefore FY and YK shall be as KY and YM. FY therefore is double to YK: and CE being divided into two equal parts in Z, ZF, shall be double to KD: and for that cause ZD triple to DY: But to the Right Line DO CD is triple: Therefore the Right Line DO is greater than DY: And for the like cause Y the Center of the inscribed Figure approacheth nearer the Base than the point O. And because as CD is to DO, so is the part taken away ZD to the part taken away DY; the remaining part CZ shall be to the remaining part YO, as CD is to DO; that is YO shall be the third part of CZ; that is, the sixth part of CE. Again we will, by the same reason, demonstrate the Cylinders of the circumscribed Figure to exceed one another equally, and that the excess is equal to the least, and that their Centers of Gravity are constituted in equal distances upon the Beam KZ: and likewise that the Rings equal to those same Cylinders are in like manner disposed on another Beam KG, the half of the said KZ, and that therefore the Center of Gravity of the circumscribed Figure, which let be R, so divideth the Beam, as that ZR is to RK, as KR is to RG. Therefore ZR shall be double to RK: But CZ is equal to the Right Line KD, and not double to it. The whole CD shall be lesser than triple to DR: Wherefore the Right Line DR is greater than DO; that is to say, the Center of the circumscribed Figure recedeth from the Base more than the point O. And because ZK is triple to KR; and KD with twice ZC is triple to KD; the whole CD with CZ shall be triple to DR: But CD is triple to DO: Wherefore the remaining part CZ shall be triple to the remaining part RO; that is, OR

is

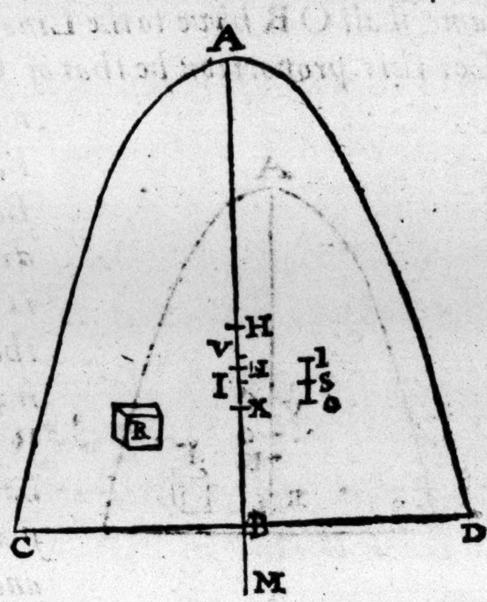
is the sixth part of EC: *Which was the Proposition.*
This being pre-demonstrated, we will prove that

PROPOSITION.

The Center of Gravity of the Parabolick Conoid doth so divide the Axis, as that the part towards the Vertex is double to the remaining part towards the Base.

Let there be a Parabolick Conoid whose Axis let be AB *divided in* N *so as that* AN *be double to* NB. *It is to be proved that the Center of Gravity of the Conoid is the point* N. *For if it be not* N, *it shall be either above or below it. First let it be below; and let it be* X: *And set off upon some place by it self the Line* LO *equal to* NX; *and let* LO *be divided at pleasure in* S: *and look what proportion* BX *and* OS *both together have to* OS, *and the same shall the Conoid have to the Solid* R. *And in the Conoid let Figures be described by Cylinders having equal Altitudes, so, as that that which lyeth between the Center of Gravity and the point* N *be less than* LS: *and let the excess of the Conoid above it be less than the Solid* R: *and that this may be done is clear. Take therefore the inscribed, whose Center of Gravity let be* I: *now* IX *shall be greater than* SO: *And because that as* XB *with* SO *is to* SO, *so is the Conoid to the Solid* R: (*and* R *is greater than the excess by which the Conoid exceeds the inscribed Figure:*) *the proportion of the Conoid to the said excess shall be greater than both* BX *and* OS *unto* SO: *And, by Division, the inscribed Figure shall have greater proportion to the said excess than* BX *to* SO: *But* BX *hath to* XI *a proportion yet less than to* SO: *Therefore the inscribed Figure shall have much greater proportion to the rest of the proportions than* BX *to* XI: *Therefore what proportion the inscribed Figure hath to the rest of the portions, the same shall a certain other Line have to* XI: *which shall necessarily be greater than* BX: *Let it, therefore, be* MX. *We have therefore the Center of Gravity of the Conoid* X: *But the Center of Gravity of the Figure inscribed in it is* I: *of the rest of the portions by which the Conoid exceeds the inscribed Figure the Center of Gravity shall be in the Line* XM, *and in it that point in which it shall be so terminated, that look what proportion the inscribed Figure hath to the excess by which the Conoid exceeds it, the same it shall have to* XI: *But it hath been proved, that this proportion is that which* MX *hath to* XI: *Therefore* M *shall be the Center of Gravity of those proportions by which the Conoid exceeds the inscribed Figure: Which certainly cannot be. For if along by* M *a Plane be drawn equidistant to the Base of the Conoid, all those proportions shall be towards one and the*

the same part, and not by it divided. Therefore the Center of Gravity of the said Conoid is not below the point N: *Neither is it above. For, if it may, let it be* H: *and again, as before, set the Line* LO *by it self equal to the said* HN, *and divided at pleasure in* S: *and the same proportion that* BN *and* SO *both together have to* SL, *let the Conoid have to* R: *and about the Conoid let a Figure be circumscribed consisting of Cylinders, as hath been said: by which let it be exceeded a less quantity than that of the Solid* R: *and let the Line betwixt the Center of Gravity of the circumscribed Figure and the point* N *be lesser than* SO: *the remainder* VH *shall be greater than* SL. *And because that as both* BN *and* OS *is to* SL, *so is the Conoid to* R: (*and* R *is greater than the excess by which the circumscribed Figure exceeds the Conoid:*) *Therefore* BN *and* SO *hath less proportion to* SL *than the Conoid to the said excess. And* BV *is lesser than both* BN *and* SO; *and* VH *is greater than* SL: *much greater proportion, therefore, hath the Conoid to the said proportions, than* BV *hath to* VH. *Therefore whatever proportion the Conoid hath to the said proportions, the same shall a Line greater than* BV *have to* VH. *Let the same be* MV: *And because the Center of Gravity of the circumscribed Figure is* V, *and the Center of the Conoid is* H: *and since that as the Conoid to the rest of the proportions, so is* MV *to* VH, M *shall be the Center of Gravity of the remaining proportions: which likewise is impossible: Therefore the Center of Gravity of the Conoid is not above the point* N: *But it hath been demonstrated that neither is it beneath: It remains, therefore, that it necessarily be in the point* N *it self. And the same might be demonstrated of Conoidal Plane cut upon an Axis not erect. The same in other terms, as appears by what followeth:*

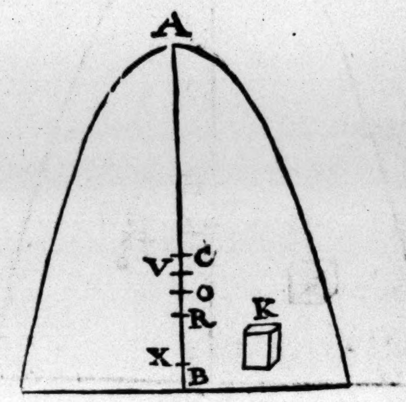


PROPOSITION.

The Center of Gravity of the Parabolick Conoid falleth betwixt the Center of the circumscribed Figure and the Center of the inscribed.

Let

Let there be a Conoid whose Axis is AB, *and the Center of the circumscribed Figure* C, *and the Center of the inscribed* O. *I say the Center of the Conoid is betwixt the points* C *and* O. *For if not, it shall be either above them, or below them, or in one of them. Let it be below, as in* R. *And because* R *is the Center of Gravity of the whole Conoid; and the Center of Gravity of the inscribed Figure is* O: *Therefore of the remaining proportions by which the Conoid exceeds the inscribed Figure the Center of Gravity shall be in the Line* OR *extended towards* R, *and in that point in which it is so determined, that, what proportion the said proportions have to the inscribed Figure, the same shall* OR *have to the Line falling betwixt* R *and that falling point. Let this proportion be that of* OR *to* RX. *Therefore* X *falleth either without the Conoid or within, or in its Base. That it falleth without, or in its Base it is already manifest to be an absurdity. Let it fall within: and because* XR *is to* RO, *as the inscribed Figure is to the excess by which the Conoid exceeds it; the same proportion that* BR *hath to* RO, *the same let the inscribed Figure have to the Solid* K: *Which necessarily shall be lesser than the said excess. And let another Figure be inscribed which may be exceeded by the Conoid a less quantity than is* K, *whose Center of Gravity falleth betwixt* O *and* C. *Let it be* V. *And, because the first Figure is to* K *as* BR *to* RO, *and the second Figure, whose Center* V *is greater than the first, and exceeded by the Conoid a less quantity than is* K; *what proportion the second Figure hath to the excess by which the Conoid exceeds it, the same shall a Line greater than* BR *have to* RV. *But* R *is the Center of Gravity of the Conoid; and the Center of the second inscribed Figure* V: *The Center therefore of the remaining proportions shall be without the Conoid beneath* B: *Which is impossible. And by the same means we might demonstrate the Center of Gravity of the said Conoid not to be in the Line* CA. *And that it is none of the points betwixt* C *and* O *is manifest. For say, that there other Figures described, greater something than the inscribed Figure whose Center is* O, *and less than that circumscribed Figure whose Center is* C, *the Center of the Conoid would fall without the Center of these Figures: Which but now was concluded to be impossible: It rests therefore that it be betwixt the Center of the circumscribed and inscribed Figure. And if so, it shall necessarily be in that point which divideth the Axis, so as that the part towards the Vertex is double to the remainder; since* N *may circumscribe and inscribe Figures, so, that those Lines which fall between their*

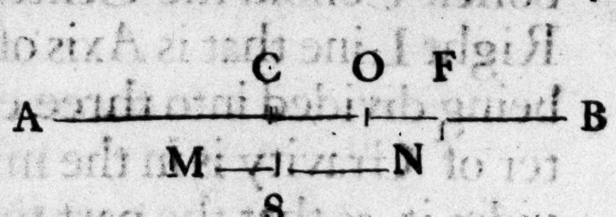


their Centers and the said points, may be lesser than any other Line. To express the same in other terms, we have reduced it to an impossibility, that the Center of the Conoid should not fall betwixt the Centers of the inscribed and circumscribed Figures.

PROPOSITION.

Supposing three proportional Lines, and that what proportion the least hath to the excess by which the greatest exceeds the least, the same should a Line given have to two thirds of the excess by which the greatest exceeds the middlemost: and moreover, that what proportion that compounded of the greatest, and of double the middlemost, hath unto that compounded of the triple of the greatest and middlemost, the same hath another Line given, to the excess by which the greatest exceeds the middle one; both the given Lines taken together shall be a third part of the greatest of the proportional Lines.

L*Et* AB, BC, *and* BF, *be three proportional Lines; and what proportion* BF *hath to* FA, *the same let* MS *have to two thirds of* CA. *And what proportion that compounded of* AB *and the double of* BC *hath to that compounded of the triple of both* AB *and* BC, *the same let another, to wit* SN, *have to* AC. *Because therefore that* AB, BC, *and* CF, *are proportionals,* AC *and* CF *shall, for the same reason, be likewise so. Therefore, as* AB *is to* BC, *so is* AC *to* CF:



and as the triple of AB *is to the triple of* BC, *so is* AC *to* CF: *Therefore, what proportion the triple of* AB *with the triple of* BC *hath to the triple of* CB, *the same shall* AC *have to a Line less than* CF. *Let it be* CO. *Wherefore by Composition and by Conversion of proportion,* OA *shall have to* AC, *the same proportion, as triple* AB *with Sexcuple* BC, *hath to triple* AB *with triple* BC. *But* AC *hath to* SN *the same proportion, that triple* AB *with triple* BC *hath to* AB *with double* BC: *Therefore, ex equali,* OA *to* NS *shall have the same proportion, as triple* AB *with Sexcuple* BC *hath to* AB *with*

double B C *: But triple* A B *with sexcuple* B C, *are triple to* A B *with double* B C. *Therefore* A O *is triple to* S N.

Again, because O C *is to* C A *as triple* C B *is to triple* A B *with triple* C B *: and because as* C A *is to* A F, *so is triple* A B *to triple* B C: Therefore, ex equali, *by perturbed proportion, as* O C *is to* C F, *so shall triple* A B *be to triple* A B *with treble* B C *: And, by Conversion of proportion, as* O F *is to* F C, *so is triple* B C *to triple* A B *with triple* B C *: And as* C F *is to* F B, *so is* A C *to* C B, *and triple* A C *to triple* C B *: Therefore,* ex equali, *by Perturbation of proportion, as* O F *is to* F B, *so is triple* A C *to the triple of both* A B *and* A C *together. And because* F C *and* C A *are in the same proportion as* C B *and* B A; *it shall be that as* F C *is to* C A, *so shall* B C *be to* B A. *And, by Composition, as* F A *is to* A C, *so are both* B A *and* B C *to* B A *: and so the triple to the triple : Therefore as* F A *is to* A C, *so the compound of triple* B A *and triple* B C *is to triple* A B. *Wherefore, as* F A *is to two thirds of* A C, *so is the compound of triple* B A *and triple* B C *to two thirds of triple* B A; *that is, to double* B A *: But as* F A *is to two thirds of* A C, *so is* F B *to* M S *: Therefore, as* F B *is to* M S, *so is the compound of triple* B A *and triple* B C *to double* B A *: But as* O B *is to* F B, *so was Sexcuple* A B *to triple of both* A B *and* B C *: Therefore,* ex equali, O B *shall have to* M S *the same proportion as Sexcuple* A B *hath to double* B A. *Wherefore* M S *shall be the third part of* O B *: And it hath been demonstrated, that* S N *is the third part of* A O *: It is manifest therefore, that* M N *is a third part likewise of* A B *: And this is that which was to be demonstrated.*

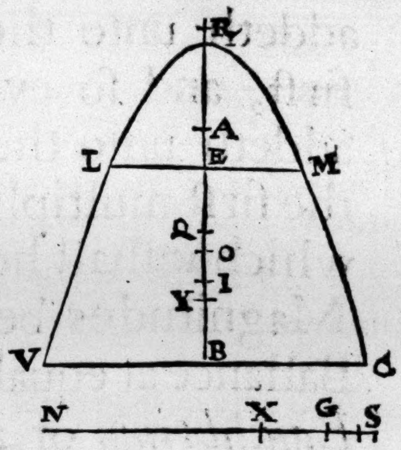
PROPOSITION.

Of any *Frustum* or Segment cut off from a Parabolick Conoid the Center of Gravity is in the Right Line that is Axis of the *Frustum*; which being divided into three equal parts the Center of Gravity is in the middlemost and so divides it, as that the part towards the lesser Base hath to the part towards the greater Base, the same proportion that the greater Base hath to the lesser.

F*Rom the Conoid whose Axis is* R B *let there be cut off the Solid whose Axis is* B E; *and let the cutting Plane be equidistant to the Base : and let it be cut in another Plane along the Axis erect upon the Base, and let it be the Section of the Parabola* V R C *:* R B *shall be the Diameter of the proportion, or the equidistant Diameter*

LM, VC:

LM, VC: *they shall be ordinately applyed. Divide therefore* EB *into three equal parts, of which let the middlemost be* QY: *and divide this so in the point* I *that* QI *may have the same proportion to* IY, *as the Base whose Diameter is* VC *hath to the Base whose Diameter is* LM; *that is, that the Square* VC *hath to Square* LM. *It is to be demonstrated that* I *is the Center of Gravity of the* Frustrum LMC. *Draw the Line* NS, *by the by, equall to* BR: *and let* SX *be equal to* ER: *and unto* NS *and* SX *assume a third proportional* SG: *and as* NG *is to* GS, *so let* BQ *be to* IO. *And it nothing matters whether the point* O *fall above or below* LM. *And because in the Section* VRC *the Lines* LM *and* VC *are ordinately applyed, it shall be that as the Square* VC *is to the Square* LM, *so is the Line* BR *to* RE: *And as the Square* VC *is to the Square* LM, *so is* QI *to* IY: *and as* BR *is to* RE, *so is* NS *to* SX: *Therefore* QI *is to* IY, *as* RS *is to* SX. *Wherefore as* GY *is to* YI, *so shall both* NS *and* SX *be to* SX: *and as* EB *is to* YI, *so shall the compound of triple* NS *and triple* SX *be to* SX: *But as* EB *is to* BY, *so is the compound of triple* NS *and* SX *both together to the compound of* NS *and* SX: *Therefore, as* EB *is to* BI, *so is the compound of triple* NS *and triple* SX *to the compound of* NS *and double* SX. *Therefore* NS, SX, *and* SG *are three proportional Lines: And as* SG *is to* GN, *so is the assumed* OI *to two thirds of* EB; *that is, to* NX: *And as the compound of* NS *and double* SX *is to the compound of triple* NS *and triple* SX, *so is another assumed Line* IB *to* BE; *that is, to* NX. *By what therefore hath been above demonstrated, those Lines taken together are a third part of* NS; *that is, of* RB: *Therefore* RB *is triple to* BO: *Wherefore* O *shall be the Center of Gravity of the Conoid* VRC. *And let it be the Center of Gravity of the* Frustrum LRM *of the Conoid: Therefore the Center of Gravity of* VLMC *is in the Line* OB, *and in that point which so terminates it, that as* VLMC *of the* Frustrum *is to the proportion* LRM, *so is the Line* AO *to that which intervenes betwixt* O *and the said point. And because* RO *is two thirds of* RB; *and* RA *two thirds of* RE; *the remaining part* AO *shall be two thirds of the remaining part* EB. *And because that as the* Frustum VLMC *is to the proportion* LRM, *so is* NG *to* GS: *and as* NG *to* GS, *so is two thirds of* EB *to* OI: *and two thirds of* EB *is equal to the Line* AO: *it shall be that as the* Frustum VLMO *is to the proportion* LRM, *so is* AO *to* OI. *It is manifest therefore that of the* Frustum VLMC *the Center of Gravity is the point* I, *and so divideth the Axis, as that the part towards the lesser Base is to the part towards the greater*

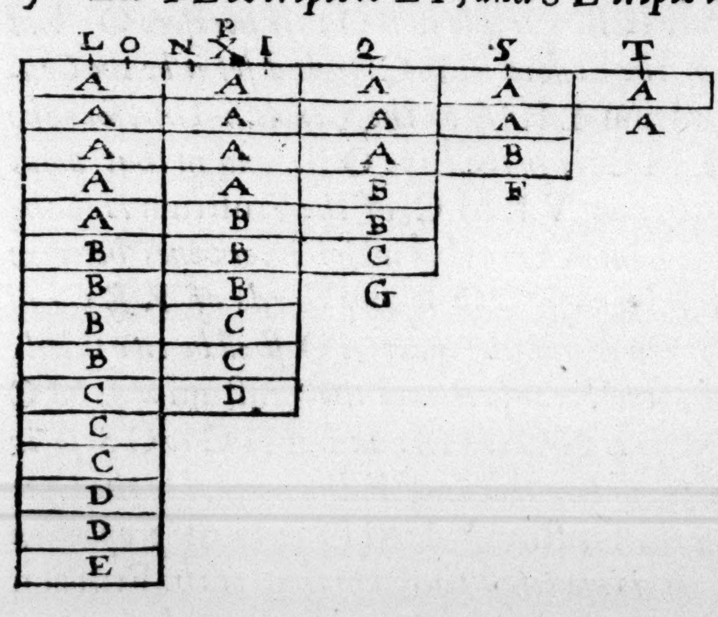


ter, as the double of the greater Base together with the Lesser is to the double of the lesser together with the greater. Which is the Proposition more elegantly expressed.

PROPOSITION.

If any number of Magnitudes so disposed to one another, as that the second addeth unto the first the double of the first, the third addeth unto the second the triple of the first, the fourth addeth unto the third the quadruple of the first, and so every one of the following ones addeth unto the next unto it the magnitude of the first multiplyed according to the number which it shall hold in order; if, I say, these Magnitudes be suspended ordinarily on the Ballance at equal distances; the Center of the *Equilibrium* of all the compounding Magnitudes shall so divide the Beam, as that the part towards the lesser Magnitudes is triple to the remainder.

Let the Beam be L T, *and let such Magnitudes as were spoken of hang upon it; and let them be* A, F, G, H, K; *of which* A *is in the first pla ce suspended at* T. *I say, that the Center of the* Equilibrium *so cuts the Beam* T L *as that the part towards* T *is triple to the rest. Let* T L *be triple to* L I; *and* S L *triple to* L P: *and* Q L *to* L N, *and* L P *to* L O: I P, P N, N O, *and* O L *shall be equal. And in* F *let a Magnitude be placed double to* A; *in* G *another trebble to the same; in* H *another Quadruple; and so of the rest: and let those Magnitudes be taken in which there is* A; *and let the same be done in the Magnitudes* F, G, H, K. *And because in* F *the remaining Magnitude, to wit* B, *is equal to* A; *take it*



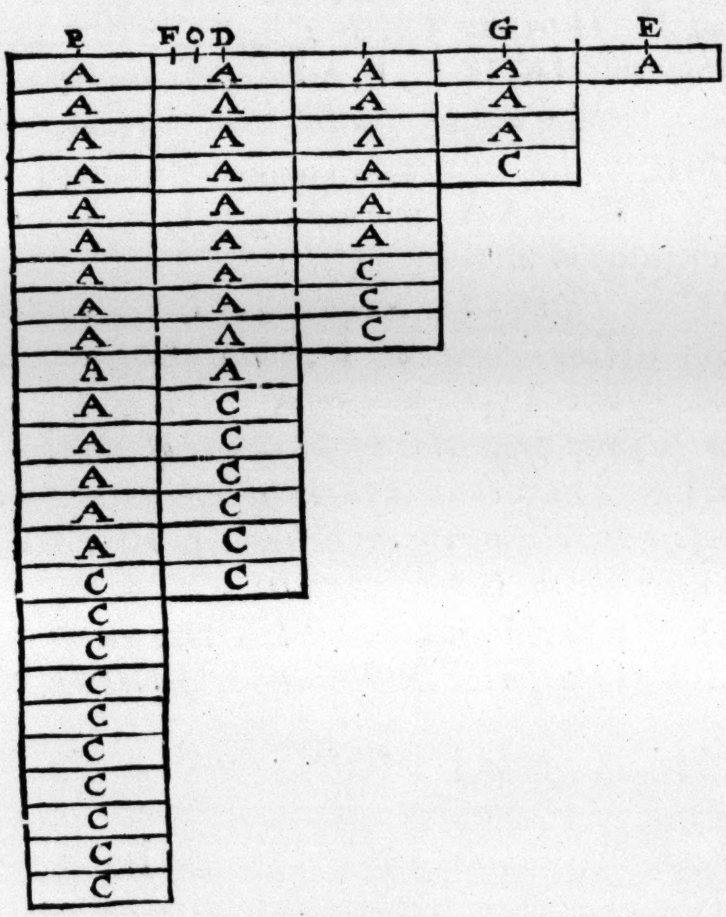
double in G, *triple in* H, *&c. and let those Magnitudes be taken in which there is* B: *and in the same manner let those be taken in which is* C, D, *and* E: *now all those in which there is* A *shall be equal to* K: *and the compound of all the* B B *shall equal* H; *and the compound of* C C *shall equal* G; *and the compound of all the* D D *shall equal* F; *and* E *shall equal* A. *And because* T I *is double to* I L, I *shall be the point of the* Equilibrium *of the Magnitudes composed of all the* A A: *and likewise since* S P *is double to* P L, P *shall be the point of the* Equilibrium *of the compost of* B B: *and for the same cause* N *shall be the point of the* Equilibrium *of the compost of* C C: *and* O *of the compound of* D D: *and* L *that of* E. *Therefore* T L *is a Beam on which at equal distances certain Magnitudes* K, H, G, F, A *do hang. And again* L I *is another Ballance, on which, at distances in like manner equal, do hang such a number of Magnitudes, and in the same order equal to the former. For the compound of all the* A A, *which hang on* I, *is equal to* K *hanging at* L; *and the compost of all* B B, *which is suspended at* P, *is equal to* H *hanging at* P; *and likewise the compound of* C C, *which hangeth at* N *do equal* G; *and the compost of* D, *which hang on* O, *are equal to* F; *and* E, *hanging on* L, *is equal to* A. *Wherefore the Ballances are divided in the same proportion by the Center of the compounds of the Magnitudes. And the Center of the compound of the said Magnitudes is one. Therefore the common point of the Right Line* T L, *and of the Right Line* L I *shall be the Center, which let be* X. *Therefore as* T X *is to* X L, *so shall* L X *be to* X I; *and the whole* T L *to the whole* L I. *But* T L *is triple to* L I: *Wherefore* T X *shall also be triple to* X L.

PROPOSITION.

If any number of Magnitudes be so taken, that the second addeth unto the first the triple of the first, and the third addeth unto the second the quintuple of the first, and the fourth addeth unto the third the septuple of the first, and so the rest, every one encreasing above the next to it, and proceedeth still to a new multiplex of the first Magnitude according to the consequent odd numbers, like as the Squares of Lines equally exceeding one another do proceed, whereof the excess is equal to the least, and if they be suspended on a Ballance at equal Distances, the Center of *Equilibrium* of all the compound Magnitudes so divideth the Beam that

that the part towards the lesser Magnitudes is more than triple the remaining part; and also one may take a distance that is to the same less than triple.

IN the Ballance BE *let there be Magnitudes, such as were spoken off, from which let there be other Magnitudes taken away that were to one another as they were disposed in the precedent, and let it be of the compound of all the* AA*: the rest in which are* C *shall be distributed in the same order, but the greatest deficient. Let* ED *be triple to* DB*; and* GF *triple to* FB. D *shall be the Center of the* Equilibrium *of the compound consisting of all the* AA*; and* F *that of the compound of all the* CC. *Wherefore the Center of the compound of both* AA *and* CC *falleth between* D *and* F. *Let it be* O. *It is therefore manifest that* EO *is more than triple to* OB*; but* GO *less than triple to the same* OB*: Which was to be demonstrated.*

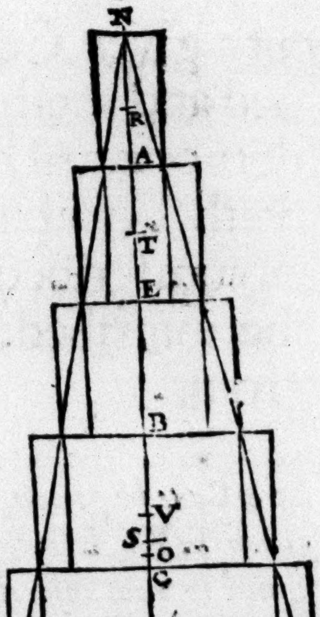


PROP.

PROPOSITION.

If to any Cone or portion of a Cone a Eigure consisting of Cylinders of equal heights be inscribed and another circumscribed; and if its Axis be so divided as that the part which lyeth betwixt the point of division and the Vertex be triple to the rest; the Center of Gravity of the inscribed Figure shall be nearer to the Base of the Cone than that point of division: and the Center of Gravity of the circumscribed shall be nearer to the Vertex than that same point.

Take therefore a Cone, whose Axis is NM. *Let it be divided in* S *so, as that* NS *be triple to the remainder* SM. *I say, that the Center of Gravity of any Figure inscribed, as was said, in a Cone doth consist in the Axis* NM, *and approacheth nearer to the Base of the Cone than the point* S: *and that the Center of Gravity of the Circumscribed is likewise in the Axis* NM, *and nearer to the Vertex than is* S. *Let a Figure therefore be supposed to be inscribed by the Cylinders whose Axis* MC, CB, BE, EA *are equal. First therefore the Cylinder whose Axis is* MC *hath to the Cylinder whose Axis is* CB *the same proportion as its Base hath to the Base of the other (for their Altitudes are equal.) But this proportion is the same with that which the Square* CN *hath to the Square* NB. *And so we might prove, that the Cylinder whose Axis is* CB *hath to the Cylinder whose Axis is* BE *the same proportion, as the Square* BN *hath to the Square* NE: *and the Cylinder whose Axis is* BE *hath to the Cylinder whose Axis is* EA *the same proportion that the Square* EN *hath to the Square* NA. *But the Lines* NC, NB, EN, *and* NA *equally exceed one another, and their excess equalleth the least, that is* NA. *Therefore they are certain Magnitudes, to wit, inscribed Cylinders having consequently to one another the same proportion as the Squares of Lines that equally exceed one another, and the excess*



cess of which is equal to the least: and they are so disposed on the Beam T I *that their several Centers of Gravity consist in it, and that at equal distances. Therefore by the things above demonstrated it appeareth that the Center of Gravity of all so composed Magnitudes do so divide the Balance* T I, *that the part towards* T *is more than triple to the remainder. Let this Center be* O. T O *therefore is more than triple to* O I. *But* T N *is triple to* I M. *Therefore the whole* M O *will be less than a fourth part of the whole* M N, *whose fourth part was supposed to be* M S. *It is manifest, therefore, that the point* O *doth nearer approach the Base of the Cone than* S. *And let the circumscribed Figure be composed of the Cylinders whose Axis* M C, C B, B E, E A *and* A N *are equal to each other, and, like as in those inscribed, let them be to one another as the Squares of the Lines* M N, N C, B N, N E, A N, *which equally exceed one another, and the excess is equal to the least* A N. *Wherefore, by the premises, the Center of Gravity of all the Cylinders so disposed, which let be* V, *doth so divide the Beam* R I, *that the part towards* R, *to wit* R V, *is more than triple to the remaining part* V I: *but* T V *shall be less than triple to the same. But* N T *is triple to all* I M: *Therefore all* V M *is more than the fourth part of all* M N, *whose fourth part was supposed to be* M S. *Therefore the point* V *is nearer to the Vertex than the Point* S. *Which was to be demonstrated.*

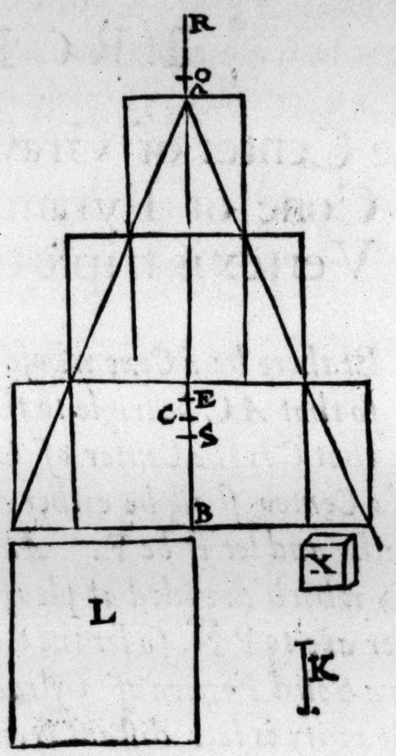
PROPOSITION.

About a given Cone a Figure may be circumscribed and another inscribed consisting of Cylinders of equal height, so, as that the Line which lyeth betwixt the Center of Gravity of the circumscribed, and the Center of Gravity of the inscribed, may be lesser than any Line given.

Let a Cone be given, whose Axis is A B; *and let the Right Line given be* K. *I say; Let there be placed by the Cylinder* L *equal to that inscribed in the Cone, having for its Altitude half of the Axis* A B: *and let* A B *be divided in* C, *so as that* A C *be triple to* C B: *And as* A C *is to* K, *so let the Cylinder* L *be to the Solid* X. *And about the Cone let there be a Figure circumscribed of Cylinders that have equal Altitude, and let another be inscribed, so as that the circumscribed exceed the inscribed a less quantity than the Solid* X. *And let the Center of Gravity of the circumscribed be* E, *which falls above* C: *and let the Center of the inscribed be* S, *falling beneath* C.

I say,

I say now, that the Line ES *is lesser than* K. *For if not, then let* CA *be supposed equal to* EO. *Because therefore* OE *hath to* K *the same proportion that* L *hath to* X; *and the inscribed Figure is not less than the Cylinder* L; *and the excess with which the said Figure is exceeded by the circumscribed is less than the Solid* X: *therefore the inscribed Figure shall have to the said excess greater proportion than* OE *hath to* K: *But the proportion of* OE *to* K *is not less than that which* OE *hath to* ES *with* ES. *Let it not be less than* K. *Therefore the inscribed Figure hath to the excess of the circumscribed Figure above it greater proportion than* OE *hath to* ES. *Therefore as the inscribed is to the said excess, so shall it be to the Line* ES. *Let* ER *be a Line greater than* EO; *and the Center of Gravity of the inscribed Figure is* S; *and the Center of the circumscribed is* E. *It is manifest therefore, that the Center of Gravity of the remaining proportions by which the circumscribed exceedeth the inscribed is in the Line* RE, *and in that point by which it is so terminated, that as the inscribed Figure is to the said proportions, so is the Line included betwixt* E *and that point to the Line* ES. *And this proportion hath* RE *to* ES. *Therefore the Center of Gravity of the remaining proportions with which the circumscribed Figure exceeds the inscribed shall be* R, *which is impossible. For the Plane drawn thorow* R *equidistant to the Base of the Cone doth not cut those proportions. It is therefore false that the Line* ES *is not lesser than* K. *It shall therefore be less. The same also may be done in a manner not unlike this in Pyramides, as we could demonstrate.*



COROLLARY.

Hence it is manifest, that a given Cone may circumscribe one Figure and inscribe another consisting of Cylinders of equal Altitudes so, as that the Lines which are intercepted betwixt their Centers of Gravity and the point which so divides the Axis of the Cone, as that the part towards the Vertex is triple to the rest, are less than any given Line.

F*Or, since it hath been demonstrated, that the said point dividing the Axis, as was said, is alwaies found betwixt the Centers of Gravity*

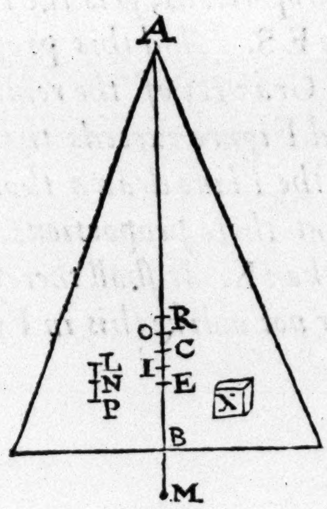
of

of the Circumscribed and inscribed Figures : and that it's possible, that there be a Line in the middle betwixt those Centers that is less than any Line assigned ; it followeth that the same given Line be much less that lyeth betwixt one of the said Centers and the said point that divides the Axis.

PROPOSITION.

The Center of Gravity divideth the Axis of any Cone or Pyramid so, that the part next the Vertex is triple to the remainder.

Let there be a Cone whose Axis is A B. And in C let it be divided, so that A C be triple to the remaining part C B. It is to be proved, that C is the Center of Gravity of the Cone. For if it be not, the Cone's Center shall be either above or below the point C. Let it be first beneath, and let it be E. *And draw the Line* L P, *by it self, equal to* C E ; *which divided at pleasure in* N. *And as both* B E *and* P N *together are to* P N, *so let the Cone be to the Solid* X : *and inscribe in the Cone a Solid Figure of Cylinders that have equal Bases, whose Center of Gravity is less distant from the point* C *than is the Line* L N, *and the excess of the Cone above it less than the Solid* X. *And that this may be done is manifest from what hath been already demonstrated.*



Now let the inscribed Figure be such as was required, whose Center of Gravity let be I. *The Line* I E *therefore shall be greater than* N P *together with* L P. *Let* C E *and* I C *less* L N *be equal : And because both together* B E *and* N P *is to* N P *as the Cone to* X : *and the excess by which the Cone exceeds the inscribed Figure is less than the Solid* X : *Therefore the Cone shall have greater proportion to the said* X *S than both* B E *and* N P *to* N P : *and, by Division, the inscribed Figure shall have greater proportion to the excess by which the Cone exceeds it, than* B E *to* N P : *But* B E *hath less proportion to* E I *than to* N P *with* I E. *Let* N P *be greater. Then the inscribed Figure hath to the excess of the Cone above it much greater proportion than* B E *to* E I. *Therefore as the inscribed Figure is to the said excess, so shall a Line bigger than* B E *be to* E I. *Let that Line be* M E. *Because, therefore,* M E *is to* E I *as the inscribed Figure is to the excess of the Cone above the said Figure, and* D *is the Center of Gravity of the Cone, and* I *the Center of Gravity of the inscribed Figure : Therefore*

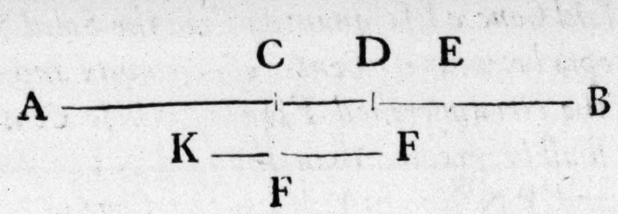
M *shall*

M shall be the Center of Gravity of the remaining proportions by which the Cone exceeds the inscribed Figure. Which is impossible. Therefore the Center of Gravity of the Cone is not below the point C. Nor is it above it. For if it may be, let it be R. And again assume L P cut at pleasure in N: And as both B C and N P together are to N L, so let the Cone be to X. And let a Figure be, in like manner, circumscribed about the Cone, which exceeds the said Cone a less quantity than the Solid X. And let the Line which intercepts betwixt its Center of Gravity and C, be lesser than N P. Now take the circumscribed Figure, whose Center let be O; the remainder O R shall be greater than the said N L. And because, as both together B C and P N is to N L, so is the Cone to X: And the excess by which the circumscribed exceeds the Cone is lesser than X: And B O is lesser than B C and P N together: And O R greater than L N: The Cone therefore shall have much greater proportion to the remaining proportions by which it was exceeded by the circumscribed Figure, than B O to O R. Let it be as M O is to O R. M O shall be greater than B C; and M shall be the Center of Gravity of the proportions by which the Cone is exceeded by the circumscribed Figure. Which is inconvenient. Therefore the Center of Gravity of the Cone is not above the point C. But neither is it below it; as hath been proved. Therefore it shall be C it self. And so in like manner may it be demonstrated in any Pyramid.

PROPOSITION.

If there were four Lines continual proportionals; and as the least of them were to the excess by which the greatest exceeds the least, so a Line taken at pleasure should be to $\frac{3}{4}$ the excess by which the greatest exceeds the second; and as the Line equal to these (*viz.* to the greatest, double of the second, and triple of the third) is to the Line equal to the quadruple of the fourth, the quadruple of the second, and the quadruple of the third, so should another Line taken be to the excess of the greatest above the second: these two Lines taken together shall be a fourth part of the greatest of the proportionals.

FOR let AB, BC, BD, and BE be four proportional Lines. And as BE is to EA, so let FG be to $\frac{3}{4}$ of AC. And as the Line equal to AB and to double BC and to triple BD is to the Line equal to the quadruples of AB, BC, and BD, so let HG be to AC. It is to be proved, that HF is a fourth part of AB. Forasmuch therefore as AB, BC, BD, and BE are proportionals, AC, CD, and DE shall be in the same proportion: And as the quadruple of the said AB, BC, and BD is to AB with the double of BC and triple of BD, so is the quadruple of AC, CD, and DE; that is, the quadruple of AE; to AC with the double of CD, and triple of DE. And so is AC to HG. Therefore as the triple of AE is to AC, with the double of CD and triple of DE, so is $\frac{3}{4}$ of AC to HG. And as the triple of AE is to the triple of EB, so is $\frac{3}{4}$ AC to GF: Therefore, by the Converse of the twenty fourth of the fifth, As triple AE is to AC with double CD and triple DB, so is $\frac{3}{4}$ of AC to HF: And as the quadruple of AE is to AC with the double of CD and triple of DB; that is, to AB with CB and BD, so is AC to HF. And, by Permutation, as the quadruple of AE is to AC, so is AB with CB and BD to HF. And as AC is to AE, so is AB to AB with CB and BD. Therefore, ex æquali, by Perturbed proportion, as quadruple AE is to AE, so is AB to HF. Wherefore it is manifest that HF is the fourth part of AB.

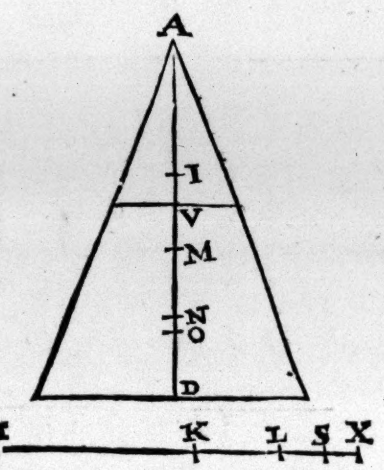


PROPOSITION.

The Center of Gravity of the *Frustum* of any Pyramid or Cone, cut equidistant to the Plane of the Base, is in the Axis, and doth so divide the same, that the part towards the lesser Base is to the remainder, as the triple of the greater Base, with the double of the mean Space betwixt the greater and lesser Base, together with the lesser Base is to the triple of the lesser Base, together with the same double of the mean Space, as also of the greater Base.

From

FRom a Cone or Pyramid whose Axis is AD, and equidistant to the Plane of the Base, let a Frustum be cut whose Axis is VD. And as the triple of the greatest Base with the double of the mean and least is to the triple of the least and double of the mean and greatest, so is VO to OD. It is to be proved that the Center of Gravity of the Frustum is in O. Let VM be the fourth part of VD. Set the Line HX by the by, equal to AD: and let KX be equal to AV: and unto HXK let XL be a third proportional, and XS a fourth. And as HS is to SX, so let MD be to the Line taken from O towards A: which let be ON. And because the greater Base is in proportion to that which is mean betwixt the greater and lesser as DA to AV; that is, as HX, to XK, but the said mean is to the least as KX to XL; the greater, mean, and lesser Bases shall be in the same proportion as HX, XK, and XL. Wherefore, as triple the greater Base, with double the mean and lesser, is to triple the least with double the mean and greatest; that is, as VO is to OD; so is triple HX with double XK and XL to triple XL, with double XK and XH: And by Composition and Converting the proportion, OD shall be to VD, as HX, with double XK and triple XL, to quadruple HX, XK, and XL. There are, therefore, four proportional Lines, HX, XK, XL, and XS: And as XS is to SH, so is the Line taken NO to $\frac{3}{4}$ of DV, to wit, to DM; that is, to $\frac{3}{4}$ of HK: And as HX with double XK and triple XL is to quadruple HX, XK and XL; so is another Line taken OD to DV; that is, to HK. Therefore, by the things demonstrated, DN shall be the fourth part of HX; that is, of AD. Wherefore the point N shall be the Center of Gravity of the Cone or Pyramid whose Axis is AD. Let the Center of Gravity of the Pyramid or Cone whose Axis is AV be I. It is therefore manifest that the Center of Gravity of the Frustum is in the Line IN inclining towards the part N, and in that point of it which with the point N include a Line to which IM hath the same proportion that the Frustum cut hath to the Pyramid or Cone whose Axis is AV. It remaineth therefore to prove that IN hath the same proportion to NO, that the Frustum hath to the Cone whose Axis is AV. But as the Cone whose Axis is DA is to the Cone whose Axis is AV, so is the Cube DA to the Cube DV; that is, the Cube HX to the Cube XK: But this is the same proportion that HX hath to XS. Wherefore, by Division, as HS is to SX, so shall the Frustum whose



Axis

Axis is DV *be to the Cone or Pyramid whose Axis is* V *A. And as* HS *is to* SX, *so also is* MD *to* ON. *Wherefore the* Frustum *is to the Pyramid whose Axis is A*V, *as* MD *to* NO. *And because A*N *is* $\frac{3}{4}$ *of* AD; *and* AI *is* $\frac{1}{4}$ *of* AV; *the remainder* IN *shall be* $\frac{3}{4}$ *of the remainder* VD. *Wherefore* IN *shall be equal to* MD. *And it hath been demonstrated that* MD *is to* NO, *as the* Frustum *to the Cone* AV. *It is manifest, therefore, that* IN *hath likewise the same proportion to* NO: *Wherefore the Proposition is manifest.*

FINIS.

GALILEUS, HIS MECHANICKS:

OF THE BENEFIT DERIVED FROM THE SCIENCE OF MECHANICKS, AND FROM ITS INSTRUMENTS.

I Judged it extreamly necessary, before our descending to the Speculation of Mechanick Instruments, to consider how I might, as it were, set before your eyes in a general Discourse, the many benefits that are derived from the said Instruments: and this I have thought my self the more obliged to do, for that (if I am not mistaken) I have seen the generality of Mechanitians deceive themselves in going about to apply Machines to many operations of their own nature impossible; by the successe whereof they have been disappointed, and others likewise frustrate of the hope which they had conceived upon the promise of those presumptuous undertakers: of which mistakes I think I have found the principall cause to be the belief and constant opinion these

Artificers

Artificers had, and still have, that they are able with a small force to move and raise great weights; (in a certain manner with their Machines cozening nature, whose Instinct, yea most positive constitution it is, that no Resistance can be overcome, but by a Force more potent then it:) which conjecture how false it is, I hope by the ensuing true and necessary Demonstrations to evince.

In the mean time, since I have hinted, that the benefit and help derived from Machines is not, to be able with lesse Force, by help of the Machine to move those weights, which, without it, could not be moved by the same Force: it would not be besides the purpose to declare what the Commodities be which are derived to us from such like faculties, for if no profit were to be hoped for, all endeavours employed in the acquist thereof will be but lost labour.

Proceeding therefore according to the nature of these Studies, let us first propose four things to be considered. First, the weight to be transferred from place to place; and secondly, the Force and Power which should move it; thirdly, the Distance between the one and the other Term of the Motion; Fourthly, the Time in which that mutation is to be made: which Time becometh the same thing with the Dexterity, and Velocity of the Motion; we determining that Motion to be more swift then another, which in lesse Time passeth an equal Distance.

Now, any determinate Resistance and limited Force whatsoever being assigned, and any Distance given, there is no doubt to be made, but that the given Force may carry the given Weight to the determinate Distance; for, although the Force were extream small, yet, by dividing the Weight into many small parts, none of which remain superiour to the Force, and by transferring them one by one, it shall at last have carried the whole Weight to the assigned Term: and yet one cannot at the end of the Work with Reason say, that that great Weight hath been moved, and transported by a Force lesse then it self, howbeit indeed it was done by a Force, that many times reiterated that Motion, and that Space, which shall have been measured but only once by the whole Weight. From whence it appears, that the Velocity of the Force hath been as many times Superiour to the Resistance of the weight, as the said Weight was superiour to the Force; for that in the same Time that the moving Force hath many times measured the intervall between the Terms of the Motion, the said Moveable happens to have past it onely once: nor therefore ought we to affirm a great Resistance to have been overcome by a small Force, contrary to the constitution of Nature. Then onely may we say the Natural Constitution is overcome, when the lesser Force transfers the greater Resistance, with a Velocity of Motion like to that where-

wherewith it ſelf doth move; which we affirm abſolutely to be impoſſible to be done with any Machine imaginable. But becauſe it may ſometimes come to paſſe, that having but little Force, it is required to move a great Weight all at once, without dividing it in pieces, on this occaſion it will be neceſſary to have recourſe to the Machine, by means whereof the propoſed Weight may be transferred to the aſſigned Space by the Force given. But yet this doth not hinder, but that the ſame Force is to move, meaſuring that ſame Space, or another equall to it, as many ſeverall times as it is exceeded by the ſaid Weight. So that in the end of the action we ſhall ſind that we have received from the Machine no other benefit then only that of tranſporting the ſaid Weight with the given Force to the Term given all at once. Which Weight, being divided into parts, would without any Machine have been carried by the ſame Force, in the ſame Time, through the ſame Intervall. And this ought to paſſe for one of the benefits taken from the Mechanicks: for indeed it frequently happens, that being ſcanted in Force but not Time, we are put upon moving great Weights unitedly or in groſſe: but he that ſhould hope, and attempt to do the ſame by the help of Machines without increaſe of Tardity in the Moveable, would certainly be deceived, and would declare his ignorance of the uſe of Mechanick Inſtruments, and the reaſon of their effects.

Another benefit is drawn from the Inſtruments, which dependeth on the place wherein the operation is to be made: for all Inſtruments cannot be made uſe of in all places with equall convenience. And ſo we ſee (to explain our ſelves by an example) that for drawing of Water out of a Well, we make uſe of onely a Rope and a Bucket fitted to receive and hold Water, wherewith we draw up a determinate quantity of Water, in a certain Time, with our limited ſtrength: and he that ſhould think he could with a Machine of whatſoever Force, with the ſame ſtrength, and in the ſame Time, take up a great quantity of Water, is in a groſſe Errour. And he ſhall find himſelf ſo much the more deceived, the more he ſhall vary and multiply his Inventions: Yet neverthleſſe we ſee Water drawn up with other Engines, as with a Pump that drinks up Water in the Hold of Ships; where you muſt note that the Pump was not imployed in thoſe Offices, for that it draws up more Water in the ſame Time, and with the ſame ſtrength then that which a bare Bucket would do, but becauſe in that place the uſe of the Bucket or any ſuch like Veſſel could not effect what is deſired, namely to keep the Hold of the Ship quite dry from every little quantity of Water; which the Bucket cannot do, for that it cannot dimerge and dive, where there is not a conſiderable depth of Water. And thus we ſee the Holds of Ships by the

ſaid Inſtrument kept dry, when Water cannot but onely obliquely be drawn up, which the ordinary uſe of the Bucket would not effect, which riſeth and deſcends with its Rope perpendicularly.

The third is a greater benefit, haply, then all the reſt that are derived from Mechanick Inſtruments, and reſpects the aſſiſtance which is borrowed of ſome Force exanimate, as of the ſtream of a River, or elſe animate, but of leſſe expence by far, then that which would be neceſſary for maintaining humane ſtrength: as when to turn Mills, we make uſe of the Current of a River, or the ſtrength of a Horſe, to effect that, which would require the ſtrength of five or ſix Men. And this we may alſo advantage our ſelves in raiſing Water, or making other violent Motions, which muſt have been done by Men, if there were no other helps; becauſe with one ſole Veſſel we may take Water, and raiſe, and empty it where occaſion requires; but becauſe the Horſe, or ſuch other Mover wanteth Reaſon, and thoſe Inſtruments which are requiſite for holding and emptying the Veſſel in due time, returning again to fill it, and onely is endued with Force, therefore it's neceſſary that the Mechanitian ſupply the naturall defect of that Mover, furniſhing it with ſuch devices and inventions, that with the ſole application of it's Force the deſired effect may follow. And therein is very great advantage, not becauſe that a Wheel or other Machine can enable one to tranſport the ſame VVeight with leſſe Force, and greater Dexterity, or a greater Space than an equall Force, without thoſe Inſtruments, but having Judgement and proper Organs, could have done; but becauſe that the ſtream of a River coſteth little or nothing, and the charge of keeping of an Horſe or other Beaſt, whoſe ſtrength is greater then that of eight, or it may be more Men, is far leſſe then what ſo many Men would be kept for.

Theſe then are the benefits that may be derived from Mechanick Inſtruments, and not thoſe which ignorant Engineers dream of, to their own diſgrace, and the abuſe of ſo many Princes, whilſt they undertake impoſſible enterprizes; of which, both by the little which hath been hinted, and by the much which ſhall be demonſtrated in the Progreſſe of this Treatiſe, we ſhall come to aſſure our ſelves, if we attentively heed that which ſhall be ſpoken.

DEFI-

DEFINITIONS.

THat which in all Demonstrative Sciences is necessary to be observed, we ought also to follow in this Discourse, that is; to propound the Definitions of the proper Terms of this Art, and the primary Suppositions, from which, as from seeds full of fecundity, may of consequence spring and result the causes, and true Demonstrations, of the Nature of all the Mechanick Engines which are used, for the most part about the Motions of Grave Matters, therefore we will determine, first, what is *GRAVITIE*.

VVe call *GRAVITIE* then, That propension of moving naturally downwards, which is found in solid Bodies, caused by the greater or lesse quantity of matter, whereof they are constituted.

MOMENT is the propension of descending, caused not so much by the Gravity of the moveable, as by the disposure which divers Grave Bodies have in relation to one another; by means of whichMoment,we oft see a Body less Grave counterpoise another of greater Gravity: as in the Stiliard, a great VVeight is raised by a very small counterpoise, not through excess of Gravity, but through the remotenesse from the point whereby the Beam is upheld, which conjoyned to the Gravity of the lesser weight adds thereunto Moment, and *Impetus* of descending, wherewith the Moment of the other greater Gravity may be exceeded. *MOMENT* then is that IMPETUS of descending, compounded of Gravity, Position, and the like, whereby that propension may be occasioned.

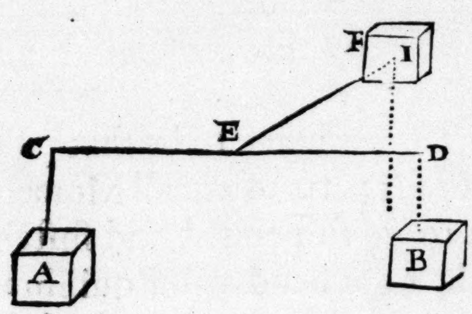
The *CENTER* of *GRAVITY* we define to be that point in every Grave Body,about which consist parts of equall Moment: so that, imagining some Grave Body to be suspended and sustained by the said point, the parts on the right hand will Equilibrate those on the left, the Anteriour, the Posteriour, and those above those below; so that be it in any whatsoever site, and position, provided it be suspended by the said *CENTER*, it shall stand still: and this is that point which would gladly unite with the universall Center of Grave Bodies,namely withthat of the Earth, if it might thorow some free *Medium* descend thither. From whence we take these Suppositions.

SUPPOSITIONS.

ANy Grave Body, (as to what belongeth to it's proper vertue) moveth downwards, so that the Center of it's Gravity never strayeth out of that Right Line which is produced from the said Center placed in the first Term of the Motion unto the universal Center of Grave Bodies. Which is a Supposition very manifest, because that single Center being obliged to endeavour to unite with the common Center, it's necessary, unlesse some impediment intervene, that it go seeking it by the shortest Line, which is the Right alone: And from hence may we secondarily suppose

Every Grave Body putteth the greatest stresse, and weigheth most on the Center of it's Gravity, and to it, as to its proper seat, all *Impetus*, all Ponderosity, and, in some, all Moment hath recourse.

We lastly suppose the Center of the Gravity of two Bodies equally Grave to be in the midst of that Right Line which conjoyns the said two Centers; or that two equall weights, suspended in equall distance, shall have the point of *Equilibrium* in the common Center, or meeting of those equal Distances. As for Example, the Distance C E being equall to the Distance E D, and there being by them two equall weights suspended, A and B, we suppose the point of *Equilibrium* to be in the point E, there being no greater reason for inclining to one, then to the other part. But here is to be noted, that the Distances ought to be measured with Perpendicular Lines, which from the point of Suspension E, fall on the Right Lines, that from the Center of the Gravity of the VVeights A and B, are drawn to the common Center of things Grave; and therefore if the Distance E D were transported into E F, the weight B would not counterpoise the weight A, because drawing from the Centers of Gravity two Right Lines to the Center of the Earth, we shall see that which cometh from the Center of the VVeight I, to be nearer to the Center E, then the other produced from the Center of the weight A. Therefore our saying that equal VVeights are suspended by [or at] equal Distances, is to be understood to be meant when as the Right Lines that go from their Centers & to seek out the common Center of Gravity, shall be equidistant from that Right Line, which is produced from the said



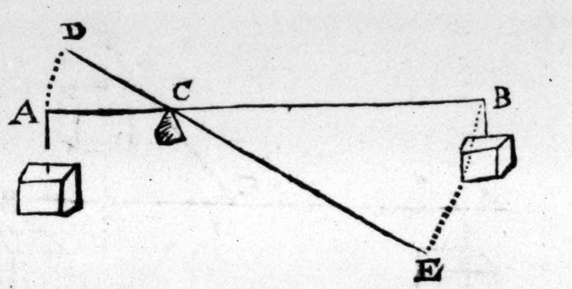
Term

Term of those Distances, that is from the point of Suspension, to the same Center of the Earth.

These things determined and supposed, we come to the explication of a Principle, the most common and materiall of the greater part of Mechanick Instruments: demonstrating, that unequall Weights weigh equally when suspended by [or at] unequal Distances, which have contrary proportion to that which those weights are found to have, See the Demonstration in the beginning of the second Dialogue of Local-Motions.

Some Advertisements about what hath been said.

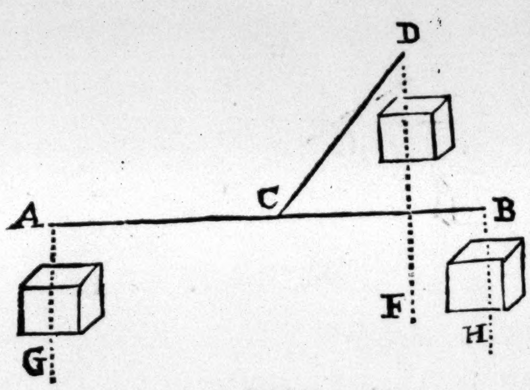
Now being that Weights unequall come to acquire equall Moment, by being alternately suspended at Distances that have the same proportion with them; I think it not fit to over-passe with silence another congruity and probability, which may confirm the same truth; for let the Ballance A B, be considered, as it is divided into unequal parts in the point C, and let the Weights be of the same proportion that is between the Distances B C, and C A, alternately suspended by the points A, and B: It is already manifest, that the one will counterpoise the other, and consequently, that were there added to one of them



a very small Moment of Gravity, it would preponderate, raising the other, so that an insensible VVeight put to the Grave B, the Ballance would move and descend from the point B towards E, and the other extream A would ascend into D, and in regard that to weigh down B, every small Gravity is sufficient, therefore not keeping any accompt of this insensible Moment, we will put no difference between one Weights *sustaining*, and one Weights *moving* another. Now, let us consider the Motion which the Weight B makes, descending into E, and that which the other A makes in ascending into D, we shall without doubt find the Space B E to be so much greater than the Space A D, as the Distance B C is greater than C A, forming in the Center C two angles D C A, and E C B, equall as being at the Cock, and consequently two Circumferences A D and B E alike; and to have the same proportion to one another, as have the Semidiameters B C, and C A, by which they are described: so that then the Velocity of the Motion of the descending Grave B cometh to be so much Superiour to the Velocity of the other ascending Moveable A, as the Gravity of this exceeds the Gravity of that; and it not being possible

possible that the VVeight A should be raised to D, although slowly, unlesse the other VVeight B do move to E swiftly, it will not be strange, or inconsistent with the Order of Nature, that the Velocity of the Motion of the Grave B, do compensate the greater Resistance of the VVeight A, so long as it moveth slowly to D, and the other descendeth swiftly to E, and so on the contrary, the VVeight A being placed in the point D, and the other B in the point E, it will not be unreasonable that that falling leasurely to A, should be able to raise the other hastily to B, recovering by its Gravity what it had lost by it's Tardity of Motion. And by this Discourse we may come to know how the Velocity of the Motion is able to encrease Moment in the Moveable, according to that same proportion by which the said Velocity of the Motion is augmented.

There is also another thing, before we proceed any farther, to be considered; and this is touching the Distances, whereat, or wherein VVeights do hang: for it much imports how we are to understand Distances equall, and unequall; and, in sum, in what manner they ought to be measured: for that A B being the Right Line, and two equall VVeights being suspended at the very ends thereof, the point C being taken in the midst of the said Line, there shall be an *Equilibrium* upon the same: And the reason is for that the Distance C B is equal to C A.

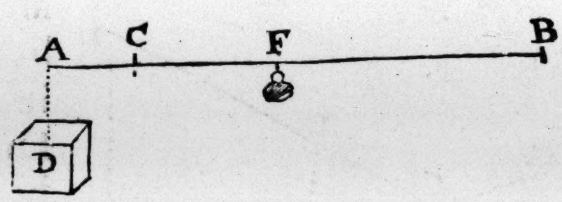


But if elevating the Line C B, moving it about the point C, it shall be transferred into CD, so that the Ballance stand according to the two Lines A C, and C D, the two equall Weights hanging at the Terms A and D, shall no longer weigh equally on that point C, because the distance of the Weight placed in D, is made lesse then it was when it hanged in B. For if we consider the Lines, along [or by] which the said Graves make their Impulse, and would descend, in case they were freely moved, there is no doubt but that they would make or describe the Lines A G, D F, B H: Therefore the Weight hanging on the point D, maketh it's Moment and *Impetus* according to the Line D F: but when it hanged in B, it made *Impetus* in the Line B H: and because the Line D F is nearer to the Fulciment C, then is the Line B H Therefore we are to understand that the Weights hanging on the points A and D, are not equi-distant from the point C, as they be when they are constituted according to their Right Line A C B: And lastly, we are to take notice, that the Distance is to be measured by Lines,

Lines, which fall at Right Angles on those whereon the Weights hang, and would move, if so be they were permitted to descend freely.

Of the BALLANCE and LEAVER.

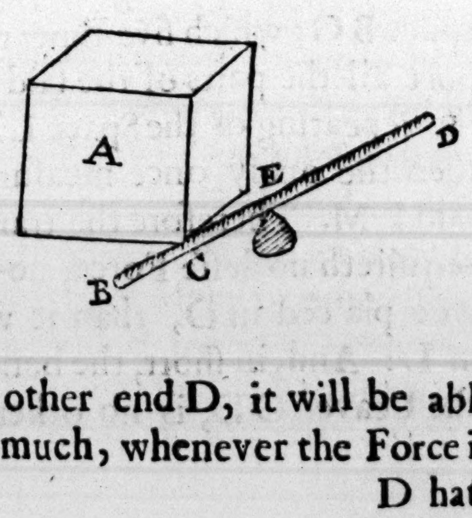
HAving understood by certain Demonstration, one of the first Principles, from which, as from a plentiful Fountain, many of the Mechanical Instruments are derived, we may take occasion without any difficulty to come to the knowledge of the nature of them: and first speaking of the Stiliard, an Instrument of most ordinary use, with which divers Merchandizes are weighed, sustaining them, though very heavy, with a very small counterpoise, which is commonly called the Roman or Plummet, we shall prove that there is no more to be done in such an operation, but to reduce into act and practice



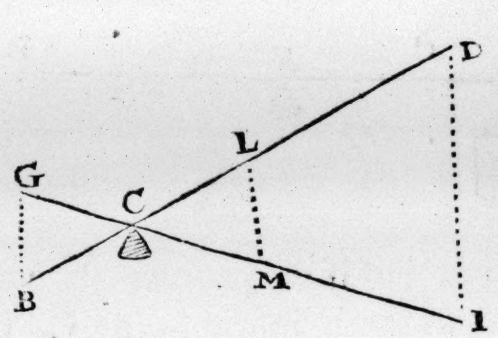
what hath been above contemplated. For if we propose the Ballance A B, whose Fulciment or Lanquer is in the point C, by which, at the small Distance C A, hangeth the heavy Weight D, and if along the other greater C B, (which we call the Needle of the Stiliard) we should suppose the Roman F, though of but little weight in comparison of the Grave Body D to be slipped to and fro, it shall be possible to place it so remotely from the Lanquet C, that the same proportion may be found between the two Weights D and F, as is between the Distances F C, and C A: and then shall an *Equilibrium* succeed; unequall Weights hanging at Distances alternately proportional to them.

Nor is this Instrument different from that other called *Vectis*, and vulgarly the * Leaver, wherewith great Weights are moved by small Force; the application of which is according to the Figure prefixed; wherein the Leaver is represented by the Bar of wood or other solid matter, *B* C D, let the heavy Weight to be raised be A, and let the steadfast support or Fulciment on which the Leaver rests and moves be supposed to be E, and putting one end of the Leaver under the Weight A, as may be seen in the point C, encreasing the Weight or Force at the other end D, it will be able to lift up the Weight A, though not much, whenever the Force in D hath

* If of Iron, it is called a Crow, if of wood, a Bar or Hand-spike.



D hath the ſame proportion to the Reſiſtance made by the Weight A, in the point C: as the Diſtance B C hath to the Diſtance C D, whereby it's clear, that the nearer the Fulciment E ſhall approach to the Term B, encreaſing the proportion of the Diſtance D C to the Diſtance C B, the more may one diminiſh the Force in D which is to raiſe the Weight A. And here it is to be noted, which I ſhall alſo in its place remember you of, that the benefit drawn from all Mechanical Inſtruments, is not that which the vulgar Mechanitians do perſwade us, to wit, ſuch, that thereby Nature is overcome, and in a certain manner deluded, a ſmall Force over-powring a very great Reſiſtance with help of the Leaver; for we ſhall demonſtrate, that without the help of the length of the Leaver, the ſame Force, in the ſame Time, ſhall work the ſame effect. For taking the ſame Leaver B C D, whoſe reſt or Fulciment is in C, let the Diſtance C D be ſuppoſed, for example, to be in quintuple proportion to the Diſtance C B, & the ſaid Leaver to be moved till it come to I C G: In the Time that the Force ſhall have paſſed the Space D I, the Weight ſhall have been moved from B to G: and becauſe the Diſtance D C, was ſuppoſed quintuple to the other C B, it is manifeſt from the things demonſtrated, that the Weight placed in B may be five times greater then the moving Force ſuppoſed to be in D: but now, if on the contrary, we take notice of the * Way paſſed by the Force from D unto I, whilſt the Weight is moved from B unto G, we ſhall find likewiſe the Way D I, to be quintuple to the Space B G. Moreover if we take the Diſtance C L, equal to the Diſtance C B, and place the ſame Force that was in D, in the point L, and in the point B the fifth part onely of the Weight that was put there at firſt, there is no queſtion, but that the Force in L being now equal to this Weight in B, and the Diſtances L C and C B being equall, the ſaid Force ſhall be able, being moved along the Space L M to transfer the VVeight equall to it ſelf, thorow the other equall Space B G: which five times reiterating this ſame action, ſhall tranſport all the parts of the ſaid VVeight to the ſame Term G: But the repeating of the Space L M, is certainly nothing more nor leſſe then the onely once meaſuring the Space D I, quintuple to the ſaid L M. Therefore the transferring of the VVeight from B to G, requireth no leſſe Force, nor leſſe Time, nor a ſhorter Way if it wee pla ced in D, than it would need if the ſame were applied in L: And, in ſhort, the benefit that is derived from the length of the Leaver C D, is no other, ſave the enabling us to move that



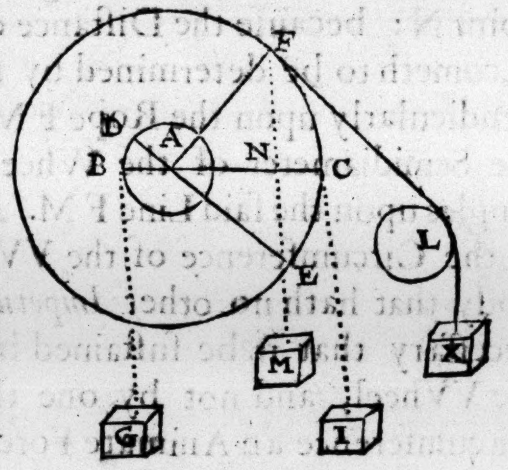
* Or Space.

Body

Body all at once, which would not have been moved by the same Force, in the same Time, with an equall Motion, save onely in pieces, without the help of the Leaver.

Of the CAPSTEN and of the CRANE.

THe Instruments which we are now about to declare, have immediate dependence upon the Leaver, nay, are no other but a perpetual Vectis or Leaver. For if we shall suppose the Leaver B A C to be sustained in the point A, and the VVeight G to hang at the point B, the Force being placed in C; It is manifest, that transferring the Leaver unto the points D A E, the VVeight G doth alter according to the Distance B D, but cannot much farther continue to raise it, so that if it were required to elevate it yet higher, it would be necessary to stay it by some other Fulciment

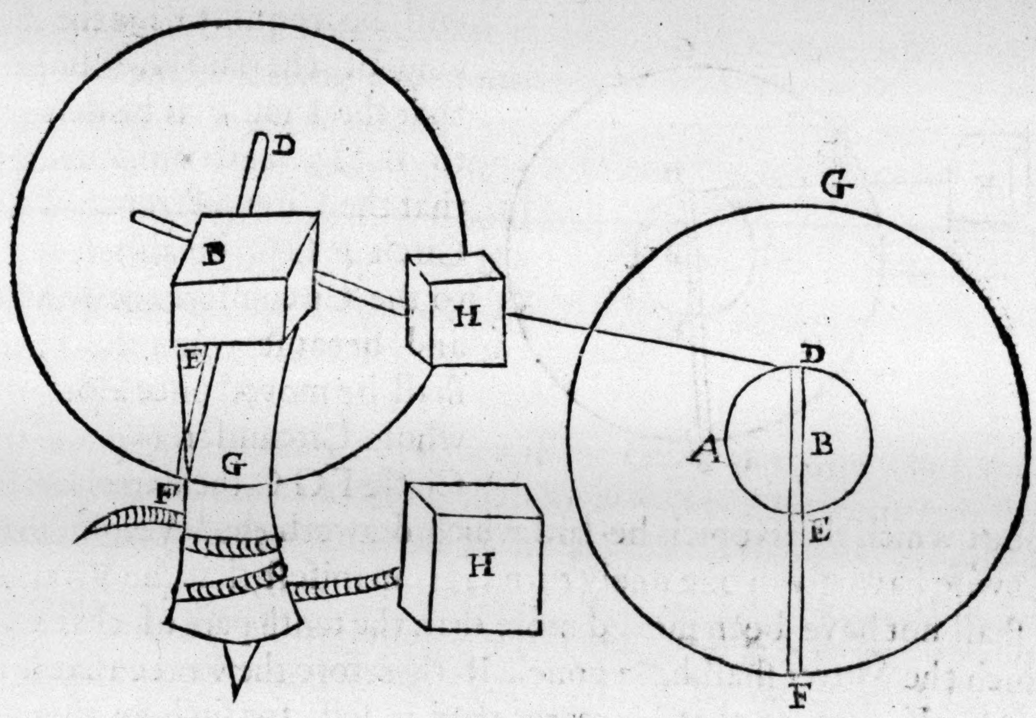


in this Position, and to remit or return the Leaver to its former Position B A C, and suspending the VVeight anew thereat, to raise it once again to the like height B D; and in this manner repeating the work, many times one shall come with an interrupted Motion to effect the drawing up of the VVeight, which for many respects will not prove very beneficial: whereupon this difficulty hath bin thought on, and remedied, by finding out a way how to unite together almost infinite Leavers, perpetuating the operation without any interruption; and this hath been done by framing a VVheel about the Center A, according to the Semidiameter A C, and an Axis or Nave, about the same Center, of which let the Line A B be the Semidiameter; and all this of very tough wood, or of other strong and solid matter, afterwards sustaining the whole Machine upon a Gudgeon or Pin of Iron planted in the point A, which passeth quite thorow, where it is held fast by two fixed Fulciments, and the Rope D B G, at which the weight G hangeth, being be-laid or wound about the Axis or Barrell, and applying another Rope about the greater VVheel, at which let the other Grave I be hanged: It is manifest, that the length C A having to the other A B the self-same proportion that the Weight G hath to the VVeight I, it may sustain the Grave G, and with any little Moment more shall move it: and because the Axis turning round together with the VVheel, the Ropes that sustain the Weights are alwaies pendent and contingent with the extream Circumferences of that VVheel and

Axis, ſo that they ſhall conſtantly maintain alike Site and Poſition in reſpect of the Diſtances B A and A C, the Motion ſhall be perpetuated, the Weight I deſcending, and forcing the other G to aſcend. Where we are to obſerve the neceſſity of be-laying or winding the Rope about the Wheel, that ſo the Weight I may hang according to the Line that is tangent to the ſaid Wheel: for if one ſhould ſuſpend the ſaid Weight, ſo as that it did hang by the point F, cutting the ſaid Wheel, as is ſeen along the Line F N M, the Motion would ceaſe, the Moment of the Weight M being diminiſhed; which would weigh no more then if it did hang by the point N: becauſe the Diſtance of its Suſpenſion from the Center A, cometh to be determined by the Line A N, which falleth perpendicularly upon the Rope F M, and is no longer terminated by the Semidiameter of the Wheel A F, which falleth at unequall Angles upon the ſaid Line F M. A violence therefore being offered in the Circumference of the VVheel by a Grave and Exanimate Body that hath no other *Impetus* then that of Deſcending, it is neceſſary that it be ſuſtained by a Line that is contingent with the VVheel, and not by one that cutteth it. But if in the ſame Circumference an Animate Force were employed, that had a Moment or Faculty of making an *Impulſe* on all ſides, the work might be effected in any whatever place of the ſaid Circumference. And thus being placed in F, it would draw up the VVeight by turning the VVheel about, pulling not according to the Line F M downwards, but ſide-waies according to the Contingent Line F L, which maketh a Right Angle, with that which is drawn from the Center A unto the point of Contact F: ſo, that if in this manner one do meaſure the Diſtance from the Center A to the Force placed in F, according to the Line A F perpendicular to F L, along which the *Impetus* is made, a man ſhall not in any part have altered the uſe of the ordinary Leaver. And we muſt note, that the ſame would be poſſible to be done likewiſe with an Exanimate Force, in caſe that a way were found out to cauſe that its Moment might make Impulſe in the point F, drawing according to the Contingent Line F L: which would be done by adjoyning beneath the Line F L a turning Pulley, making the Rope wound about the VVheel to paſſe along upon it, as it is ſeen to do by the Line F L X, ſuſpending at the end thereof the VVeight X equall to the other I, which exerciſing its Force according to the Line F L, ſhall alwaies keep a Diſtance from the Center A equall unto the Semidiameter of the VVheel. And from what hath been declared we will gather for a Concluſion, That in this Inſtrument the Force hath alwaies the ſame proportion to the Weight, as the Semidiameter of the Axis or Barrell hath to the Semidiameter of the Wheel.

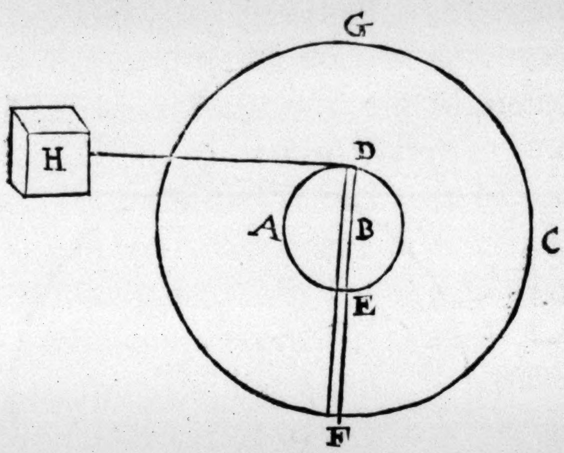
From

From the Instrument last described, the other Instrument which we call the Crane is not much different, as to form, nay, differeth nothing, save in the way of applying or employing it: For that the Capsten moveth and is constituted perpendicular to the Horizon, and the Crane worketh with its Moment parallel to the same Ho-



rizon. For if upon the Circle D A E we suppose an Axis to be placed Column-wise, turning about the Center B, and about which the Rope G H, fastened to the Weight that is to be drawn, is belaid, and if the Bar F E B D be let into the said Axis [*by the Mortace B*] and the Force of a Man, of an Horse, or of some other Animal apt to draw, be applyed at its end F, which moving round, passeth along the Circumference F G C, the Crane shall be framed and finished, so that by carrying round the Bar F B D, the Barrell or Axis E A D shall turn about, and the Rope which is twined about it, shall constrain the Weight H to go forward: And because the point of the Fulciment about which the Motion is made, is the point B, and the Moment keeps at a Distance from it according to the Line B F, and the Resistor at the Distance B D, the Leaver F B D is formed, by vertue of which the Force acquireth Moment equall to the Resistance, if so be, that it be in proportion to it, as the Line B D is to B F, that is, as the Semidiameter of the Axis to the Semidiameter of the Circle, along whose Circumference the Force moveth. And both in this, and in the other Instrument we are to observe that which hath been frequently mentioned, that is, That the benefit which is derived from these Machines, is not that which the generality of the Vulgar promise themselves from the Mechanicks; namely, that being too hard for Nature, its possible

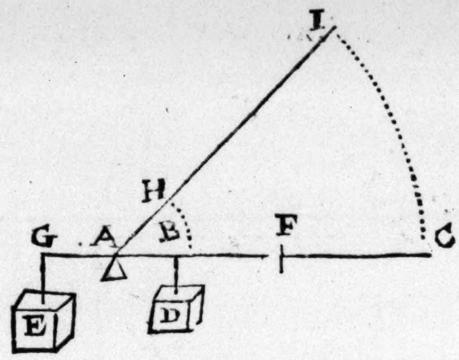
with a Machine to overcome a Resistance, though great, with a small Force, in regard, that we shall manifestly prove that the same Force placed in F, might in the same Time conveigh the same VVeight, with the same Motion, unto the same Distance, without any Machine at all: For supposing, for example, that the Resistance of the Grave H be ten times greater than the Force placed in F, it will be requisite for the moving of the said Resistance, that the Line F B be decuple to B D; and consequently, that the Circumference of the Circle F G C be also decuple to the Circumference E A D: and because when the Force shall be moved once along the whole Circumference of the Circle F G C, the Barrel EAD, about which the Rope is be-laid which draweth the VVeight, shall likewise have given one onely turn; it is manifest, that the VVeight H shall not have been moved more than the tenth part of that way which the Mover shall have gone. If therefore the Force that is to move a Resistance that is greater than it self, for such an assigned Space by help of this Machine, must of necessity move ten times as far, there is no doubt, but that dividing that VVeight into ten parts, each of them shall be equall to the Force, and consequently, might have been transported one at a Time, as great a Space as that which it self did move, so that making ten journeys, each equal to the Circumference E A D, it shall not have gone any farther than if it did move but once alone about the Circumference F G C; and shall have conveighed the same Weight H to the same Distance. The benefit therefore that is to be derived from these Machines is, that they carry all the Weight together, but not with lesse Labour, or with greater Expedition, or a greater VVay than the same Force might have done conveying it by parcels.



Of Pullies.

THe Instruments, whose Natures are reducible unto the Ballance, as to their Principle and Foundation, and others little differing from them, have been already described; now for the understanding of that which we have to say touching Pullies, it is requisite, that we consider in the first place another way to use the Leaver, which will conduce much towards the investigation of the Force of Pullies, and towards the understanding of other Mechanical Effects. The use of the Leaver above declared supposed the

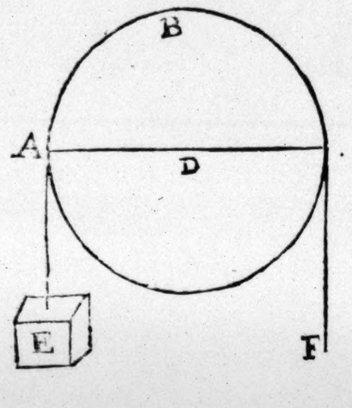
the VVeight to be at one extream, and the Force at the other, and the Fulciment placed in ſome point between the extreams: but we may make uſe of the Leaver another way, yet, placing, as we ſee, the Fulciment in the extream A, the Force in the other extream C, and ſuppoſing the VVeight D to hang by ſome point in the midſt, as here we ſee by the point B, in this example it's manifeſt, that if the VVeight did hang at a point Equi-diſtant from the two extreams A and C, as at the point F, the labour of ſuſtaining it would be equally divided betwixt the two points A and C, ſo that half the VVeight would be felt by the Force C, the other half being ſuſtained by the Fulciment A: but if the Grave Body ſhall be hanged at another place, as at B, we ſhall ſhew that the Force in C is ſufficient to ſuſtain the VVeight in B, as it hath the ſame proportion to it, that the Diſtance A B hath to the Diſtance A C. For Demonſtration of which, let us imagine the Line B A to be continued right out unto G, and let the Diſtance B A be equall to A G, and let the VVeight hanging at G, be ſuppoſed equall to D: It is manifeſt, that by reaſon of the equality of the VVeights D and E, and of the Diſtances G A and A B, the Moment of the VVeight E ſhall equalize the Moment of the VVeight D, and is ſufficient to ſuſtain it: Therefore whatever Force ſhall have Moment equall to that of the VVeight E, and that ſhall be able to ſuſtain it, ſhall be ſufficient likewiſe to ſuſtain the VVeight D: But for ſuſtaining the VVeight E, let there be placed in the point C ſuch a Force, whoſe Moment hath that proportion to the VVeight E, that the Diſtance G A hath to the Diſtance A C, it ſhall be ſufficient to ſuſtain it: Therefore the ſame Force ſhall likewiſe be able to ſuſtain the VVeight D, whoſe Moment is equall to that of E: But look what Proportion the Line G A hath to the Line A C, and A B alſo hath the ſame to the ſaid A C, G A having been ſuppoſed equall to A B: And becauſe the VVeights E and D are equall, each of them ſhall have the ſame proportion to the Force placed in C: Therefore the Force in C is concluded to equall the Moment of the VVeight D, as often as it hath unto it the ſame proportion that the Diſtance B A hath to the Diſtance C A. And by moving the VVeight, with the Leaver uſed in this manner, it is gathered in this alſo, as well as in the other Inſtruments, that what is gained in Force is loſt in Velocity: for the Force C raiſing the Leaver, and transferring it to A I, the VVeight is moved the Space B H, which is as much leſſer than the Space C I paſſed by the Force, as the Diſtance A B is leſſer than



than the Distance A C; that is, as the Force is lesse than the Weight.

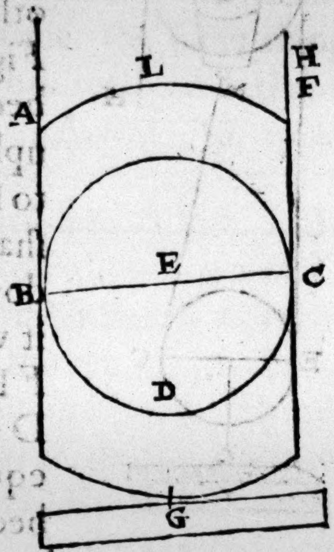
These Principles being declared, we will passe to the Contemplation of Pullies, the composition and structure of which, together with their use, shall be described by us. And first let us suppose the * Little Pulley A B C, made of Mettall or hard Wood, voluble about it's Axis which passeth thorow it's Center B, and about this Pulley let the Rope E A B C be put, at one end of which let the Weight E hang, and at the other let us suppose the Force F. I say, that the Weight being sustained by a Force equall to it self in the upper Nut or Pulley A B C, bringeth some benefit, as the moving or sustaining of the said Weight with the Force placed in F: For if we shall understand, that from the Center D, which is the place of the Fulciment, two Lines be drawn out as far as the Circumference of the Pulley in the points A and C, in which the pendent Cords touch the Circumference, we shall have a Ballance of equal Arms which determine the Distance of the two Suspensions from the Center and Fulciment D: Whereupon it is manifest, that the Weight hanging at A cannot be sustained by a lesser Weight hanging at G, but by one equal to it; such is the nature of equal Weights hanging at equal Distances. And although in moving downwards, the Force E cometh to turn about the Pulley A B C, yet there followeth no alteration of the Altitude or Respect, that the Weight and Force have unto the two Distances A D and D C, nay, the Pulley encompassed becometh a Ballance equal to A C, but perpetuall. Whence we may learn, how childishly *Aristotle* deceiveth himself, who holds, that by making the small Pulley A B C bigger, one might draw up the Weight with a lesser Force; he considering that upon the enlargement of the said Pulley, the Distance B C encreased, but not considering that there was as great an encrease of the other Distance of the Weight, that is, the other Semidiameter D A. The benefit therefore that may be drawn from the Instrument abovesaid, is nothing at all, as to the diminution of the labour: and if any one should ask how it happens, that on many occasions of raising VVeights, this means is made use of to help the Axis, as we see, for example, in drawing up the VVater of VVells; it is answered, that that is done, because that by this means the manner of employing the Force is found more commodious: for being to pull downwards, the proper Gravity of our Arms and other parts help us, whereas if we were to draw the same VVeight upwards with a meer Rope, by the sole strength

*Called by some a Nut.



of

of the Members and Muscles, and as we use to say, by Force of Armes, besides the extern Weight we are to lift up the Weight of our own Armes, in which greater pains is required. Conclude we, therefore, that this upper Pulley doth not bring any Facility to the Force simply considered, but onely to the manner of applying it; but if we shall make use of the like Machine in another manner, as we are now about to declare; we may raise the Weight with diminution of Forces: For let the Pulley B D C be voluble about the Center E placed in it's Frame B L C, at which hang the Grave G; and let the Rope A B D C F passe about the Pulley; of which let the end A be fastned to some fixed stay, and in the other F let the Force be placed; which moving towards H shall raise the Machine B L C, and consequently the Weight G: and in this operation I say, that the Force in F is the half of the Weight sustained by it.

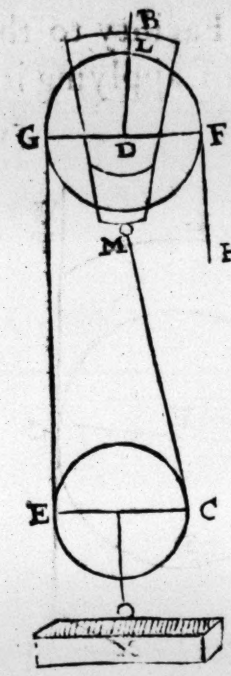


For the said Weight being kept to Rights by the two * Ropes A B and F C, it is manifest, that the Labour is equally shared betwixt the Force F and the Fulciment A: and more subtilly examining the nature of this Instrument, if we but continue forth the Diameter B E C, we shall see a Leaver to be made, at the midst of which, that is at the point E, the Grave doth hang, and the Fulciment cometh to be at the end B, and the Force in the Term C: whereupon, by what hath been above demonstrated, the Force shall have the same proportion to the Weight, that the Distance E B hath to the Distance; Therefore it shall be the half of the said Weight: And because the Force rising towards A, the Pulley turneth round, therefore that Respect or Constitution which the Fulciment B and Center E, on which the Weight and Term C, in which the Force is employed do depend, shall not change all the while; but yet in the Circumduction the Terms B and C happen to vary in number, but not in vertue, others and others continually succeeding in their place, whereby the Leaver B C cometh to be perpetuated. And here (as hath been done in the other Instruments, and shall be in those that follow) we will not passe without considering how that the journey that the Force maketh, is double to the Moment of the Weight. For in case the Weight shall be moved so far, till that the Line B C come to arrive with it's points B and C, at the points A and F, it is necessary that the two equal Ropes be distended in one sole Line F H, and consequently, when the Weight shall have ascended along the Intervall B A, the Force shall have been moved twice as far, that is, from F unto H. Then considering that the

* Or, two ends of the same Rope.

Force

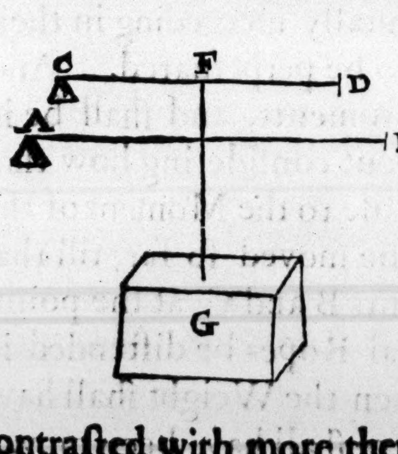
Force in *F*, that it may raise the Weight, must move upwards, which to exanimate Movers, as being for the most part Grave Bodies, is altogether impossible, or at least more laborious, than the making of the same *Force* downwards: Therefore to help this inconvenience, a Remedy hath been found by adjoyning another Nut or Pulley above, as in the adjacent *Figure* is seen, where the Rope C E *F* hath been made to pass about the upper Pulley *F* G upheld by the Hook L, so that the Rope passing to H, and thither transferring the *Force* E, it shall be able to move the Weight X by pulling downwards, but not that it may be lesser than it was in E: *For* the Motions of the *Force* F H, hanging at the equal Distances *F* D and D G of the upper Pulley, do alwaies continue equal; nor doth that upper Pulley (as hath been shewn above) come to produce any diminution in the Labour. Moreover it having been necessary by the addition of the upper Pulley to introduce the Appendix B, by which it is sustained, it will prove of some benefit to us to raise the other A, to which one end of the Rope was fastned, transferring it to a Ring annexed to the lower part of the *Frame* of the upper Pulley, as we see it done in M. Now finally, this Machine compounded of upper and lower Pullies, is that which the Greeks call Τροχίλιον.



In Latine *Trochlea.*

We have hitherto explained, how by help of Pullies one may double the *Force*, it remaineth that with the greatest brevity possible, we shew the way how to encrease it according to any Multiplicity. And first we will speak of the Multiplicity according to the even numbers, and then the odde: To shew how we may multiply the *Force* in a quadruple Proportion, we will propound the following Speculation as the Soul of all that followeth.

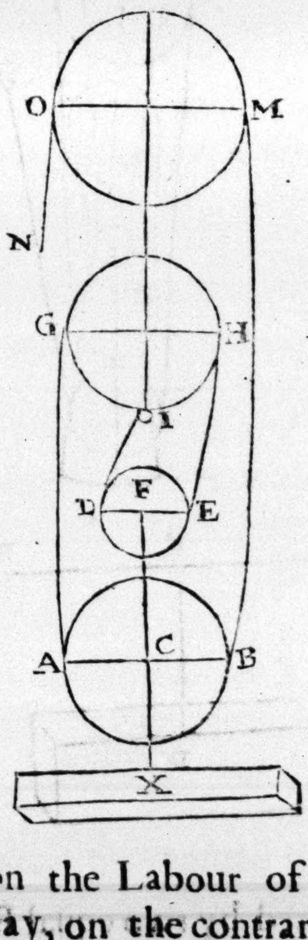
Take two Leavers, A B, C D, with the *Fulciments* in the extreams A and C; and at the middles of each of them let the Grave G hang, sustained by two *Forces* of equal Moment placed in B and D. I say, that the Moment of each of them will equal the Moment of the fourth part of the Weight G. *For* the two *Forces* B and D bearing equally, it is manifest, that the *Force* D hath not contrasted with more then one half of the Weight G. But if the Force D do by benefit of the Leaver D C sustain the half of the



Weight

Weight G hanging at *F*, it hath been already demonstrated, that the said *Force* D hath to the Weight so by it sustained, that same proportion which the Distance *F* C hath to the Distance C D: Which is subduple proportion: Therefore the Moment D is subduple to the Moment of half of the Weight G sustained by it: Wherefore it followeth, that it is the fourth part of the Moment of the whole Weight. And in the same manner the same thing is demonstrated, of the Moment *B*; and it is but reasonable, that the Weight G being sustained by the four points, A, *B*, C, D, each of them should feel an equall part of the Labour.

Let us come now to apply this Consideration to Pullies, and let the Weight X be supposed to hang at the two Pullies A B and D E entwining about them, and about the uppermost Pulley G H, the Rope, as we see, I D E H G A B, sustaining the whole Machine in the point K. Now I say, that placing the Force in L, it shall be able to sustain the Weight X, if so be, it be equal to the fourth part of it. For if we do imagine the two Diameters D E and A B, and the Weights hanging at the middle points F and C, we shall have two Leavers like to those before described, the Fulciments of which answer to the points D and A. VVhereupon the Force placed in B, or if you will, in L, shall be able to sustain the VVeight X, being the fourth part of it: And if we adde another Pulley above the other two, making the Rope or Cord to pass along L M N, transferring the Force L into N, it shall be able to bear the same Weight gravitating downwards, the upper Pulley neither augmenting or diminishing the Force, as hath been declared. And we will likewise note, that to make the Weight ascend the four * Ropes B L, E H, D I, and A G ought to pass, whereupon the Mover will be to begin, as much as those Ropes are long; and yet neverthelesss the Weight shall move but only as much as the length of one of them: So that we may say by way of advertisement, and for confirmation of what hath been many times spoken, namely, that look with what proportion the Labour of the Mover is diminished, the length of the Way, on the contrary, is encreased with the same proportion.

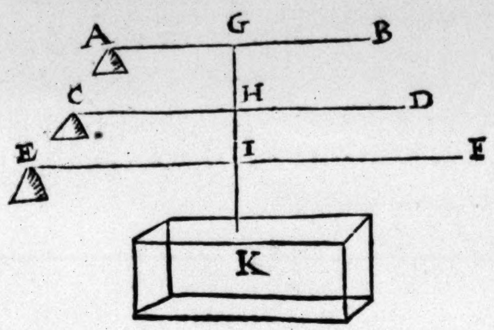


* Or four parts of the same Rope

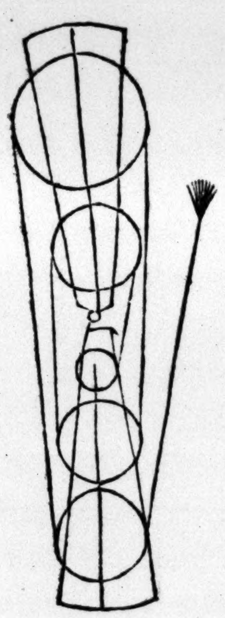
But if we would encrease the Force in sexcuple proportion, it will be requisite that we adjoyn another * small Pulley or Gyrill to the inferiour Pulley: which that you may the better understand

* The word *Girilla* signifieth a Shiver, Rundle, or small Wheel of a Pulley, translated by we sometimes Pulley, sometimes Nut or Girill.

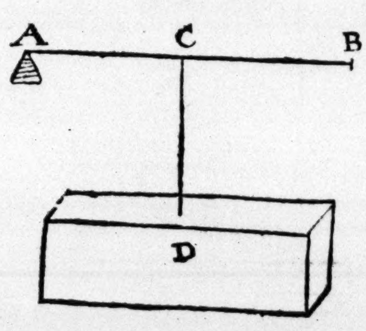
we will set before you the present Contemplation. Suppose, therefore, that A B, C D, and E F are three Leavers; and that on the middle points of them G, H, and I the Weight K doth hang in common, so that every one of them shall sustain the third part of it: And because the Power in B, sustaining with the Leaver B A the pendent Weight in G, hapneth to be the half of the said Weight, and it hath been already said, that it sustaineth the third part of the Weight K: Therefore the Moment of the Force B is equal to half of the third part of the Weight K; that is, to the sixth part of it: And the same shall be demonstrated of the other Forces D and F: From whence we may easily gather, that putting three Gyrils or Rundles into the inferiour Pulley, and two or three into the uppermost, we may multiply the Force according to our * *Senarius*. And if we would encrease it according to any other even Number, the Gyrils of the Pulley below must be multiplyed according to the half of that Number, according to which the Force is to be multiplyed, circumposing the Rope about the Pulleys, so as that one of the ends be fastned to the upper Pulley, and let the Force be in the other; as in this Figure adjoyning may manifestly be gathered.



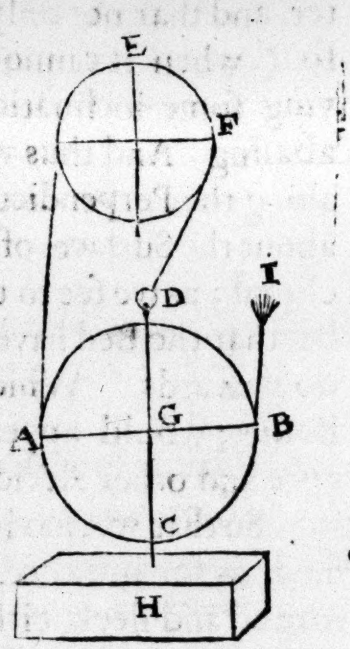
*Or in Sexcuple proportion.



Now passing to the Declaration of the manner how to multiply the Force according to the odd Numbers, and beginning at the triple proportion: first, let us propose the present Contemplation, as that, on the understanding of which the knowledge of all the VVork in hand doth depend. Let therefore the Leaver be A B, its Fulciment A, and from the middle of it, that is, at the point C let the Grave D be hanged; and let it be sustained by two equal Forces; and let one of them be applied to the point C, and the other to the term B. I say, that each of those Powers have Moment equal to the third part of the VVeight D. For the Force in C sustaineth a VVeight equal to it self, being placed in the same Line in which the VVeight D doth hang & Gravitate: But the Force



Force in B ſuſtaineth a part of the VVeight D double to it ſelf, its Diſtance from the Fulciment A, that is, the Line B A being double to the Diſtance A C at which the Grave hangeth: But becauſe the two Forces in B and C are ſuppoſed to be equal to each other: Therefore the part of the Weight D, which is ſuſtained by the Force in B, is double to the part ſuſtained by the Force in C. If therefore, of the Grave D two parts be made, the one double to the remainder, the greater is ſuſtained by the Force in B, and the leſſer by the Force in C: But this leſſer is the third part of the Weight D: Therefore the Moment of the Force in C is equal to the Moment of the third part of the VVeight D; to which, of conſequence, the Force B ſhall be equal, we having ſuppoſed it equal to the other Force C: Wherefore our intention is manifeſt, which we were to demonſtrate, how that each of the two Powers C and B is equal to the third part of the VVeight D. Which being demonſtrated, we will paſs forwards to the Pulleys, and will deſcribe the inferiour Gyrils of A C B, voluble about the Center G, and the Weight H hanging thereat, we will draw the other upper one E F, winding about them both the Rope D F E A C B I, of which let the end D be faſtned to the inferiour Pulley, and to the other I let the Force be applyed: Which, I ſay, ſuſtaining or moving the Weight H, ſhall feele no more than the third part of the Gravity of the ſame. For conſidering the contrivance of this Machine, we ſhall find that the Diameter A B ſupplieth the place of a Leaver, in whoſe term B the Force I is applied, and in the other A the *F*ulciment is placed, at the middle G the Grave H is hanged, and another *F*orce D applied at the ſame place: ſo that the Weight is faſtned to the * three Ropes I B, *F* D, and E A, which with equal Labour ſuſtain the VVeight. Now, by what hath already been contemplated, the two *F*orces D and B being applied, one, to the midſt of the Leaver A B, and the other to the extream term B, it is manifeſt, that each of them holdeth no more but the third part of the VVeight H: Therefore the Power I, having a Moment equal to the third part of the VVeight H, ſhall be able to ſuſtain and move it: but yet the VVay of the *F*orce in I ſhall be triple to the Way that the VVeight ſhall paſs; the ſaid Force being to diſtend it ſelf according to the Length of the three Ropes I B, *F* D, and E A, of which one alone meaſureth the VVay of the VVeight H.



* Or three parts of one Rope.

Of the SCREW.

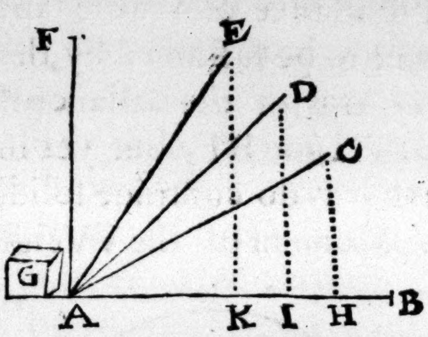
AMongst the rest of Mechanick Instruments for sundry uses found out by the Wit of Man, the Screw doth, in my opinion, both for Invention and for Utility, hold the first place, as that which is appositely accommodated, and so contrived not only to move, but also to stay and press with very great Force, that taking up but little room, it worketh those effects which other Instruments cannot, unless they were reduced to a great Machine. The Screw therefore being of most ingenious and commodious contrivance, we ought deservedly to be at some pains in explaining, with all the plainness that is possible, the Original and Nature of it. The which that we may do, we will begin at a Speculation, which, though at first blush it may appear somewhat remote from the consideration of this Instrument, yet is the *Basis* and Foundation thereof.

No doubt, but that Natures operation in the Motions of Grave Bodies is such, that any whatever Body that hath a Gravity in it hath a propension of moving, being at liberty, towards the Center, and that not only * by the Right Line perpendicularly, but also (when it cannot do otherwise) by any other Line, which having some inclination towards the Center goeth more and more abasing. And thus we see the Water not only to fall downwards along the Perpendicular from some eminent place, but also to run about the Surface of the Earth along Lines though very little enclined; as we see in the Course of Rivers, the Waters of which, if so be that the Bed have any the least declivity, go freely declining downwards. Which very effect, like as it is discerned in all Fluid Bodies, would appear also in hard Bodies, if so be, that their Figure and other Accidental and Extern Impediments did not hinder it. So that we, having a Superficies very well smoothed and polished, as for instance, that of a Looking-glass, and a Ball exactly rotund and sleek, either of Marble, or of Glass, or of any other Matter apt to be polished, this being placed upon that Superficies shall trundle along, in case that this have any, though very small, inclination; and shall lie still only upon that Superficies which is exactly levelled and parallel to the Plane of the Horizon: as is that, for example, of a Lake or standing Water being frozen, upon which the said Spherical Body would stand still, but in a condition of being moved by every small Force. For we having supposed that if that Plane did incline but an hairs breadth only, the said Ball would move along it spontaneously towards the part declining, and on the opposite would have a Resistance, nay, would not be able without some Violence to move towards the part rising

* ... along.

rising or ascending: it of necessity remaineth manifest, that in the Superficies which is exactly equilibrated, the said Ball remaineth indifferent and dubious between Motion and Rest, so that every small Force is sufficient to move it, as on the contrary, every small Resistance, and no greater than that of the meer Air that environs it, is able to hold it still.

From whence we may take this Conclusion for indubitable, That Grave Bodies, all Extern and Adventitious Impediments being removed, may be moved along the Plane of the Horizon by any never so small Force: but when the same Grave is to be thrown along an Ascending Plane, then, it beginning to strive against that ascent, having an inclination to the contrary Motion, there shall be required greater Violence, and still greater the more Elevation that same Plane shall have. As for example, the Moveable G, being posited upon the Line A B parallel to the Horizon, it shall, as hath been said, be indifferent on it either to Motion or Rest, so that it may be moved by a very small Force: But if we shall have the Planes Elevated, they shall not be driven along without Violence; which Violence will be required to be greater to move it along the Line A D, than along A C; and still greater along A E than along A D: The which hapneth, because it hath greater *Impetus* of going downwards along A E than along A D, and along A D than along A C. So that we may likewise conclude Grave Bodies to have greater Resistance upon Planes differently Elevated, to their being moved along the same, according as one shall be more or less elevated than the other; and, in fine, that the greatest Resistance of the same Grave to its being raised is in the Perpendicular A F. But it will be necessary to declare exactly what proportion the Force must have to the Weight, that it may be able to carry it along several elevated Planes, before we proceed any farther, to the end that we may perfectly understand all that which remains to be spoken.

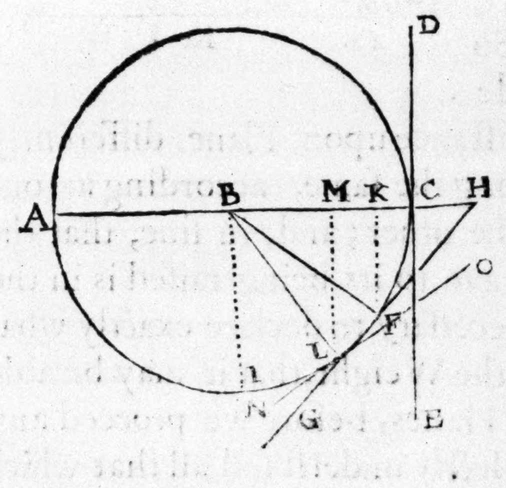


Letting, therefore, Perpendiculars fall from the points C, D, and E unto the Horizontal Line A B, which let be C H, D I, and E K: it shall be demonstrated that the same Weight shall be moved along the Plane A B with lesser Force than along the Perpendicular A F, (where it is raised by a Force equal to it self) according to the proportion by which the Perpendicular C H is less than A C: and that along the Plane A D, the Force hath the same proportion to the Weight, that the Perpendicular I D hath to D A: and, lastly, that in the Plane A E the Force to the Weight observeth the proportion of E K and E A.

The

The present Speculation hath been attempted by *Pappus Alexandrinus* in *Lib.* 8. *de Collection. Mathemat.* but, if I be in the right, he hath not hit the mark, and was overseen in the Assumption that he maketh, where he supposeth that the Weight ought to be moved along the Horizontal Line by a *Force* given; which is false: there needing no sensible *Force* (removing the Accidental Impediments, which in the Theory are not regarded) to move the given Weight along the Horizon, so that he goeth about in vain afterwards to seek with what *Force* it is to be moved along the elevated Plane. It will be therefore better, the *Force* that moveth the Weight upwards perpendicularly, (which equalizeth the Gravity of that Weight which is to be moved) being given, to seek the *Force* that moveth it along the Elevated Plane: Which we will endeavour to do in a Method different from that of *Pappus*.

Let us therefore suppose the Circle A I C, and in it the Diameter A B C, and the Center B, and two VVeights of equal Moment in the extreams B and C; so that the Line A C being a Leaver, or Ballance moveable about the Center B, the VVeight C shall come to be sustained by the VVeight A. But if we shall imagine the Arm of the Ballance B C to be inclined downwards according to the Line B F, but yet in such a manner that the two Lines *A B* and *B F* do continue solidly conjoyned in the point *B*, in this case the Moment of the VVeight C shall not be equal to the Moment of the VVeight *A*, for that the Distance of the point F from the Line of Direction, which goeth according to B I, from the *F*ulciment B unto the Center of the Earth, is diminished: But if from the point *F* we erect a Perpendicular unto B C, as is *F* K, the Moment of the VVeight in *F* shall be as if it did hang by the Line K *F*, and look how much the Distance K B is diminished by the Distance B *A*, so much is the Moment of the Weight *F* diminished by the Moment of the *W*eight *A*.

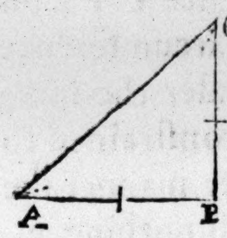


*A*nd in this fashion inclining the *W*eight more, as for instance, according to B L, its Moment shall still diminish and shall be as if it did hang at the Distance *B* M, according to the *L*ine M *L*, in which point *L* it shall be sustained by a *W*eight placed in *A*, so much less than it self, by how much the Distance B *A* is greater than the Distance *B* M. See therefore that the *W*eight placed in the extream of the *L*eaver B C, in inclining downwards along the Circumference C *F L* I, cometh to diminish its Moment and *Impetus* of going downwards from time to time, more

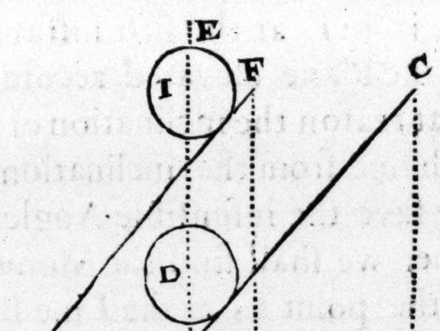
more and leſs, as it is more or leſs ſuſtained by the Lines B F and B L: But the conſidering that this Grave deſcending, and ſuſtained by the Semidiameters B F and B L is one while leſs, and another while more conſtrained to paſs along the Circumference C F L, is no other, than if we ſhould imagine the ſame Circumference C F L I to be a Superficies ſo curved, and put under the ſame Moveable: ſo that bearing it ſelf thereon it were conſtrained to deſcend along thereby; for if in the one and other manner the Moveable deſcribeth the ſame Courſe or Way, it will nothing import whether, if ſuſpended at the Center B, it is ſuſtained by the Semidiameter of the Circle, or elſe, whether that Fulciment being taken away, it proceed along the Circumference C F L I: So that we may confidently affirm, that the Grave deſcending downwards from the point C along the Circumference C F L I, its Moment of Deſcent in the point C is total and entire, becauſe it is not in any part ſuſtained by the Circumference: And there is not in that firſt point C, any indiſpoſition to Motion different from that, which being at liberty, it would make along the Perpendicular and Contingent Line D C E: But if the Moveable ſhall be placed in the point F, then its Gravity is in part ſuſtained, and its Moment of Deſcent is diminiſhed by the Circular Path or Way that is placed under it, in that proportion wherewith the *L*ine *B* K is overcome by *B* C: But if when the Moveable is in F, at the firſt inſtant of ſuch its Motion, it be as if it were in the Plane elevated according to the Contingent *L*ine G F H, for that reaſon the inclination of the Circumference in the point F differeth not from the inclination of the Contingent *L*ine F G any more ſave the inſenſible Angle of the Contact. And in the ſame manner we ſhall find the Moment of the ſaid Moveable to diminiſh in the point *L*, as the *L*ine B M is diminiſhed by B C; ſo that in the Plane contingent to the Circle in the point *L*, as for inſtance, according to the *L*ine H *L* O, the Moment of Deſcent diminiſheth in the Moveable with the ſame proportion. If therefore * upon the Plane H G the Moment of the Moveable be diminiſhed by the total *Impetus* which it hath in its Perpendicular D C E, according to the proportion of the *L*ine K B to the *L*ine B C, and B F, being by the Solicitude of the Triangles K B F and K F H the ſame proportion betwixt the *L*ines K F and F H, as betwixt the ſaid K B and *B* F, we will conclude that the proportion of the entire and abſolute Moment, that the Moveable hath in the Perpendicular to the Horizon to that which it hath upon the Inclined Plane HF, hath the ſame proportion that the *L*ine H F hath to the *L*ine F K; that is, that the *L*ength of the Inclined Plane hath to the Perpendicular which ſhall fall from it unto the Horizon. So that paſſing to a more diſtinct Figure, ſuch as this here preſent, the Moment of Deſcending which the Moveable

* Or along.

able hath upon the inclined Plane C A hath to its total Moment wherewith it gravitates in the Perpendicular to the Horizon C P the same proportion that the said Line P C hath to C A. And if thus it be, it is manifest, that like as the Force that sustaineth the Weight in the Perpendiculation P C ought to be equal to the same, so for sustaining it in the inclined Plane C A, it will suffice that it be so much lesser, by how much the said Perpendicular C P wanteth of the Line C A: and because, as sometimes we see, it sufficeth, that the Force for moving of the Weight do insensibly superate that which sustaineth it, therefore we will infer this universal Proposition, [That upon an Elevated Plane the Force hath to the Weight the same proportion, as the Perpendicular let fall from the Plane unto the Horizon hath to the Length of the said Plane.]

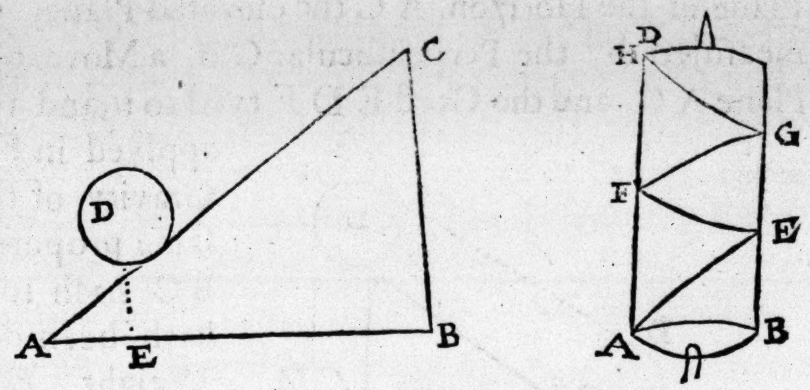


Returning now to our first Intention, which was to investigate the Nature of the Screw, we will consider the Triangle A B C, of which the Line A B is Horizontal, B C perpendicular to the said Horizon, and A C a Plane elevated; upon which the Moveable D shall be drawn by a Force so much less than it, by how much the Line B C is shorter than C A: But to elevate or raise the said Weight along the said Plane A C, is as much as if the Triangle C A B standing still, the Weight D be moved towards C, which is the same, as if the same Weight never removing from the Perpendicular A E, the Triangle did press forwards towards H. For if it were in the Site F H G, the Moveable would be found to have mounted the height A I. Now, in fine, the primary Form and Essence of the Screw is nothing else but such a Triangle A C B, which being forced forwards, shall work it self under the Grave Body to be raised, and lifteth it up, as we say, by the * head and shoulders. And this was its first Original: For its first Inventor (whoever he was) considering how that the Triangle A B C going forwards raiseth the Weight D, he might have framed an Instrument like to the said Triangle, of a very solid Matter, which being thrust forwards did raise up the proposed Weight: But afterwards considering better, how that that same Machine might be reduced into a much lesser and more commodious Form, taking the same Triangle he twined and wound it about the Cylinder A B C D in such a fashion, that the height of the said Triangle, that is the Line C B, did make the Height of the Cylinder, and the Ascending Plane did beget upon the



* *Levar in capo* signifieth to lift on high by force

the ſaid Cylinder the Helical Line deſcribed by the Line AEFGH, which we vulgarly call the Wale of the Screw, which was produced by the Line A C. And in this manner is the Inſtrument made, which is by the Greeks called Κόχλος, and by us a Screw; which winding about cometh to work and inſinuate with its Wales under the Weight, and with facility raiſeth it. And we having demonſtrated, That upon [*or along*]

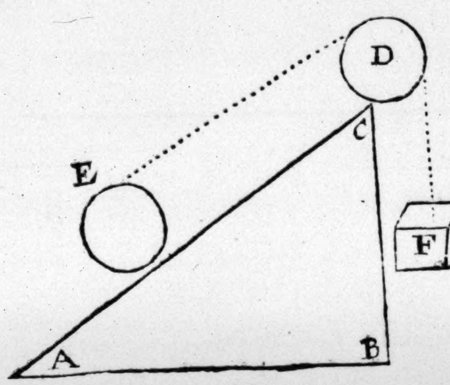


* Κόχλος, in Latine *Cochlea*, any Screw winding like the Shell of a Snail.

the elevated Plane the Force hath the ſame proportion to the VVeight, that the perpendicular Altitude of the ſaid Plane hath to its Length; ſo, ſuppoſing that the Force in the Screw A B C D is multiplied according to the proportion by which the Length of the whole VVale exceedeth the Altitude C B, from hence we come to know that making the Screw with its Helix's more thick or cloſe together, it becometh ſo much the more forceable, as being begot by a Plane leſs elevated, and whoſe Length regards its own Perpendicular Altitude with greater proportion. But we will not omit to advertiſe you, that deſiring to find the Force of a propoſed Screw, it will not be needful that we meaſure the Length of all its VVales, and the Altitude of the whole Cylinder, but it will be enough if we ſhall but examine how many times the Diſtance betwixt two ſingle and Contiguous terms do enter into one ſole Turn of the ſame VVale, as for example, how many times the Diſtance A F is contained in the Length of the Turn A E F: For this is the ſame proportion that the Altitude C B hath to all the VVale.

If all that be underſtood which we have hitherto ſpoken touching the Nature of this Inſtrument, I do not doubt in the leaſt but that all the other circumſtances may without difficulty be comprehended: as for inſtance, that inſteed of making the VVeight to mount upon the Screw if one accommodates its Nut with the Helix incavated or made hollow, into which the Male Screw that is the VVale entring, & then being turned round it raiſeth and lifteth up the Nut or Male Screw together with the VVeight which was hanged thereat. Laſtly, we are not to paſs over that Conſideration with ſilence which at the beginning hath been ſaid to be neceſſary for us to have in all Mechanick Inſtruments, to wit, That what is gained in Force by their aſſiſtance, is loſt again in Time,

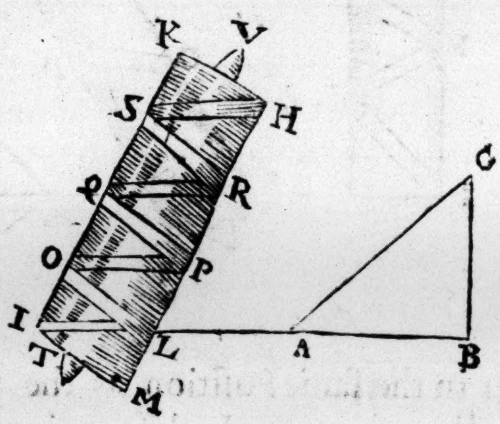
and in the Velocity: which peradventure, might not have seemed to some so true and manifest in the present Contemplation; nay, rather it seems, that in this case the Force is multiplied without the Movers moving a longer way than the Moveable: In regard, that if we shall in the Triangle A B C suppose the Line A B to be the Plane of the Horizon, A C the elevated Plane, whose Altitude is measured by the Perpendicular C B, a Moveable placed upon the Plane A C, and the Cord E D *F* tyed to it, and a *F*orce or Weight applyed in *F* that hath to the Gravity of the Weight E the same proportion that the Line B C hath to C A; by what hath been demonstrated, the Weight *F* shall descend downwards, drawing the Moveable E along the elevated Plane; nor shall the Moveable E measure a greater Space when it shall have passed the whole Line A *C*, than that which the said Grave *F* measureth in its descent downwards. But here yet it must be advertised, that although the Moveable E shall have passed the whole Line A C, in the same Time that the other Grave *F* shall have been abased the like Space, nevertheless the Grave E shall not have retired from the common Center of things Grave more than the Space of the Perpendicular *C* B: but yet the Grave *F* descending Perpendicularly shall be abased a Space equal to the whole Line A *C*. And because Grave Bodies make no Resistance to Transversal Motions, but only so far as they happen to recede from the *C*enter of the Earth; Therefore the Moveable E in all the Motion A *C* being raised no more than the length of the Line *C* B, but the other *F* being abased perpendicularly the quantity of all the Line A *C*: Therefore we may deservedly affirm that Way of the *F*orce E maintaineth the same proportion to the *F*orce *F* that the *L*ine A *C* hath to *C* B; that is, the Weight E to the Weight *F*. It very much importeth, therefore, to consider by [*or along*] what *L*ines the Motions are made, especially in exanimate Grave Bodies, the Moments of which have their total Vigour, and entire Resistance in the *L*ine Perpendicular to the Horizon; and in the others transversally Elevated and Inclined they feel the more or less Vigour, *Impetus*, or Resistance, the more or less those Inclinations approach unto the Perpendicular Inclination.



Of

Of the SCREW of ARCHIMEDES to draw Water.

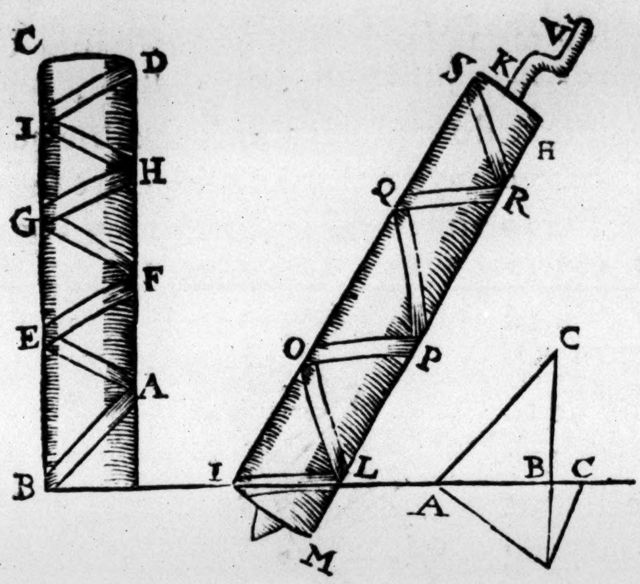
I Do not think it fit in this place to pass over with Silence the Invention of *Archimedes* to raise Wa er with the Screw, which is not only marvellous, but miraculous: for we shall find that the Water ascendeth in the Screw continually descending; and in a given Time, with a given Force doth raise an unspeakable quantity therof. But before we proceed any farther, let us declare the use of the Screw in making Water to rise: And in the ensuing Figure, let us consider the Line I L O P Q R S H being wrapped or twined about the Collumn M I K H, which Line you are to suppose to be a Chanel thorow which the Water may run: If we shall put the end I into the Water, making the Screw to stand leaning, so as the point L may be lower than the first I, as the Diagram sheweth, and shall turn it round about on the two Axes, T and V, the Water shall run thorow the Chanel, till that in the end it shall discharge forth at the mouth H.



Now I say, that the Water, in its conveyance from the point I to the point H, doth go all the way descending, although the point H be higher than the point I. Which that it is so, we will declare in this manner. We will describe the Triangle A C B, which is that of which the Screw H I is generated, in such sort that the Chanel of the Screw is represented by the Line A C, whose Ascent and Elevation is determined by the Angle C A B; that is to say, if so be, that that Angle shall be the third or fourth part of a Right Angle, then the Elevation of the Chanel A C shall be according to $\frac{1}{3}$, or $\frac{1}{4}$ of a Right Angle. And it is manifest, that the Rise of that same Chanel A C will be taken away debasing the point C as far as to B: for then the Chanel A C shall have no Elevation. And debasing the point C a little below B, the Water will naturally run along the Chanel A C downwards from the point A towards C. Let us therefore conclude, that the Angle A being $\frac{1}{3}$ of a Right Angle, the Chanel A C shall no longer have any Rise, debasing it on the part C for $\frac{1}{3}$ of a Right Angle.

These things understood, let us infold the Triangle about the Column, and let us make the Screw B A E F G, &c. which if it shall be placed at Right Angles with the end B in the Water, turning it about, it shall not this way draw up the Water, the Chanel about the Column being elevated, as may be seen by the part B A:

But although the Column ſtand erect at Right-Angles, yet for all that, the Riſe along the Screw, folded about the Column, is not of a greater Elevation than of $\frac{1}{3}$ of a Right Angle, it being generated by the Elevation of the Chanel A C: Therefore if we incline the Column but $\frac{1}{3}$ of the ſaid Right Angle, and a little more, as we ſee I K H M, there is a Tranſition and Motion along the Chanel I L: Therefore the Water from the point I to the point L ſhall move deſcending, and the Screw being turned about, the other parts of it ſhall ſucceſſively diſpoſe or preſent themſelves to the Water in the ſame Poſition as the part I L: Whereupon the Water ſhall go ſucceſſively deſcending, and in the end ſhall be found to be aſcended from the point I to the point H. Which how admirable a thing it is, I leave ſuch to judge who ſhall perfectly have underſtood it. And by what hath been ſaid, we come to know, That the Screw for raiſing of Water ought to be inclined a little more than the quantity of the Angle of the Triangle by which the ſaid Screw is deſcribed.



Of the Force of the

HAMMER, MALLET, *or* BEETLE.

THe Inveſtigation of the cauſe of the Force of theſe Percutients is neceſſary for many Reaſons: and firſt, becauſe that there appeareth in it much more matter of admiration than is obſerved in any other Mechanick Inſtrument whatſoever. For ſtriking with the Hammer upon a Nail, which is to be driven into a very tough Poſt, or with the Beetle upon a Stake that is to penetrate into very ſtiffe ground, we ſee, that by the ſole vertue of the blow of the Percutient both the one and the other is thruſt forwards: ſo that without that, only laying the Beetle upon the Nail or Stake it will not move then, nay, more, although you ſhould lay upon them a Weight very much heavier than the ſaid Beetle. An effect truly admirable, and ſo much the more worthy of Contemplation, in that, as I conceive, none of thoſe who have

hitherto

hitherto discoursed upon it, have said any thing that hits the mark; which we may take for a certain Sign and Argument of the Obscurity and difficulty of this Speculation. For *Aristotle*, or others, who would reduce the cause of this admirable Effect unto the length of the *Manubrium*, or Handle, may, in my judgement, be made to see their mistake in the effect of those Instruments, which having no Handle, yet percuss, either in falling from on high downwards, or by being thrown with Velocity sidewaies. Therefore it is requisite, that we have recourse to some other Principle, if we would find out the truth of this business; the cause of which, although it be of its own nature somewhat obscure, and of difficult consideration, yet neverthelefs we will attempt with the greatest perspicuity possible to render it clear and obvious, shewing, for a close of all, that the Principle and Original of this Effect is derived from no other Fountain than this, from which the reasons of all other Mechanick Effects do proceed: and this we will do by setting before your eyes that very thing which is seen to befall in every other Mechanick Operation, *scilicet*, That the Force, the Resistance, and the Space by which the Motion is made, do go alternately with such proportion operating, and with such a rate answering to each other, that a Resistance, equal to the Force, shall be moved by the said Force along an equal Space, with Velocity equal to that with which it is moved. Likewise, That a Force that is less by half than a Resistance shall be able to move it, so that it be moved with double Velocity, or, if you will, for a Distance twice as great as that which the moved Resistance shall pass: and, in a word, it hath been seen in all the other Instruments, that any, never so great, Resistance may be moved by every small Force given, provided, that the Space, along which the Resistance shall move, have the same proportion that is found to be betwixt the said great Resistance and the Force: and that this is according to the necessary Order and Constitution of Nature: So that inverting the Discourse, and Arguing the contrary way, what wonder shall it be, if that Power that shall move a small Resistance a great way, shall carry one an hundred times bigger an hundredth part of that Distance? Certainly none at all: nay, it would be absurd, yea, impossible that it should be otherwise. Let us therefore consider, what the Resistance of the Beetle unto Motion may be in that point where it is to strike, and how far, if it do not strike, it would be carryed by the received Force beyond that point: and again, what Resistance to Motion there is in him who striketh, and how much by that same Percussion he is moved: and, having found that this great Resistance goeth forwards by a percussion so much less than the Beetle driven by the *Impetus* of him that moveth it would do, by how much that same great Resistance is greater than that of

the

the Beetle; we ſhall ceaſe to wonder at the Effect, which doth not in the leaſt exceed the terms of Natural Conſtitutions, and of what hath been ſpoken. Let us, for better underſtanding, give an example thereof in particular Terms. There is a Beetle, which having four degrees of Reſiſtance, is moved by ſuch a Force, that being freed from it in that term where it maketh the Percuſſion, it would, meeting with no ſtop, go ten Paces beyond it, and in that term a great poſt being oppoſed to it, whoſe Reſiſtance to Motion is as four thouſand, that is, a thouſand times greater than that of the Beetle, (but yet is not immoveable) ſo that it without meaſure or proportion exceeds the Reſiſtance of the Beetle, yet the Percuſſion being made on it, it ſhall be driven forwards, though indeed no more but the thouſandth part of the ten Paces which the Beetle ſhall be moved: and thus in an inverted method, changing that which hath been ſpoken touching the other Mechanical Effects, we may inveſtigate the reaſon of the Force of the Percutient. I know that here ariſe difficulties and objections unto ſome, which they will not eaſily be removed from, but we will freely remit them to the * Problems Mechanical, which we ſhall adjoyn in the end of this Diſcourſe.

* Theſe Problems he here promiſeth were never yet extant.

THE

THE BALLANCE OF Signeur GALILEO GALILEI;

In which, in immitation of *Archimedes* in the Problem of the Crown, he sheweth how to find the proportion of the Alloy of Mixt-Metals; and how to make the said Instrument.

As it is well known, by such who take the pains to read old Authors, that *Archimedes* detected the Cheat of the Goldsmith in the Crown of * *Hieron*, so I think it hitherto unknown what method this Great Philosopher observed in that Discovery: for the opinion, that he did perform it by putting the Crown into the Water, having first put into it such another Mass of pure Gold, and another of Silver severally, and that from the differences in their making the Water more or less rise and run over, he came to know the Mixture or Alloy of the Gold with the Silver, of which that Crown was compounded; seems a thing (if I may speak it) very gross, and far from exactness. And it will seem so much the more dull to such who have read and understood the exquisite Inventions of so Divine a Man amongst the Memorials that are extant of him; by which it is very manifest that all other VVits are inferiour to that of *Archimedes*. Indeed I believe, that Fame divulging it abroad, that *Archimedes* had discovered that same Fraud by means of the VVater, some VVriter of those Times committed the memory thereof to Posterity, and that this person, that he might add something to that little which he had heard by common Fame, did relate that *Archimedes* had made use of the VVater in that manner, as since hath been by the generality of men believed.

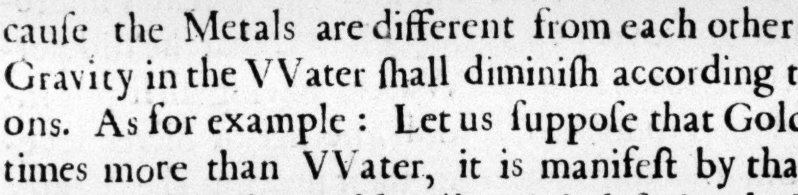
* King of *Sicily*, and Kinsman to that Great Mathematician.

Plutarch in Vit. Marcel.

But in regard I know, that that method is altogether fallacious, and falls short of that exactness which is required in Mathematical Matters, I have often thought in what manner, by help of the VVater, one might exactly find the Mixture of two Metals, and in the end, after I had diligently perused that which *Archimedes* demonstrateth in his Books *De insidentibus aquæ*, and those others

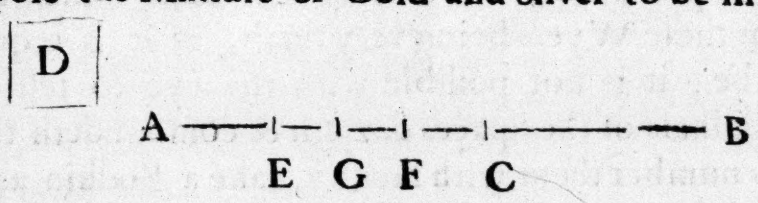
De æquiponderantium, there came into my thoughts a Rule which exquiſitely reſolveth our Queſtion; which Rule I believe to be the ſame that *Archimedes* made uſe of, ſeeing that beſides the uſe that is to be made of the VVater, the exactneſs of the VVork dependeth alſo upon certain Demonſtrations found by the ſaid *Archimedes*.

The way is by help of a Ballance, whoſe Conſtruction and Uſe ſhall be ſhewn by and by, after we ſhall have declared what is neceſſary for the knowledge thereof. You muſt know therefore, that the Solid Bodies that ſink in the VVater weigh ſo much leſs in the VVater than in the Air, as a Maſs of VVater equal to the ſaid Solid doth weigh in the Air: which hath been demonſtrated by *Archimedes*. But, in regard his Demonſtration is very mediate, becauſe I would not be over long, laying it aſide, I ſhall declare the ſame another way. Let us conſider, therefore, that putting into the VVater *v.g.* a Maſs of Gold, if that Maſs were of VVater it would have no weight at all: For the VVater moveth neither upwards, nor downwards in the VVater: It remains, therefore, that the Maſs of Gold weigheth in the VVater only ſo much as the Gravity of the Gold exceeds the Gravity of the VVater. And the like is to be underſtood of other Metals. And becauſe the Metals are different from each other in Gravity, their Gravity in the VVater ſhall diminiſh according to ſeveral proportions. As for example: Let us ſuppoſe that Gold weigheth twenty times more than VVater, it is manifeſt by that which hath been ſpoken, that the Gold will weigh leſs in the VVater than in the Air by a twentieth part of its whole weight. Now, let us ſuppoſe that Silver, as being leſs Grave than Gold, weigheth 12 times more than VVater: this then, being weighed in the VVater, ſhall diminiſh in Gravity the twelfth part of its whole weight. Therefore the Gravity of Gold in the VVater decreaſeth leſs than that of Silver; for that diminiſheth a twentieth part, and this a twelfth. If therefore in an exquiſite Ballance we ſhall hang a Metal at the one Arm, and at the other a Counterpoiſe that weigheth equally with the ſaid Metal in the VVater, leaving the Counterpoiſe in the Air, to the end that it may equivalate and compenſate the Metal, it will be neceſſary to hang it nearer the Perpendicular or Cock. As for example, Let the Ballance be A B, its Perpendicular C, and let a Maſs of ſome Metal be ſuſpended at B, counterpoiſed by



the Weight D: putting the Weight B into the Water, the Weight D in A would weigh more: therefore that they may weigh

weigh equally it would be neceſſary to hang it nearer to the Perpendicular C, as *v. gr.* in E: and look how many times the Diſtance C A ſhall contain A E, ſo many times ſhall the Metal weigh more than the Water. Let us therefore ſuppoſe that the VVeight in B be Gold, and that weighed in the VVater it withdraws the Counterpoiſe D into E; and then doing the ſame with pure Silver, let us ſuppoſe that its Counterpoiſe, when afterwards it is weighed in the VVater, returneth to F: which point ſhall be nearer to the point C, as Experience ſheweth, becauſe the Silver is leſs grave than the Gold: And the Diſtance that is between A and F ſhall have the ſame Difference with the Diſtance A E, that the Gravity of the Gold hath with that of the Silver. But if we have a Mixture of Gold and Silver, it is clear, that by reaſon it participates of Silver, it ſhall weigh leſs than the pure Gold; and by reaſon it participates of Gold, it ſhall weigh more than the pure Silver: and therefore being weighed in the Air, and deſiring that the ſame Counterpoiſe ſhould counterpoiſe it, when that Mixture ſhall be put into the VVater it will be neceſſary to draw the ſaid Counterpoiſe more towards the Perpendicular C, than the point E is, which is the term of the Gold; and more from C than F is, which is the term of the pure Silver; Therefore it ſhall fall between the points E and F: And the proportion into which the Diſtance E F ſhall be divided, ſhall exactly give the proportion of the two Metals which compound that Mixture. As for example: Let us ſuppoſe the Mixture of Gold and Silver to be in B, counterpoiſed in the Air by D, which Counterpoiſe when the Compound Metal is put into the VVater returneth into G: I ſay now, that the Gold and the Silver which compound this Mixture are to one another in the ſame proportion, as the Diſtance F G is to the Diſtance G E.



But you muſt know that the Diſtance G F terminated in the mark of the Silver, ſhall denote unto us the quantity of the Gold, and the Diſtance G E, terminated in the mark of the Gold, ſhall ſhew us the quantity of the Silver: inſomuch that if F G ſhall prove double to G E, then that Mixture ſhall be two parts Gold, and one part Silver: and in the ſame method proceeding in the examination of other Mixtures, one ſhall exactly find the quantity of the ſimple Metals.

To compoſe the Ballance, therefore, take a Rod at leaſt a yard long, (and the longer it is, the exacter the Inſtrument ſhall be) and divide it in the midſt, where place the Perpendicular: then adjuſt the Arms that they may ſtand in *Equilibrium*, by filing or

ſhaving that leſs which weigheth moſt; and upon one of the Arms note the terms to which the Counterpoiſes of ſimple Metals return when they ſhall be weighed in the Water: taking care to weigh the pureſt Metals that can be found. This being done, it remaineth that we find out a way, how we may with facility diſcover the proportion, according to which, the Diſtances between the terms of the ſimple and pure Metals are divided by the Marks of the Mixt Metals: Which ſhall be effected in this manner.

We are to have two very ſmall Wires drawn thorow the ſame drawing-Iron, one of Steel, the other of Braſs, and above the terms of the ſimple Metals we muſt wind the Steel Wyer; as for example: above the point E, the term of the pure Gold, we are to wind the Steel VVyer, and under it the other Braſs VVyre, and having made ten folds of the Steel VVyer, we muſt make ten more with that of Braſs, and thus we are to continue to do with ten of Steel, and ten of Braſs, untill that the whole Space between the points E and F, the terms of the pure Metals, be full; cauſing thoſe two terms to be alwaies viſible and perſpicuous: and thus the Diſtance E F ſhall be divided into many equal parts, and numbred by ten and ten. And if at any time we would know the proportion that is between F G and G E, we muſt count the Wyers F G, and the Wyers G E: and finding the Wyers F G to be, for example, 40, and the Wyers G E, 21: we will ſay that there is in the mixt Metal 40 parts of Gold, and 21 of Silver. But here you muſt note, that there is ſome difficulty in the counting, for thoſe Wyers being very ſmall, as it is requiſite for exactneſs ſake, it is not poſſible with the eye to tell them, becauſe the ſmalneſs of the Spaces dazleth & confoundeth the Sight. Therefore to number them with facility, take a Bodkin as ſharp as a Needle and ſet it into an handle, or a very fine pointed Pen-knife, with which we may eaſily run over all the ſaid Wyers, and this way partly by help of hearing, partly by the impediments the hand ſhall feel at every VVyer, thoſe VVyers ſhall be counted; the number of which, as I ſaid before, ſhall give us the exact quantity of the ſimple Metals, of which the Mixt-Metal is compounded: taking notice that the Simple anſwer alternately to the Diſtances. As for example, in a Mixture of Gold and Silver, the VVyers that ſhall be towards the term of Gold ſhall ſhew us the quantity of the Silver: And the ſame is to be underſtood of other Metals.

Annotations

Annotations of *Dominico Mantovani* upon the Ballance of *Signore Galileo Galilei*.

First, I conceive that the difficulty of Numbring the VVyres is removed by wrapping about the Ballance ten of Steel, and then ten of Brass, which being divided by tens, there only remains that tenth part to be numbred, in which the term of the Mixt Metal falleth. For although *Signore Galileo*, who is Author of this Invention, makes mention of two VVyres, one of Steel, the other of Brass, yet he doth not say, that we are to take * ten of the one, and ten of the other: which it may be hapneth by the negligence of him that hath transcribed it; although I must confess that the Copy which came to my hands was of his own writing.

* *Galileus* saith it expresly in this Copy which I follow, but might omit it in the Copy which came to the hands of *Mantovani*.

Secondly, it is supposed in this Problem that the Composition of two Metals do retain the same proportion of Mass in the Mixture as the two Simple Metals, of which it is compounded, had at first. I mean, that the Simple Metals retain and keep in the Composition (after that they are incorporated and commixed) the same proportion in Mass that the Simple Metals had when they were separated: VVhich in the Case of *Signore Galileo*, touching the Commixtion of Gold and Silver, I do neither deny, nor particularly confess. But if one would, for example, unite 101 pounds of Copper with 21 pounds of Tin, to make thereof 120 pounds of Bell-Metal, (I abate two pounds, supposed to be wasted in the Melting) I do think that 120 pounds of Compound Metal will have a less Bulk than the 100 pounds of pure Copper, and the 20 pounds of Tin unmixt, that is, before they were incorporated and melted into one Mass, and that the Composition is more grave *in Specie* than the single Copper, and the single Brass: and in the Case of *Signore Galileo* the Composition of Gold and Silver is supposed to be lighter *in Specie* than the pure Gold, and heavier *in Specie* than the pure Silver. Of which it would be easie to make some such like experiment, melting together, *v. gr.* 10 pounds of Lead with 5 pounds of Tin, and observing whether those 15 pounds, or whatever the Mixture maketh, do give the difference betwixt the weight in the Water to the weight in the Air, in the proportion that the 15 pounds of the two Metals dis-united gave before: I do not say, the same difference, because I pre-suppose that they will waste in melting down, and that the Compound will be less than 15 pounds, therefore I say in proportion.

Thirdly, He doth also suppose, that one ought to take the

Simple Metals, that is, the Gold and the Silver, each of the same weight as the Mixture, although he doth not say so; which may be collected in that he marketh the ballance only betwixt the Terms of the Gold and the Silver, which is the cause of the great facility in resolving the Problem by only counting the Wyers.

One might take the pure Gold, and pure Silver of the same weight, in respect of one another, but yet different from the weight of the Mixture, that is, either more or less grave than the Mixt Metal: and being equal in weight to one another they might shew the proportion of the Mass of the Gold to that of the Silver; but yet with this difference, that the more grave will shew the said proportion more exactly than the small and less grave. But the Simple and pure Metals not being of the same weight as the Compound, it will be necessary, having found the proportion of the Mass of the Gold to that of the Silver; to find by numbers proportionally the exact quantity of each of the two Metals compounding the Mixture.

A man may likewise use the quantity of the simple Metals according to necessity and convenience, although of different Weights, both as to each other, and to the Mixture, provided that each of them be pure in its kind: but then we must afterwards by numbers find the proportion of the Masses of the two Simple ones of equal weight (which is soon done, taking them of equal weight as was said before) and then according to this proportion to find, by means of the Weight, and of the Mass of the Compound Metal, the distinct quantity of each of the two Simple ones that make the Composition: of each of which Cases examples might be given. But to conclude, if the pure Gold, and pure Silver, and the Mixt Metal should be of equal Mass, they would be unequal in Weight, and it would not need to weigh them in the Water, for being of equal Bulk, the differences of their Weights in the Air and in the Water would be also equal: for the difference of the weight of any Body in the Air to its weight in the Water, is alwaies equal to the Weight of so much VVater as equalleth the same Body in Mass, by *Archimedes* his fifth Proposition, *De ijs quæ vehuntur in aqua.*

And last of all, the Simple and pure Metals may have the same proportion in Gravity, mutually or reciprocally, as their Bodies have in Bulk: In which case, as well the Mass, found by help of the weight in VVater, or by any other meanes, as their VVeight in the Air shall shew the proportion of their Specifical Gravities; as their VVeights in the VVater do when their VVeights in the Air are equal; but yet alternately weighed: that is to say, the Specifical Gravity of the Gold shall have such proportion to the

Specifical

Specifical Gravity of the Silver, as the Mass of the Silver hath to the Mass of the Gold; that is, as the difference betwixt the VVeight in VVater and VVeight in Air of the Silver, hath to the difference betwixt the VVeight in VVater and VVeight in Air of the Gold.

VVith this same Ballance one may with facility measure the Mass or Magnitude of any Body, in any manner whatsoever Irregular in manner following, namely:

VVe will have at hand a Solid Body of a substance more grave *in Specie* than the VVater; as for instance of Lead; or if it were of VVood, or other matter more light *in Specie* than the VVater, it may be made heavier by fastning unto it Lead, or some other thing that makes it sink in the VVater, and let us take some known Measure, and with it measure the Irregular Solid; as for instance, the Roman Palm, the Geometrical Foot, or any other known measure, or part of the same, as the half Foot, the quarter of a Foot, or any such like part known; then let it be weighed in the Air, and suppose that it weigh 10 pounds; let the same Measure be weighed in the Air, and suppose that it weigh 8 pounds: and substract 8 pounds, the VVeight in the VVater, from 10 pounds, the Weight in the Air, and there remaineth 2 pounds for the Weight of a Body of Water equal in Magnitude to the Measure known. Now, if we would measure a Statue of Marble, let it be weighed first in the Air, and then in the Water, and substract the Weight in the *W*ater from the *W*eight in the Air, and the remainder shall be the weight of so much *W*ater as equalleth the Statue in Mass; which being divided by the difference betwixt the *W*eight in *W*ater and the *W*eight in Air of the Measure known, the Quotient will give how many times the Statue containeth the same given Measure. As for example; if the Statue in Air weigh 100 pounds, and in the *W*ater 80 pounds, 80 pounds being substracted from 100 there resteth 20 pounds for the *W*eight of so much *W*ater in Mass as equalleth the Statue. But because the difference betwixt the *W*eight in *W*ater, and the *W*eight in Air equal in Magnitude to the Measure known, was supposed to be 2 pounds; divide 18 pounds by two pounds, and the Quotient is 9, for the number of times that the proposed Statue containeth the given Measure. The same Method may be observed, if it were required, to measure a Statue, or other Mass of any kind of Metal: only it must be advertised, that all the holes must be stopt, that the *W*ater may not enter into the Body of the Statue: but he that desireth only the Solid content of the Metal of the said Statue must open the holes, and with Tunnels fill the whole cavity of the Statue with *W*ater. And if the Statue were of a Substance lighter *in Specie* than the *W*ater; as, for example, of

*W*ax,

Wax, it will be requisite to add unto the Statue some Counterpoise, that maketh it sink in the *Water*, and then to measure the Counterpoise, as above, and to substract its measure from the Compound Body, and there will remain the Measure of the Statue of *Wax*. And lastly, to make use of the said Ballance, instead of seeking the numbers of the pounds of the Differences of the *Weights* of the Measure known, and of the Solids to be measured in *Water*, and in Air, we may count the *Wyers* of the Arm of the Ballance, which being very small will give the Measure exactly.

FINIS.

DISCOURSES
OF THE
MECHANICKS:
A MANUSCRIPT of
Monsieur Des-Cartes.

The Explication.

Of Engines, by help of which we may raise a very great weight with small strength.

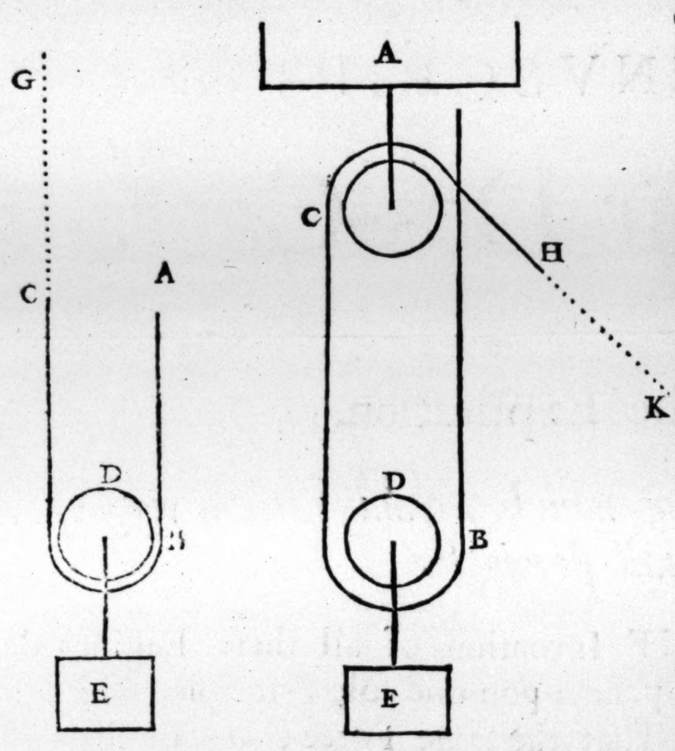
THE Invention of all these Engines depends upon one sole Principle, which is, That the same Force that can lift up a Weight, for example, of 100 pounds to the height of one foot, can lift up one of 200 pounds to the height of half a foot, or one of 400 pounds to the height of a fourth part of a foot, and so of the rest, be there never so much applyed to it: and this Principle cannot be denied if we consider, that the Effect ought to be proportioned to the Action that is necessary for the production of it; so that, if it be necessary to employ an Action by which we may raise a Weight of 100 pounds to the height of two foot, for to raise one such to the height of one foot only this same ought to weigh 200 pounds: for its the same thing to raise 100 pounds to the height of one foot, and again yet another 100 pounds to the height of one foot, as to raise one of 200 pounds to the height of one foot, and the same, also, as to raise 100 pounds to the height of two feet.

Now, the Engines which serve to make this Application of a Force which acteth at a great Space upon a *Weight* which it causeth

seth to be raised by a lesser, are the Pulley, the Inclined Plane, the Wedg, the Capsten, or Wheel, the Screw, the Leaver, and some others, for if we will not apply or compare them one to another, we cannot well number more, and if we will apply them we need not instance in so many.

The PULLEY, *Trochlea.*

LEt A B C be a Chord put about the Pulley D, to which let the Weight E be fastned; and first, supposing that two men sustain or pull up equally each of them one of the ends of the said Chord: it is manifest, that if the Weight weigheth 200 pounds, each of those men shal employ but the half thereof, that is to say, the Force that is requisite for sustaining or raising of 100 pounds, for each of them shal bear but the half of it.



Afterwards, let us suppose that A, one of the ends of this Chord, being made fast to some Nail, the other C be again sustained by a Man; and it is manifest, that this Man in C, needs not (no more than before) for the sustaining the Weight E, more Force than is requisite for the sustaining of 100 pounds: because the Nail at A doth the same Office as the Man which we supposed there before. In fine, let us suppose that this Man in C do pull the Chord to make the Weight E to rise, and it is manifest, that if he there employeth the Force which is requisite for the raising of 100 pounds to the height of two feet, he shall raise this Weight E of 200 pounds to the height of one foot: for the Chord A B C being doubled, as it is, it must be pull'd two feet by the end C, to make the Weight E rise as much, as if two men did draw it, the one by the end A, and the other by the end C, each of them the length of one foot only.

There is alwaies one thing that hinders the exactness of the Calculation, that is the ponderosity of the Chord or Pulley, and the difficulty that we meet with in making the Chord to slip, and in bearing it: but this is very small in comparison of that which

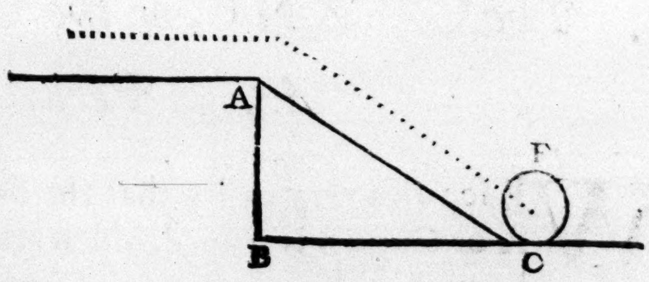
raiseth

raiseth it, and cannot be estimated save within a small matter.

Moreover, it is necessary to observe, that it is nothing but the redoubling of the Chord, and not the Pulley, that causeth this Force: for if we fasten yet another Pulley towards A, about which we pass the Chord A B C H, there will be required no less Force to draw H towards K, and so to lift up the Weight E, than there was before to draw C towards G. But if to these two Pulleys we add yet another towards D, to which we fasten the Weight, and in which we make the Chord to run or slip, just as we did in the first, then we shall need no more Force to lift up this Weight of 200 pounds than to lift up 50 pounds without the Pulley: because that in drawing four feet of Chord we lift it up but one foot. And so in multiplying of the Pulleys one may raise the greatest Weights with the least Forces. It is requisite also to observe, that a little more Force is alwaies necessary for the raising of a Weight than for the sustaining of it: which is the reason why I have spoken here distinctly of the one and of the other.

The Inclined PLANE.

IF not having more Force than sufficeth to raise 100 pounds, one would nevertheless raise this Body F, that weigheth 200 pounds, to the height of the Line B A, there needs no more but to draw or rowl it along the Inclined Plane C A, which I suppose to be twice as long as the Line A B, for by this means, for to make it arrive at the point A, we must there employ the Force that is necessary for the raising 100 pounds twice as high, and the more inclined this Plane shall be made, so much the less Force shall there need to raise the Weight F. But yet there is to be rebated from this Calculation the difficulty that there is in moving the Body F, along the Plane A C, if that Plane were laid down upon the Line B C, all the parts of which I suppose to be equidistant from the Center of the Earth.



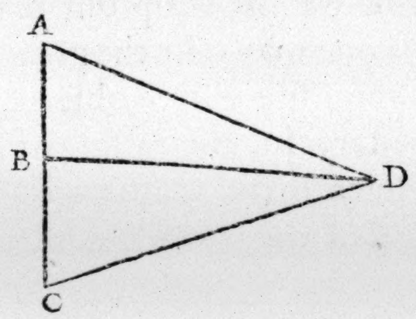
It is true, that this impediment being so much less as the Plane is more united, more hard, more even, and more polite; it cannot likewise be estimated but by guess, and it is not very considerable.

We need not neither much to regard that the Line B C being a part of a Circle that hath the same Center with the Earth, the Plane A C ought to be (though but very little) curved, and to have the Figure of part of a Spiral, described between two Circles,

which likewise have for their Center that of the Earth, for that it is not any way sensible.

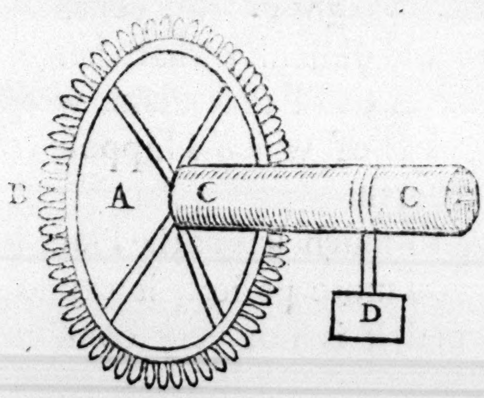
The WEDGE, *Cuneus.*

THe Force of the Wedge A B C D is easily understood after that which hath been spoken above of the Inclined Plane, for the Force wherewith we strike downwards acts as if it were to make it move according to the Line B D; and the Wood, or other thing and Body that it cleaveth, openeth not, or the Weight that it raiseth doth not rise, save only according to the Line A C, insomuch that the Force, wherewith one driveth or striketh this Wedge, ought to have the same Proportion to the Resistance of this Wood or Weight, that A C hath to A B. Or else again, to be exact, it would be convenient that B D were a part of a Circle, and A D and C D two portions of Spirals that had the same Center with the Earth, and that the Wedge were of a Matter so perfectly hard and polite, and of so small weight, as that any little Force would suffice to move it.



The CRANE, *or the* CAPSTEN, *Axis in Peritrochio.*

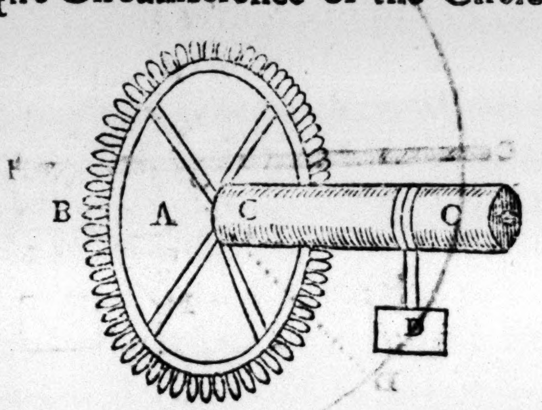
WE see also very easily, that the Force wherewith the Wheel A or Cogg B is turned, which make the Axis or Cylinder C to move, about which a Chord is rolled, to which the Weight D, which we would raise, is fastned, ought to have the same proportion to the said Weight, as the Circumference of the Cylinder hath to the Circumference of a Circle which that Force describeth, or that the Diameter of the one hath unto the Diameter of the other; for that the Circumferences have the same proportion as the Diameters: insomuch that the Cylinder C, having no more but one foot in Diameter, if the Wheel A B be six feet in its Diameter, and the VVeight D do weigh 600 pounds, it shall suffice that the Force in B shall be capable to raise 100 pounds, and so of others. One may also



also instead of the Chord that rolleth about the Cylinder C, place there a small VVheel with teeth or Coggs, that may turn another greater, and by that means multiply the power of the Force as much as one shall please, without having any thing to deduct of the same, save only the difficulty of moving the Machine, as in the others.

The SCREW, *Cochlea.*

VVHen once the Force of the Capsten and of the Inclined Plane is understood, that of the Screw is easie to be computed, for it is composed only of a Plane much inclined, which windeth about a Cylinder: and if this Plane be in such manner Inclined, as that the Cylinder ought to make *v. gr.* ten turns to advance forwards the length of a foot in the Screw, and that the bigness of the Circumference of the Circle which the Force that turneth it about doth describe be of ten feet; forasmuch as ten times ten are one hundred, one Man alone shall be able to press as strongly with this Instrument, or Screw, as one hundred without it, provided alwaies, that we rebate the Force that is required to the turning of it.



Now I speak here of Pressing rather than of Raising, or Removing, in regard that it is about this most commonly that the Screw is employed, but when we would make use of it for the raising of VVeights, instead of making it to advance into a Female Screw, we joyn or apply unto it a VVheel of many Coggs, in such sort made, that if *v. gr.* this *Wheel* have thirty Coggs, whilst the Screw maketh one entire turn, it shall not cause the *Wheel* to make more than the thirtieth part of a turn, and if the *Weight* be fastned to a Chord that rowling about the Axis of this *Wheel* shall raise it but one foot in the time that the *Wheel* makes one entire revolution, and that the greatness of the Circumference of the Circle that is described by the Force that turneth the Screw about be also of ten feet, by reason that 10 times 30 make 300, one single Man shall be able to raise a *Weight* of that bigness with this Instrument, which is called the Perpetual Screw, as would require 300 men without it.

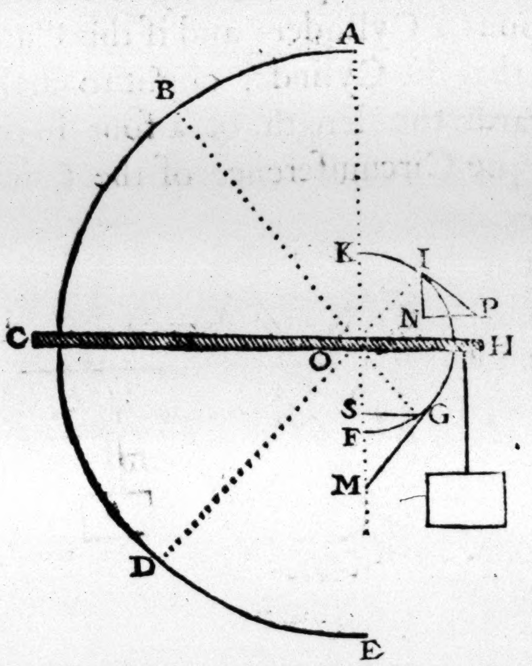
Provided, as before, that we thence deduct the difficulty that we meet with in turning of it, which is not properly caused by the Ponderosity of the *Weight*, but by the Force or Matter of the In-

ſtrument: which difficulty is more ſenſible in it than in thoſe afore-going, foraſmuch as it hath greater Force.

The LEAVER, *Vectis.*

I Have deferred to ſpeak of the Leaver until the laſt, in regard that it is of all Engines for raiſing of Weights, the moſt difficult to be explained.

Let us ſuppoſe that C H is a Leaver, in ſuch manner ſupported at the point O, (by means of an Iron Pin that paſſeth thorow it acroſs, or otherwiſe) that it may turn about on this point O, its part C deſcribing the Semicircle A B C D E, and its part H the Semicircle F G H I K; and that the Weight which we would raiſe by help of it were in H, and the Force in C, the Line C O being ſuppoſed triple of O H. Then let us conſider that in the Time whilſt the Force that moveth this Leaver deſcribeth the whole Semicircle A B C D E, and acteth according to the Line A B C D E, although that the Weight deſcribeth likewiſe the Semicircle F G H I K, yet it is not raiſed to the length of this curved Line F G H I K, but only to that of the Line F O K; inſomuch that the Proportion that the Force which moveth this Weight ought to have to its Ponderoſity ought not to be meaſured by that which is between the two Diameters of theſe Circles, or between their two Circumferences, as it hath been ſaid above of the Wheel, but rather by that which is betwixt the Circumference of the greater, and the Diameter of the leſſer. Furthermore let us conſider, that there is a neceſſity that this Force needeth not to be ſo great, at ſuch time as it is near to A, or near to E, for the turning of the Leaver, as then when it is near to B, or to D; nor ſo great when it is near to B or D, as then when it is near to C: of which the reaſon is, that the Weights do there mount leſs: as it is eaſie to underſtand, if having ſuppoſed that the Line C O H is parallel to the Horizon, and that A O F cutteth it at Right Angles, we take the point G equidiſtant from the points F and H, and the point B equidiſtant from A and C; and that having drawn G S perpendicular to F O, we obſerve that the Line F S (which ſheweth how much the Weight mounteth in the Time that the Force operates along



the

the Line AB) is much lesser than the Line SO, which sheweth how much it mounteth in the Time that the Force opperates along the Line BC.

And to measure exactly what his Force ought to be in each Point of the curved Line A B C D E, it is requisite to know that it operates there just in the same manner as if it drew the Weight along a Plane Circularly Inclined, and that the Inclination of each of the Points of this circular Plane were to be measured by that of the right Line that toucheth the Circle in this Point. As for example, when the Force is at the Point B, for to find the proportion that it ought to have with the ponderosity of the Weight which is at that time at the Point G, it is necessary to draw the Contingent Line G M, and to account that the ponderosity of the Weight is to the Force which is required to draw it along this Plane, and consequently to raise it, according to the Circle F G H, as the Line G M is to S M. Again, for as much as B O is triple of O G, the Force in B needs to be to the Weight in G but as the third part of the Line S M is unto the whole Line G M. In the self same manner, when the Force is at the Point D, to know how much the Weight weigheth at I, it is necessary to draw the Contingent Line betwixt I and P, and the right Line I N perpendicular upon the Horizon, and from the Point P taken at discretion in the Line I P, provided that it be below the Point I, you must draw P N parallel to the same Horizon, to the end you may have the proportion that is betwixt the Line I P and the third part of the Line I N, for that which betwixt the ponderosity of the Weight, and the Force that ought to be at the Point D for the moving of it: and so of others Where, nevertheless, you must except the Point H, at which the Contingent Line being perpendicular upon the Horizon, the Weight can be no other than triple the Force which ought to be in C for the moving of it: in the Points F and K, at which the Contingent Line being parallel unto the Horizon it self, the least Force that one can assign is sufficient to move the Weight. Moreover, that you may be perfectly exact, you must observe that the Lines S M and P N ought to be parts of a Circle that have for their Center that of the Earth; and G M and I P parts of Spirals drawn between two such Circles; and, lastly, that the right Lines S M and I N both tending towards the Center of the Earth are not exactly Parallels: and furthermore, that the Point H where I suppose the Contingent Line to be perpendicular unto the Horizon ought to be some small matter nearer to the Point F than to K, at the which F and K the Contingent Lines are Parallels unto the said Horizon.

This done, we may easily resolve all the difficulties of the Balance, and shew, That then when it is most exact, and for instance,

supposing

supposing it's Centre at O by which it is sustained to be no more but an indivisible Point, like as I have supposed here for the Leaver, if the Armes be declined one way or the other, that which shall be the lowermost ought evermore to be adjudged the heavier; so that the Centre of Gravity is not fixed and immoveable in each several Body, as the Ancients have supposed, which no person, that I know of, hath hitherto observed.

But these last Considerations are of no moment in Practice, and it would be good for those who set themselves to invent new Machines, that they knew nothing more of this businesse than this little which I have now writ thereof, for then they would not be in danger of deceiving themselves in their Computation, as they frequently do in supposing other Principles.

FINIS.

A LETTER OF Monsieur Des-Cartes TO THE REVEREND FATHER MARIN MERSENNE.

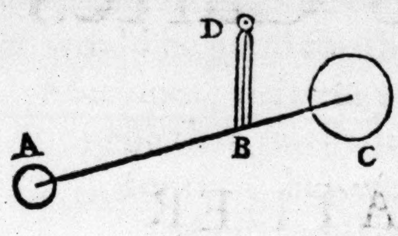
Reverend Father,

I Did think to have deferred writing unto you yet eight or fifteen dayes, to the end I might not trouble you too often with my Letters, but I have received yours of the first of *Sept.* which giveth me to understand that it is an hard matter to admit the Principle which I have supposed in my Examination of the Geostatick Question, and in regard that if it be not true, all the rest that I have inferred from it would be yet lesse true: I would not one onely day defer sending you a more particular Explication. It is requisite above all things to consider that I did speak of the Force that serveth to raise a Weight to some heighth, the which Force hath evermore two Dimensions, and not of that which serveth in each point to sustain it, which hath never more than one Dimension, insomuch that these two Forces differ as much the one from the other, as a Superficies differs from a Line, for the same Force which a Nail ought to have for the sustaining of a Weight of 100 pound one moment of time, doth also suffice for to sustain it the space of a year, provided that it do not diminish, but the same Quantity of this Force which serveth to raise the Weight to the heighth of one foot, sufficeth not (*eadem numero*) to raise it two feet; and it is not more manifest that two and two make four, than it's manifest that we are to employ double as much therein.

Now, forasmuch as that this is nothing but the same thing that I have supposed for a Principle, I cannot guesse on what the Scruple should be grounded that men make of receiving it; but I shall in this

this place ſpeak of all ſuch as I ſuſpect, which for the moſt part ariſe onely from this, that men are before-hand over-knowing in the Mechanicks; that is to ſay, that they are pre-occupied with Principles that others prove touching theſe matters, which not being abſolutely true, they deceive the more, the more true they ſeem to be.

The firſt thing wherewith a man may be pre-occupied in this buſineſſe, is, that they many times confound the Conſideration of Space, with that of Time, or of the Velocity, ſo that, for Example, in the Leaver, or (which is the ſame) the Ballance A B C D having ſuppoſed that the Arm A B is double to B C, and the VVeight in C double to the Weight in A, and alſo that they are in *Equilibrium*, inſtead of ſaying, that that which cauſeth this *Equilibrium* is, that if the Weight C did ſuſtain, or was raiſed up by the Weight A, it did not paſſe more than half ſo much Space as it, they ſay that it did move ſlower by the half: which is a fault ſo much the more prejudicial, in that it is very difficult to be known: for it is not the difference of the Velocity that is the cauſe why theſe Weights are to be one double to the other, but the difference of the Space, as appeareth by this, that to raiſe, for Example, the Weight F with the hand unto G, it is not neceſſary to employ a Force that is preciſely double to that which one ſhould have therein employed the firſt bout, to raiſe it twice as quickly, but it is requiſite to employ therein either more or leſs than the double, according to the different proportion that this Velocity may have unto the Cauſes that reſiſt it.



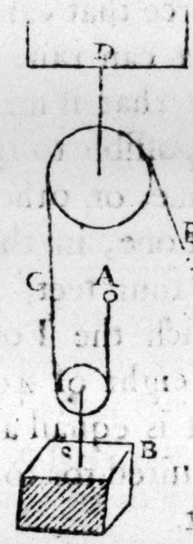
Inſtead of requiring a Force juſt double for the raiſing of it with the ſame Velocity twice as high, unto H, I ſay that it is juſt double in counting (as two and two make four) that one and one make two, for it is requiſite to employ a certain quantity of this Force to raiſe the Weight from F to G, and again alſo, as much more of the ſame Force to raiſe it from G to H.

For if I had had a mind to have joyned the Conſideration of the Velocity with that of the Space, it had been neceſſary to have aſſigned three Dimenſions to the Force, whereas I have aſſigned it no more but two, on purpoſe to exclude it. And if I have teſtified that there is ſo little of worth in any part of this ſmall Tract of the Staticks, yet I deſire that men ſhould know, that there is more in this alone than in all the reſt: for it's impoſſible to ſay any thing that is good and ſolid touching Velocity, without having rightly explained what we are to underſtand by Gravity, as alſo the whole Syſteme of the World. Now becauſe I would not undertake it,

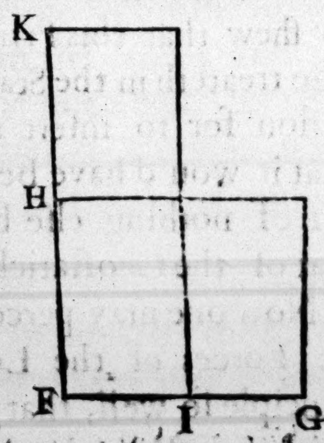
I have

I have thought good to omit this Consideration, and in this manner to single out these others that I could explain without it: for though there be no Motion but hath some Velocity, neverthelesse it is onely the Augmentations and Diminutions of this Velocity that are considerable. And now that speaking of the Motion of a Body, we suppose that it is made according to the Velocity which is most naturall to it, which is the same as if we did not consider it at all.

The other reason that may have hindred men from rightly understanding my Principle is, that they have thought that they could demonstrate without it some of those things which I demonstrate not without it: As, for example, touching the Pulley A B C, they have thought that it was enough to know that the Nail in A did sustain the half of the Weight B; to conclude that the Hand in C had need but of half so much Force to sustain or raise the Weight, thus wound about the Pulley, as it would need for to sustain or raise it without it. But howbeit that this explaineth very well, how the application of the Force at C is made unto a Weight double to that which it could raise without a Pulley, and that I my self did make use thereof, yet I deny that this is simply, because that that the Nail A sustaineth one part of the Weight B, that the Force in C, which sustaineth it, might be less than if it had been so sustained.



For if that had been true, the Rope C E being wound about the Pulley D, the Force in E might by the same reason be less than the Force in C: for that the Nail A doth not sustain the Weight less than it did before, and that there is also another Nail that sustains it, to wit, that to wich the Pulley D is fastned. Thus therefore, that we may not be mistaken in this, that the Nail A sustaineth the half of the Weight B, we ought to conclude no more but this, that by this application the one of the Dimensions of the Force that ought to be in C to raise up this Weight is diminished the one half; and that the other, of consequence, becometh double, in such sort that if the Line F G represent the Force that is required for the sustaining the Weight B in a point, without the help of any Machine, and the Quadrangle G H that which is required for the raising of it to the height of a foot, the support of the Nail A diminisheth the Dimension which is represented by the Line F G the one half, and the redoubling of the Rope A B C maketh the other Dimension to



double, which is represented by the Line F H; and so the Force that ought to be in C for the raising of the Weight B to the height of one foot is represented by the Quadrangle I K; and,as we know in Geometry, that a Line being added to, or taken from a Superficies, neither augmenteth, nor diminisheth it in the least, so the Force wherewith the Nail A sustains the VVeight B, having but one sole Dimension, cannot cause that the Force in C, considered according to its two Dimensions, ought to be less for the raising in like manner the VVeight E, than for the raising it without any Pulley.

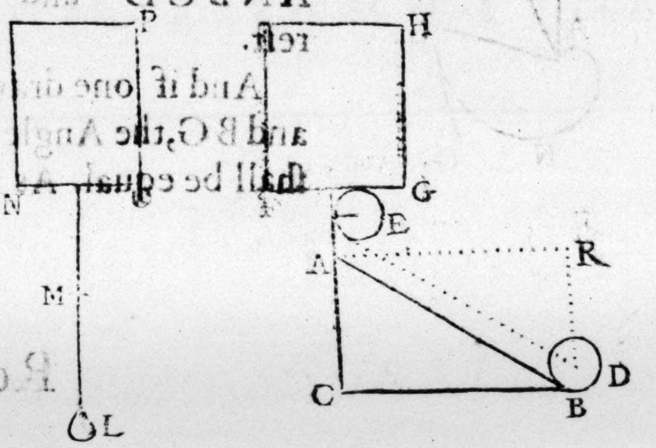
The third thing which may make men imagine some Obscurity in my Principle is, that they, it may be, have not had regard to all the words by which I explain it; for I do not say simply that the Force that can raise a VVeight of 50 pounds to the height of four feet can raise one of 200 pounds to the height of one foot; but I say that it may do it, if so be that it be applyed to it: now it is impossible to apply the same thereto, but by the means of some Machine, or other Invention that shall cause this VVeight to ascend but one, in the time whilst the Force passeth the whole length of four feet, and so that it do transform the Quandrangle, by which the Force is represented that is required to raise this VVeight of 400 pounds to the height of one foot into another that is equall and like to that which represents the Force that is required for to raise a VVeight of 50 pounds to the height of four feet.

In fine, it may be that men may have thought the worse of my Principle,because they have imagined that I have alledged the Examples of the Pulley, of the Inclined Plane,and of the Leaver, to the end that I might better perswade the truth thereof, as if it had been dubious,or else that I had so ill discoursed as to offer to assume from thence a Principle, which ought of it self to be so clear, as not to need any proof by things that are so difficult to comprehend as that; it may be, they have never been well demonstrated by any man: but neither have I made use of them, save only with a design to shew that this Principle extends it self to all matters of which one treateth in the Staticks: or, rather, I have made use of this occasion for to insert them into my Treatise, for that I conceived that it would have been too dry and barren if I had therein spoken of nothing else but of this Question, that is of no use, as of that of the Geostaticks, which I purposed to examine.

Now one may perceive, by what hath already been said, how the Forces of the Leaver and Pulley are demonstrated by my Principle so well, that there only remains the Inclined Plane, of which you shall clearly see the Demonstration by this Figure; in which C F represents the first Dimension of the Force that the

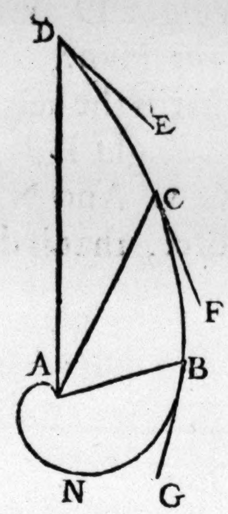
Rectangle

Rectangle F H describeth whilst it draweth the Weight D along the Plane B A, by the means of a Chord parallel to this Plane, and passing about the Pulley E, in such sort, that H G, that is the height of this Rectangle, is equal to B A, along which the Weight D is to move, whilst it mounteth to the height of the Line C A. And N O represents the first Dimension of such another Force, that is described by the Rectangle N P, in the time that it is raising the Weight L to M. And I suppose that L M is equal to B A, or double to C A; and that N O is to F G, as O P is to G H. This done, I consider that at such time as the Weight D is moved from B towards A, one may imagine its Motion to be composed of two others, of which the one carrieth it from B R towards C A, (to which operation there is no Force required, as all those suppose who treat of the Mechanicks) and the other raiseth it from B C towards R A, for which alone the Force is required: insomuch that it needs neither more nor less Force to move it along the Inclined Plane B A, than along the Perpendicular C A. For I suppose that the unevennesses, &c. of the Plane do not at all hinder it, like as it is alwaies supposed in treating of this matter.



So then the whole Force F H is employed only about the raising of D to the height of C A: and forasmuch as it is exactly equal to the Force N P, that is required for the raising of L to the Height of L M, double to C A, I conclude by my Principle that the Weight D is double to the Weight L. For in regard that it is necessary to employ as much Force for the one as for the other, there is as much to be raised in the one as in the other; and no more knowledge is required than to count unto two for the knowing that it is alike facile to raise 200 pounds from C to A, as to raise 100 pounds from L to M: since that L M is double to C A.

You tell me, moreover, that I ought more particularly to explain the nature of the Spiral Line that representeth the Plane equally enclined, which hath many qualities that render it sufficiently knowable.



For if A be the Center of the Earth, and A N B C D the Spiral Line, having drawn the Right-Lines A B, A D, and the like, there is the same proportion betwixt the Curved Line A N B and the Right Line A B, as is betwixt the Curved Line A N B C, and the Right Line A C; or betwixt A N B C D and A D: and so of the rest.

And if one draw the Tangents D E, C F, and B G, the Angles A D E, A C F, A B G, &c. shall be equal. As for the rest I will, &c.---

Reverend Father,

Your very humble Servant

DES-CARTES.

A LETTER OF Monsieur de Robberval TO Monsieur de Fermates,

Counsellour of *THOULOUSE*,

Containing certain Propositions in the MECHANICKS.

MONSIEUR,

I Have, according to my promise, sent you the Demonstration of the Fundamental Proposition of our Mechanicks, in which I follow the common method of explaining, in the first place, the Definitions and Principles of which we make use.

We in general call that Quality a Force or Power, by means of which any thing whatever doth tend or aspire into another place than that in which it is, be it downwards, upwards, or side waies, whether this Quality naturally belongeth to the Body, or be communicated to it from without. From which definition it followeth, that all Weights are a species of Force, in regard that it is a Quality, by means whereof Bodies do tend downwards. VVe often also assign the name of Force to that very thing to which the Force belongeth, as a ponderous Body is called a VVeight, but with this pre-caution, that this is in reference to the true Force, the which augmenting or diminishing shall be called a greater or lesser Force, albeit that the thing to which it belongeth do remain alwaies the same.

If a Force be suspended or fastned to a Flexible Line that is without Gravity, and that is made fast by one end unto some *Fulciment* or stay, in such sort as that it sustain the Force, drawing without

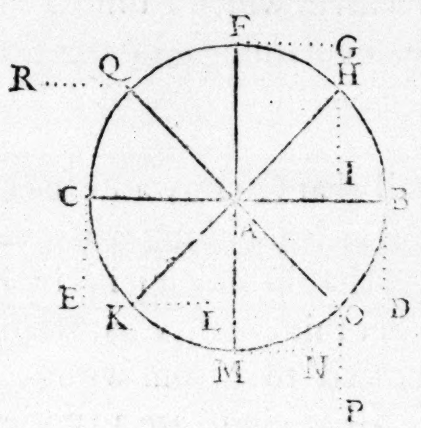
without impediment by this Line, the Force and the Line ſhall take ſome certain poſition in which they ſhall reſt, and the Line ſhall of neceſſity be ſtreight, let that Line be termed *the Pendant*, or *Line of Direction of the Force.* And let the Point by which it is faſtned to the Fulciment be called *the Point of Suſpenſion* : which may ſometimes be the Arm of a Leaver or Ballance ; and then let the Line drawn from the Center of the Fulciment of the Leaver or Ballance to the Point of Suſpenſion be named *the Diſtance* or *the Arm of the Force* : which we ſuppoſe to be a Line fixed, and conſidered without Gravity. Moreover, let the Angle comprehended betwixt the Arm of the Force and the Line of Direction be termed *the Angle of the Direction of the Force.*

AXIOM I.

AFter theſe Definitions we lay down for a Principle, that in the Leaver, and in the Ballance, Equal Forces drawing by Arms that are equal, and at equall Angles of Direction, do draw equally. And if in this Poſition they draw one againſt the other they ſhall make an *Equilibrium* : but if they draw together, or towards the ſame part, the Effect ſhall be double.

If the Forces being equal, and the Angles of Direction alſo equal, the Arms be unequal, the Force that ſhall be ſuſpended at the greater Arm ſhall work the greater Effect.

As in this Figure, the Center of the Ballance or Leaver being A,



if the Arms A B and A C are equal, as alſo the Angles A B D, and A C E, the equal Forces D and E ſhall draw equally, and make an *Equilibrium.* So likewiſe the Arm A F being equal to A B, the Angle A F G to the Angle A B D, and the Force G to D, theſe two Forces * G and D draw equally ; and in regard that they draw both one way, the Effect ſhall be double.

* In the M. S. Copy it is *C and D.*

In the ſame manner the Forces G and E ſhall make an *Equilibrium* ; as alſo I and L ſhall counterpoiſe, if (being equal) the Arms A K and A H, and the Angles A H T, and A K L be equal.

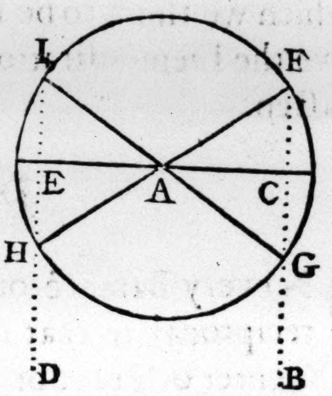
The ſame ſhall befall in the Forces P and R, if all things be diſpoſed as before. And in this caſe we make no other diſtinction betwixt Weights and other Forces ſave only this, that Weights all tend towards the Center of Grave Bodies, and Forces may be underſtood to tend all towards all parts of the Univerſe, with ſo much greater or leſſer *Impetus* than Weights. So that Weights and their

their parts do draw by Lines of Direction, which all concur in one and the same Point; and Forces and their parts may be understood to draw in such sort that all the Lines of Direction are parallel to each other.

AXIOM II.

IN the second place, we suppose that a Force and its Line of Direction abiding alwaies in the same position, as also the Center of the Ballance or Leaver, be the Arm what it will that is drawn from the Center of the Ballance to the Line of Direction, the Force drawing alwaies in the same fashion, will alwaies produce the same Effect.

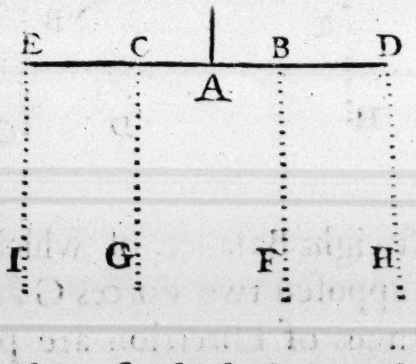
As, in this second Figure, the Center of the Ballance being A, the Force B, and the Line of Direction B *F* prolonged, as occasion shall require, in which the Arms A G, A C, and A *F* do determine, in this position let the Line B *F* be fastned to the Arm A *F*, or A C, or to another Arm drawn from the Center A to the Line of Direction * B *F*: we suppose that this *F*orce B shall alwaies work the same Effect upon the Ballance. And if drawing by the Arm A C it make an *Equilibrium* with the *F*orce D drawing by the *A*rm *A* E, when ever it shall draw by the *A*rms *A F*, or *A* G, it shall likewise make an *Equilibrium* with the *F*orce D drawing by the *A*rm *A* E. This Principle although it be not expresly found in *A*uthors, yet it is tacitly supposed by all those that have writ on this *A*rgument, and Experience constantly confirmeth it.



* In the Original it is writ, but by the mistake of the Transcriber, *a la ligne de direction* A F.

AXIOM III.

I*F* the Arms of a Ballance or Leaver are directly placed the one to the other, and that being equal they sustain equal *F*orces, of which the Angles of Direction are Right Angles, these *F*orces do alwaies weigh equally upon the Center of the Ballance, whether that they be near to the same Center, or far distant, or both conjoyned in the Center it self; as in this *F*igure the Ballance being E D, the Center A, the equal Arms A D and *A* E, let us sustain equal *F*orces H and I, of which the *A*ngles of



of Direction *A* D H and *A* E I are Right *A*ngles, we suppose that these two *F*orces I and H weigh alike upon the Center *A* as if they were nearer to the Center, at the equal Distances *A* B and A C, and we also suppose the same if these very *F*orces were suspended both together in *A*, the *A*ngles of Directions being still Right *A*ngles.

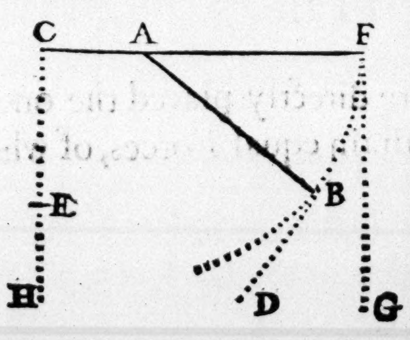
PROPOSITION I.

THese Principles agreed upon, we will easily demonstrate, in Imitation of *Archimedes*, that upon a straight Balance the *F*orces, of which and of all their parts the Lines of Direction are parallel to one another, and perpendicular to the Balance, shall couuterpoise and make an *Equilibrium*, when the said *F*orces shall be to one another in Reciprocal proportion of their Arms, which we think to be so manifest to you, that we thence shall derive the Demonstration of this Universal Proposition to which we hasten.

PROPOS. II.

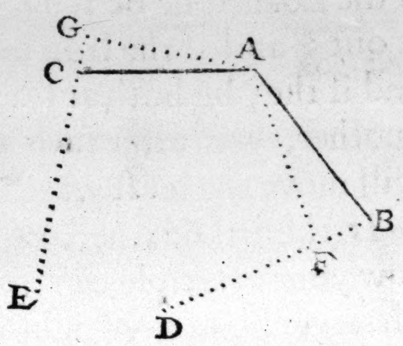
IN every Balance or Leaver, if the proportion of the *F*orces is reciprocal to that of the Perpendicular Lines drawn from the Center or Point of the *F*ulciment unto the Lines of Direction of the *F*orces, drawing the one against the other, they shall make an *Equilibrium*, and drawing on one and the same side, they shall have a like Effect, that is to say, that they shall have as much *F*orce the one as the other, to move the Balance.

In this *F*igure let the Center of the Balance be *A*, the *A*rm *A* B, bigger than *A* C, and first let the *L*ines of Direction B D, and E C be perpendicular to the *A*rms *A* B and *A* C, by which Lines the *F*orces D and E (which may be made of Weights if one will) do draw; and that there is the same rate of the *F*orce D to the Force E as there is betwixt the *A*rm *A* C to the Arm *A* B: the Forces drawing one against the other, I say, that they will make an *Equilibrium* upon the Balance C A B. For let the *A*rm C *A* be prolonged unto F, so as that *A* F may be equal to *A* B: and let C *A* F be considered as a streight Balance, of which let the Center be *A*: and let there be supposed two Forces G and H, of which and of all their parts the Lines of Direction are parallel to the Line C E, and that the Force G be equal to the Force D, and H to E, the one, to wit G, drawing



drawing upon the Arm A F, and the other, to wit H, upon the Arm A C: now, by the first Proposition, G and H shall make an *Equilibrium* upon the Balance C A F: But, by the first Principle, the Force D upon the Arm A B worketh the same effect as the Force G on the Arm A F: Therefore the Force D upon the Arm A B maketh an *Equilibrium* with the Force H upon A C: And the Force H drawing in the same manner upon the Arm *A* C as the Force E, by the same first *A*xiom, the Force D upon the Arm *A* B shall make an *Equilibrium* with the Force E upon the Arm *A* C.

Now, in the following Figure, let the Center of the Balance be *A*, the Arms A B and A C, the Lines of Direction B D and C E which are not Perpendicular to the Arms, and the Forces D and E drawing likewise by the Lines of Direction, upon which Perpendiculars are erected unto the Center A, that is A F upon B D, and A G upon E C, and that as A F is to A G, so is the Force E to the Force D: which Forces draw one against the other: I say, that they will make an *Equilibrium* upon the Balance C A B: For let the Lines A F and A G be understood to be the two Arms of a Balance G A F, upon which the Forces D and E do draw by the Lines of Direction F D and G E: These Forces shall make an *Equilibrium*, by the first part of this second Proposition; but, by the second Axiom, the Force D upon the Arm A F hath the same Effect as upon the Arm A B: Therefore the Force D upon the Arm A B maketh an *Equilibrium* with the Force E upon the Arm A C.



There are many Cases, according to the Series of Perpendiculars, but it will be easie for you to see that they have all but one and the same Demonstration.

It is also easie to demonstrate, that if the Forces draw both on one side they shall make the same Effect one as another, and that the Effect of two together shall be double to that of one alone.

OF THE

GEOSTATICKS.

THe Principle which you demand for the *Geostaticks* is, That if two equal Weights are conjoyned by a right Line fixed and void of Gravity, and that being so disposed they may descend freely, they will never rest till that the middle of the Line, that is the Center of Gravitation of the Ancients, unites it self to the common Center of Grave Bodies.

This Principle seems at the first very plausible, but when the Question concerneth a Principle, you know what Conditions are required to it, that it may be received, the principal of which are wanting in the Principle now in controversie: *scil.* that we do not know what is the radical Cause why Grave Bodies descend; and whence the Original of this Gravity ariseth: as also that we are totally ignorant of that which would arrive at the Center whither Grave Bodies do tend, nor to other places without the Surface of the Earth, of which, in regard we inhabit upon it, we have some Experiments upon which we ground our Principles.

For it may be, that Gravity is a Quality that resides in the Body it self that falleth; it may be that it is in another that attracteth that which descends, as in the Earth: It may be, and it is very likely that it is a Natural Attraction, or a Natural Desire of two Bodies to unite together, as in the Iron and Loadstone, which are such, that if the Loadstone be staid, the Iron, if nothing hinder it, will go find it out; and if the Iron be staid the Loadstone will go towards it; and if they be both at liberty, they will reciprocally approach one another, yet after such a fashion, that the strongest of the two will move the least way.

If the first be true, according to the common opinion, we see not how your Principle can subsist, for Common Sense tells us, that in whatever place a Weight is, it alwaies weigheth alike, having evermore the same Quality that maketh it to weigh, and that then a Body will repose at the Common Center of things Grave when the parts of the Body which shall be on each part of the said Center shall be of equal Ponderosity to counterpoise one another, without having any regard whether they be little or much removed from the Center. Since therefore that of these three possible Causes of Gravitation, we know not which is the right, nay, that we are not certain that it is any of them, it being possibly that there is a fourth from which one may draw Conclusions very different, it seemeth to me impossible for us to lay down other Principles in this business than those of which we are assured by a continual Experience, and a sound Judgment. As for our parts, we call those Bodies equally or unequally Grave which have an equal or unequal Force of moving towards the Common Center: and a Body is said to have the same Weight when it alwaies hath this same Force: but if this Force augmenteth or diminisheth, then, although it be the same Body, we consider it no longer as the same Weight: Now since that this hapneth to Bodies that recede or approach to the Common Center, this is it which we desire to know, but finding nothing that giveth me content upon this Subject, I will leave the Question undetermined and undescribed.

FINIS.

ARCHIMEDES HIS TRACT De Incidentibus Humido,

OR OF THE NATATION OF BODIES VPON, OR SVBMERSION IN, THE

WATER

OR OTHER LIQUIDS.

IN TWO BOOKS.

Translated from the Original Greek,

First into Latine, and afterwards into Italian, by *NICOLO TARTAGLIA*, and by him familiarly demonstrated by way of Dialogue, with *Richard Wentworth*, a Noble English Gentleman, and his Friend.

Together with the Learned Commentaries of *Federico Commandino*, who hath Restored such of the Demonstrations as, thorow the Injury of Time, were obliterated.

Now compared with the ORIGINAL, and Englished By *THOMAS SALUSBURY*, Esq.

LONDON, Printed by *W. Leybourn*, 1662.

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LONDON, Printed by W. L[illegible] 166[illegible]

ARCHIMEDES HIS TRACT *De* INCIDENTIBUS HUMIDO, OR OF The Natation of Bodies upon, or Submersion in, the Water, or other Liquids.

BOOK I.

RICARDO.

DEAR *Companion*, I have perused your *Industrious Invention*, in which I find not any thing that will not certainly hold true; but, truth is, there are many of your Conclusions of which I understand uot the Cause, and therefore, if it be not a trouble to you, I would desire you to declare them to me, for, indeed, nothing pleaseth me, if the Cause thereof be hid from me.

NICOLO. My obligations unto you are so many and great, *Honoured Companion*, that no request of yours ought to be troublesome to me, and therefore tell me what those Perticulars are of which you know not the Cause, for I shall endeavour with the utmost of my power and understanding to satisfie you in all your demands.

RIC. In the first *Direction* of the first Book of that your *Industrious Invention* you conclude, That it is impossible that the Water should wholly receive into it any material Solid Body that is lighter than it self (as to *specie*) nay, you say, That there will alwaies a part of the Body stay or remain above the Waters Surface (that is uncovered by it;) and, That as the whole Solid Body put into the Water is in proportion to that part of it that shall be immerged, or received, into the Water, so shall the Gravity of the Water be to the Gravity (*in specie*) of that same material Body: And that those Solid Bodies, that are by nature more Grave than the Water, being put into the Water, shall presently make the said Water give place; and, That they do not only wholly enter or submerge in the same, but go continually descending untill they arrive at the Bottom; and, That they sink to the Bottom so much faster, by how much they are more Grave than the Water. And, again, That those which are precisely of the same Gravity with the Water, being put into the same, are of necessity wholly received into, or immerged by it, but yet retained in the Surface of the said Water, and much less will the Water consent that it do descend to the Bottom: and, now, albeit that all these things are manifest to Sense and Experience, yet nevertheless would I be very glad, if it be possible, that you would demonstrate to me the most apt and proper Cause of these Effects.

NIC. The

NIC. The Cause of all these Effects is assigned by *Archimedes*, the *Siracusan*, in that Book *De Incidentibus* (*) *Aqua*, by me published in Latine, and dedicated to your self, as I also said in the beginning of that my *Industrious Invention*.

* *Aqua*, tanslated by me *Humido*, as the more Comprehensive word, for his Doctrine holds true in all Liquids as well as in Water, *scil.* in Wine, Oyl, Milk, *&c.*

RIC. I have seen that same *Archimedes*, and have very well understood those two Books in which he treateth *De Centro Gravitatis aequerepentibus*, or of the Center of Gravity in Figures plain, or parallel to the Horizon; and likewise those *De Quadratura Parabolae*, or, of Squaring the Parabola; but * *that* in which he treateth of Solids that Swim upon, or sink in Liquids, is so obscure, that, to speak the truth, there are many things in *it* which I do not understand, and therefore before we proceed any farther, I should take it for a favour if you would declare it to me in your Vulgar Tongue, beginning with his first *Supposition*, which speaketh in this manner.

* He speaks of but one Book, *Tartaglia* having translated no more.

SVPPOSITION I.

It is supposed that the Liquid is of such a nature, that its parts being equi-jacent and contiguous, the less pressed are repulsed by the more pressed. And that each of its parts is pressed or repulsed by the Liquor that lyeth over it, perpendicularly, if the Liquid be descending into any place, or pressed any whither by another.

NIC. EVery Science, Art, or Doctrine (as you know, *Honoured Companion*,) hath its first undemonstrable Principles, by which (they being granted or supposed) the said Science is proved, maintained, or demonstrated. And of these Principles, some are called *Petitions*, and others *Demands*, or *Suppositions*. I say, therefore, that the Science or Doctrine of those Material Solids that Swim or Sink in Liquids, hath only two undemonstrable *Suppositions*, one of which is that above alledged, the which in compliance with your desire I have set down in our Vulgar Tongue.

RIC. Before you proceed any farther tell me, how we are to understand the parts of a Liquid to be *Equijacent*.

NIC. When they are equidistant from the Center of the World, or of the Earth (which is the same, although * some hold that the Centers of the Earth and World are different.)

* The Copernicans.

RIC. I understand you not unless you give me some Example thereof in Figure.

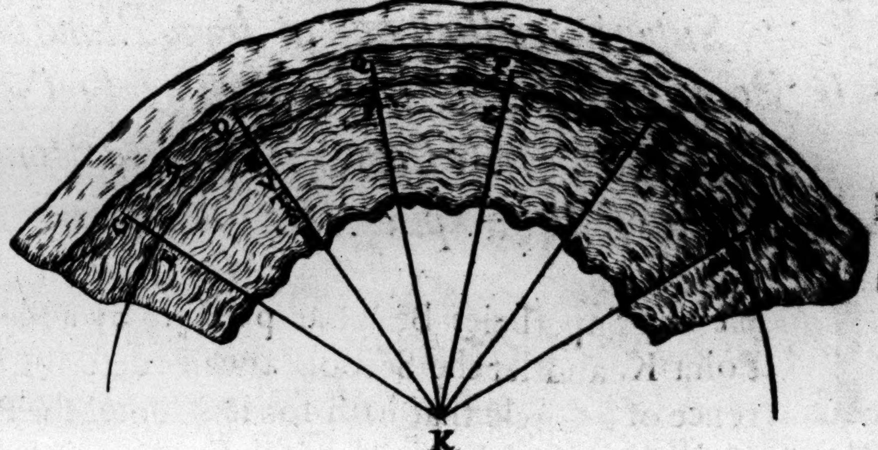
NIC. To exemplifie this particular, Let us suppose a quantity of Liquor (as for instance of Water) to be upon the Earth; then let us with the Imagination cut the whole Earth together with that Water into two equal parts, in such a manner as that the said Section may pass * by the Center of the Earth: And let us suppose that one part of the Superficies of that Section, as well of the Water as of the Earth, be the Superficies AB, and that the Center of the Earth be the point K. This being done, let us in our Imagination describe a Circle upon the said Center K, of such a bigness as that the Circumference may pass by the Superficies of the Section of the Water: Now let this Circumference be EFG: and let many Lines be drawn from the point K to the said Circumference, cutting the same, as KE, KHO, KFG, KLP, KM. Now I say, that all these parts of the said Water, terminated in that Circumference, are Equijacent, as being all equi-

* Or through.

equidistant from the point K, the Center of the World, which parts are G M, M L, L F, F H, H E.

RIC. I understand you very well, as to this particular: But tell me a little; he saith that each of the parts of the Liquid is pressed or repulsed by the Liquid that is above it, according to the Perpendicular: I know not what that Liquid is that lieth upon a part of another Perpendicularly.

NIC. Imagining a Line that cometh from the Center of the Earth penetrating thorow some Water, each part of the Water that is in that Line he supposeth to be pressed or repulsed by the Water that lieth above it in that same Line, and that that repulse is made according to the same Line, (that is, directly towards the Center of the VVorld) which Line is called a Perpendicular; because every Right-Line that departeth from any point, and goeth directly towards the VVorlds Center is called a Perpendicular. And that you may the better understand me, let us imagine the Line KHO, and in that let us imagine several parts, as suppose R S, S T, T V, V H, H O. I say, that he supposeth that the part V H is pressed by that placed above it, H O, according to the Line OK; the which



O K, as hath been said above, is called the Perpendicular passing thorow those two parts. In like manner, I say that the part T V is expulsed by the part V H, according to the said Line O K: and so the part S T to be pressed by T V, according to the said Perpendicular O K, and R S by S T: And this you are to understand in all the other Lines that were protracted from the said Point K, penetrating the said VVater, As for Example, in *K* G, *K* M, *K* L, *K* F, *K* E, and infinite others of the like kind.

RIC. Indeed, *Dear Companion*, this your Explanation hath given me great satisfaction; for, in my Judgment, it seemeth that all the difficulty of this Supposition consists in these two particulars which you have declared to me.

NIC. It doth so; for having understood that the parts E H, H F, F L, L M, and M G, determining in the Circumference of the said Circle are equijacent, it is an easie matter to understand the foresaid *Supposition* in Order, which saith, *That it is supposed that the Liquid is of such a nature, that the part thereof less pressed or thrust is repulsed by the more thrust or pressed.* As for example, if the part E H were by chance more thrust, crowded, or pressed from above downwards by the Liquid, or some other matter that was over it, than the part H F, contiguous to it, it is supposed that the said part H F, less pressed, would be repulsed by the said part E H. And thus we ought to understand of the other parts equijacent, in case that they be contiguous, and not severed. That each of the parts thereof is pressed and repulsed by the *Liquid* that lieth over it Perpendicularly, is manifest by that which was said above, to wit, that it should be repulsed, in case the *Liquid* be descending into any place, and thrust, or driven any whither by another.

RIC. I understand this Supposition very well, but yet me thinks that before the Supposition, the Author ought to have defined those two particulars, which you first declared to me, that is, how we are to understand the parts of the *Liquid* equijacent, and likewise the Perpendicular.

NIC. You

NIC. You ſay truth.

RIC. I have another queſtion to aske you, which is this, VVhy the Author uſeth the word *Liquid*, or Humid, inſtead of VVater.

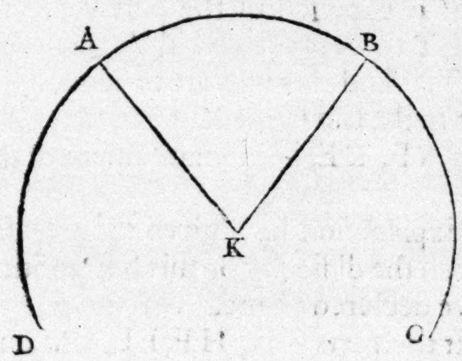
NIC. It may be for two of theſe two Cauſes; the one is, that VVater being the principal of all Liquids, therefore ſaying *Humidum* he is to be underſtood to mean the chief Liquid, that is Water: The other, becauſe that all the Propoſitions of this Book of his, do not only hold true in VVater, but alſo in every other *Liquid*, as in VVine, Oyl, and the like: and therefore the Author might have uſed the word *Humidum*, as being a word more general than *Aqua*.

RIC. This I underſtand, therefore let us come to the firſt *Propoſition*, which, as you know, in the Original ſpeaks in this manner.

PROP. I. THEOR. I.

If any Superficies ſhall be cut by a Plane thorough any Point, and the Section be alwaies the Circumference of a Circle, whoſe Center is the ſaid Point: that Superficies ſhall be Spherical.

LEt any Superficies be cut at pleaſure by a Plane thorow the Point K, and let the Section alwaies deſcribe the Circumference of a Circle that hath for its Center the Point K: I ſay, that that ſame Superficies is Sphærical. For were it poſſible that the ſaid Superficies were not Sphærical, then all the Lines drawn through the ſaid Point K unto that Superficies would not be equal. Let therefore A and B be two Points in the ſaid Superficies, ſo that drawing the two Lines K A and K B, let them, if poſſible, be unequal: Then by theſe two Lines let a Plane be drawn cutting the ſaid Superficies, and let the Section in the Superficies make the Line D A B G: Now this Line D A B G is, by our pre-ſuppoſal, a Circle, and the Center thereof is the Point K, for ſuch the ſaid Superficies was ſuppoſed to be. Therefore the two Lines K A and K B are equal: But they were alſo ſuppoſed to be unequal; which is impoſſible: It followeth therefore, of neceſſity, that the ſaid Superficies be Sphærical, that is, the Superficies of a Sphære.



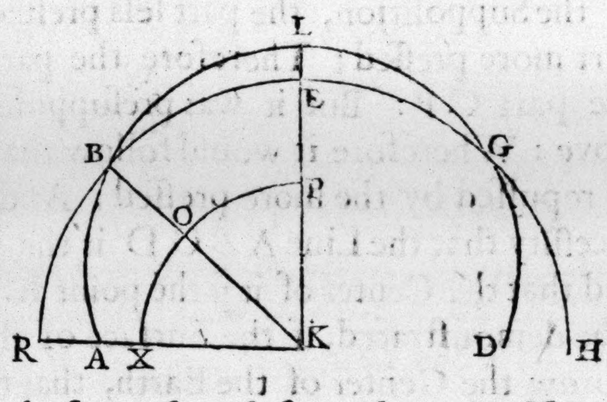
RIC. I underſtand you very well; now let us proceed to the ſecond *Propoſition*, which, you know, runs thus.

PROP.

PROP. II. THEOR. II.

The Superficies of every Liquid that is consistant and setled shall be of a Sphærical Figure, which Figure shall have the same Center with the Earth.

LEt us suppose a Liquid that is of such a consistance as that it is not moved, and that its Superficies be cut by a Plane along by the Center of the Earth, and let the Center of the Earth be the Point K: and let the Section of the Superficies be the Line A B G D. I say that the L ne A B G D is the Circumference of a Circle, and that the Center thereof is the Point K. And if it be possible that it may not be the Circumference of a Circle, the Right-Lines drawn * by the Point K to the said Line A B G D shall not be equal. Therefore let a Right-Line be taken greater than some of those produced from the Point K unto the said Line A B G D, and lesser than some other; and upon the Point K let a Circle be described at the length of that Line, Now the Circumference of this Circle shall fall part without the said Line A B G D, and part within: it having been presupposed that its Semidiameter is greater than some of those Lines that may be drawn from the said Point K unto the said Line A B G D, and lesser than some other. Let the Circumference of the described Circle be R B G H, and from B to K draw the Right-Line B K: and drawn also the two Lines K R, and K E L which make a Right-Angle in the Point K: and upon the Center K describe the Circumference X O P in the Plane and in the Liquid. The parts, therefore, of the Liquid that are * according to the Circumference X O P, for the reasons alledged upon the first *Supposition*, are equijacent, or equiposited, and contiguous to each other; and both these parts are prest or thrust, according to the second part of the *Supposition*, by the Liquor which is above them. And because the two Angles E K B and B K R are supposed equal [*by the 26. of 3. of Euclid*,] the two Circumferences or Arches B E and B R shall be equal (forasmuch as R B G H was a Circle described for satisfaction of the Oponent, and K its Center:) And in like manner the whole Triangle B E K shall be equal to the whole Triangle B R K. And because also the Triangle O P K for the same reason



* O: through.

* *i.e.* Parallel.

ſhall be equal to the Triangle O X K; Therefore (by common Notion) ſubſtracting thoſe two ſmall Triangles O P K and O X K from the two others B E K and B R K, the two Remainders ſhall be equal: one of which Remainders ſhall be the Quadrangle B E O P, and the other B R X O. And becauſe the whole Quadrangle B E O P is full of Liquor, and of the Quadrangle B R X O, the part B A X O only is full, and the reſidue B R A is wholly void of Water: It followeth, therefore, that the Quadrangle B E O P is more ponderous than the Quadrangle B R X O. And if the ſaid Quadrangle B E O P be more Grave than the Quadrangle B R X O, much more ſhall the Quadrangle B L O P exceed in Gravity the ſaid Quadrangle B R X O: whence it followeth, that the part O P is more preſſed than the part O X. But, by the firſt part of the Suppoſition, the part leſs preſſed ſhould be repulſed by the part more preſſed: Therefore the part O X muſt be repulſed by the part O P: But it was preſuppoſed that the Liquid did not move: Wherefore it would follow that the leſs preſſed would not be repulſed by the more preſſed: And therefore it followeth of neceſſity that the Line A B G D is the Circumference of a Circle, and that the Center of it is the point K. And in like manner ſhall it be demonſtrated, if the Surface of the Liquid be cut by a Plane thorow the Center of the Earth, that the Section ſhall be the Circumference of a Circle, and that the Center of the ſame ſhall be that very Point which is Center of the Earth. It is therefore manifeſt that the Superficies of a Liquid that is conſiſtant and ſetled ſhall have the Figure of a Sphære, the Center of which ſhall be the ſame with that of the Earth, by the firſt *Propoſition*; for it is ſuch that being ever cut thorow the ſame Point, the Section or Diviſion deſcribes the Circumference of a Circle which hath for Center the ſelf-ſame Point that is Center of the Earth: Which was to be demonſtrated.

RIC. I do thorowly underſtand theſe your Reaſons, and ſince there is in them no umbrage of Doubting, let us proceed to his third *Propoſition*.

PROP. III. THEOR. III.

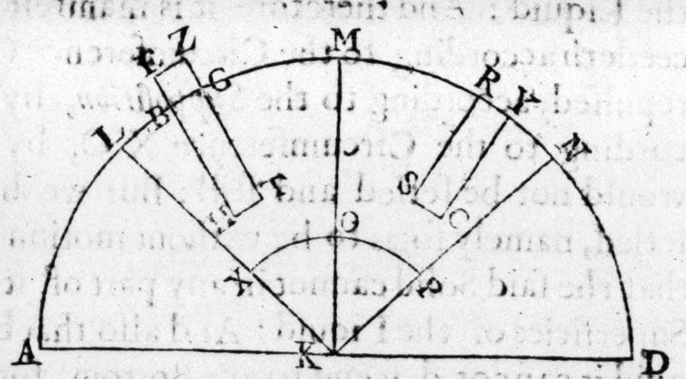
Solid Magnitudes that being of equal Maſs with the Liquid are alſo equal to it in Gravity, being demitted into the [ſetled] Liquid do ſo ſubmerge in the ſame as that they lie or appear not at all above the Surface of the Liquid, nor yet do they ſink to the Bottom.*

* I add the word ſetled, as neceſſary in making the Experiment.

NIC. In

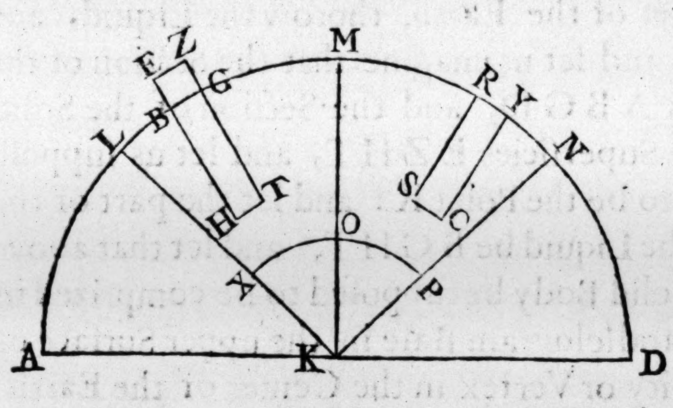
N 10. IN this *Proposition* it is affirmed that those Solid Magnitudes that happen to be equal in specifical Gravity with the Liquid being left at liberty in the said Liquid do so submerge in the same, as that they lie or appear not at all above the Surface of the Liquid, nor yet do they go or sink to the Bottom.

For supposing, on the contrary, that it were possible for one of those Solids being placed in the Liquid to lie in part without the Liquid, that is above its Surface, (alwaies provided that the said Liquid be setled and undisturbed,) let us imagine any Plane produced thorow the Center of the Earth, thorow the Liquid, and thorow that Solid Body: and let us imagine that the Section of the Liquid is the Superficies A B G D, and the Section of the Solid Body that is within it the Superficies E Z H T, and let us suppose the Center of the Earth to be the Point K: and let the part of the said Solid submerged in the Liquid be B G H T, and let that above be B E Z G: and let the Solid Body be supposed to be comprized in a Pyramid that hath its Parallelogram Base in the upper Surface of the Liquid, and its Summity or Vertex in the Center of the Earth: which Pyramid let us also suppose to be cut or divided by the same Plane in which is the Circumference A B G D, and let the Sections of the Planes of the said Pyramid be K L and K M: and in the Liquid about the Center K let there be described a Superficies of another Sphære below E Z H T, which let be X O P; and let this be cut by the Superficies of the Plane:



And let there be another Pyramid taken or supposed equal and like to that which compriseth the said Solid Body, and contiguous and conjunct with the same; and let the Sections of its Superficies be K M and K N: and let us suppose another Solid to be taken or imagined, of Liquor, contained in that same Pyramid, which let be R S C Y, equal and like to the partial Solid B H G T, which is immerged in the said Liquid: But the part of the Liquid which in the first Pyramid is under the Superficies X O, and that, which in the other Pyramid is under the Superficies O P, are equijacent or equiposited and contiguous, but are not pressed equally; for that which is under the Superficies X O is pressed by the Solid T H E Z, and by the Liquor that is contained between the two Spherical Superficies X O and L M and the Planes of the Pyramid, but that which proceeds according to P O is pressed by the Solid R S C Y, and by the Liquid

contained between the Sphærical Superficies that proceed according to P O and M N and the Planes of the Pyramid; and the Gravity of the Liquid, which is according to M N O P, shall be lesser than that which is according to L M X O; because that Solid of Liquor which proceeds according to R S C Y is less than the Solid E Z H T (having been supposed to be equal in quantity to only the part H B G T of that:) And the said Solid E Z H T hath been supposed to be equally grave with the Liquid: Therefore the Gravity of the Liquid comprised betwixt the two Sphærical Superficies L M and X O, and betwixt the sides L X and M O of the Pyramid, together with the whole Solid EZHT, shall exceed the Gravity of the Liquid comprised betwixt the other two Sphærical Superficies M N and O P, and the Sides M O and N P of the Pyramid, together with the Solid of Liquor R S C Y by the quantity of the Gravity of the part E B Z G, supposed to remain above the Surface of the Liquid: And therefore it is manifest that the part which proceedeth according to the Circumference O P is pressed, driven, and repulsed, according to the *Supposition*, by that which proceeds according to the Circumference X O, by which means the Liquid would not be setled and still: But we did presuppose that it was setled, namely so, as to be without motion: It followeth, therefore, that the said Solid cannot in any part of it exceed or lie above the Superficies of the Liquid: And also that being dimerged in the Liquid it cannot descend to the Bottom, for that all the parts of the Liquid equijacent, or disposed equally, are equally pressed, because the Solid is equally grave with the Liquid, by what we presupposed.



RIC. I do understand your Argumentation, but I understand not that Phrase *Solid Magnitudes*.

NIC. I will declare this Term unto you. *Magnitude* is a general Word that respecteth all the Species of Continual Quantity; and the Species of Continual Quantity are three, that is, the *Line*, the Superficies, and the Body; which Body is also called a Solid, as having in it self *Length*, Breadth, and Thickness, or Depth: and therefore that none might equivocate or take that Term *Magnitudes* to be meant of *Lines*, or Superficies, but only of Solid *Magnitudes*, that is, Bodies, he did specifie it by that manner of expression, as was said. The truth is, that he might have exprest that *Proposition* in this manner: *Solids (or Bodies) which being of equal Gravity with an equal Mass of the Liquid*, &c. And this *Proposition* would have been more cleer and intelligible, for it is as significant to say, a *Solid*, or, a *Body*, as to say a *Solid Magnitude*: therefore wonder not if for the future I use these three kinds of words indifferently.

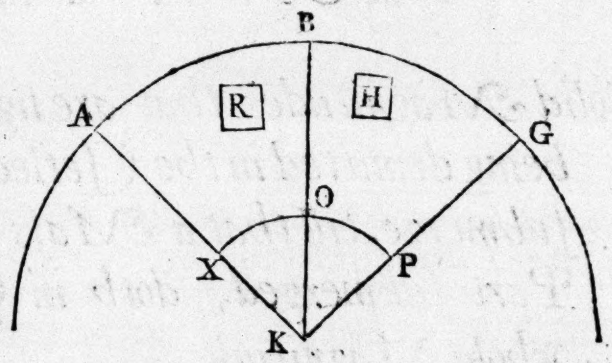
RIC. You have sufficiently satisfied me, wherefore that we may lose no time let us go forwards to the fourth *Proposition*.

PROP. IV. THEOR. IV.

Solid Magnitudes that are lighter than the Liquid, being demitted into the setled Liquid, will not totally submerge in the same, but some part thereof will lie or stay above the Surface of the Liquid.

NIC. IN this fourth *Proposition* it is concluded, that every Body or Solid that is lighter (as to Specifical Gravity) than the *Liquid*, being put into the *Liquid*, will not totally submerge in the same, but that some part of it will stay and appear without the *Liquid*, that is above its Surface.

For supposing, on the contrary, that it were possible for a Solid more light than the Liquid, being demitted in the Liquid to submerge totally in the same, that is, so as that no part thereof remaineth above, or without the said Liquid, (evermore supposing that the Liquid be so constituted as that it be not moved,) let us imagine any Plane produced thorow the Center of the Earth, thorow the Liquid, and thorow that Solid *Body*: and that the Surface of the Liquid is cut by this Plane according to the Circumference A *B* G, and the Solid *Body* according to the Figure R; and let the Center of the Earth be K. And let there be imagined a Pyramid that compriseth the Figure R, as was done in the precedent, that hath its Vertex in the Point K, and let the Superficies of that Pyramid be cut by the Superficies of the Plane A *B* G, according to A K and K *B*. And let us imagine another Pyramid equal and like to this, and let its Superficies be cut by the Superficies A *B* G according to K *B* and K G; and let the Superficies of another Sphære be described in the Liquid, upon the Center K, and beneath the Solid R; and let that be cut by the same Plane according to X O P. And, lastly, let us suppose another Solid taken * from the Liquid, in this second Pyramid, which let be H, equal to the Solid R. Now the parts of the Liquid, namely, that which is under the Spherical Superficies, that proceeds according to the Superficies or Circumference X O, in the first Pyramid, and that which is under the Spherical Superficies that proceeds according to the Circumference O P, in the second Pyramid, are equijacent, and contiguous, but are not pressed equally; for that



* That is a Mass of the Liquid.

that of the first Pyramid is pressed by the Solid R, and by the Liquid which that containeth, that is, that which is in the place of the Pyramid according to A B O X: but that part which, in the other Pyramid, is pressed by the Solid H, supposed to be of the same Liquid, and by the Liquid which that containeth, that is, that which is in the place of the said Pyramid according to P O B G: and the Gravity of the Solid R is less than the Gravity of the Liquid H, for that these two Magnitudes were supposed to be equal in Mass, and the Solid R was supposed to be lighter than the Liquid: and the Masses of the two Pyramids of Liquor that containeth these two Solids R and H are equal * by what was presupposed: Therefore the part of the Liquid that is under the Superficies that proceeds according to the Circumference O P is more pressed; and, therefore, by the *Supposition*, it shall repulse that part which is less pressed, whereby the said Liquid will not be setled: But it was before supposed that it was setled: Therefore that Solid R shall not totally submerge, but some part thereof will remain without the Liquid, that is, above its Surface, Which was the *Proposition*.

* For that the Pyramids were supposed equal.

RIC. I have very well understood you, therefore let us come to the fifth *Proposition*, which, as you know, doth thus speak.

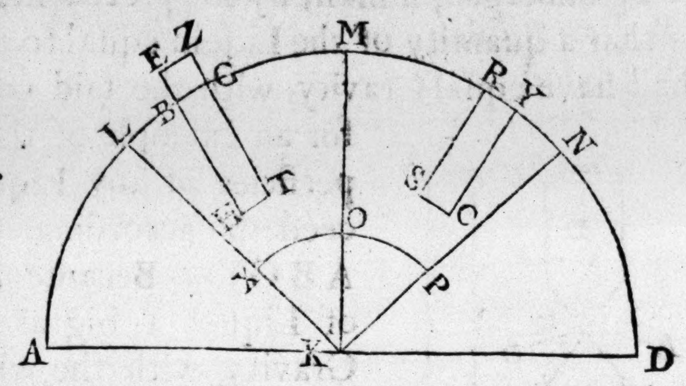
PROP. V. THEOR. V.

Solid Magnitudes that are lighter than the Liquid, being demitted in the (setled) Liquid, will so far submerge, till that a Mass of Liquor, equal to the Part submerged, doth in Gravity equalize the whole Magnitude.

NIC. IT having, in the precedent, been demonstrated that Solids lighter than the Liquid, being demitted in the *Liquid*, alwaies a part of them remains without the *Liquid*, that is above its Surface; In this fifth *Proposition* it is asserted, that so much of such a Solid shall submerge, as that a Mass of the *Liquid* equal to the part submerged, shall have equal Gravity with the whole Solid.

And to demonstrate this, let us assume all the same Schemes as before, in *Proposition* 3. and likewise let the Liquid be setled, and let the Solid E Z H T be lighter than the Liquid. Now if the said Liquid be setled, the parts of it that are equijacent are equally pressed: Therefore the Liquid that is beneath the

the Superficies that proceed according to the Circumferences X O and P O are equally pressed; whereby the Gravity pressed is equal. But the Gravity of the Liquid which is in the first Pyramid * without the Solid B H T G, is equal to the Gravity of the Liquid which is in the other Pyramid without the Liquid R S C Y: It is manifest, therefore, that the Gravity of the Solid E Z H T, is equal to the Gravity of the Liquid R S C Y: Therefore it is manifest that a Mass of Liquor equal in Mass to the part of the Solid submerged is equal in Gravity to the whole Solid.



* *Without*, i.e. that being deducted.

RIC. This was a pretty Demonstration, and because I very well understand it, let us lose no time, but proceed to the sixth *Proposition*, speaking thus.

PROP. VI. THEOR. VI.

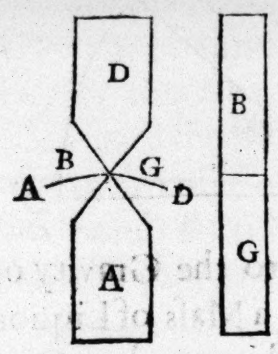
Solid Magnitudes lighter than the Liquid being thrust into the Liquid, are repulsed upwards with a Force as great as is the excess of the Gravity of a Mass of Liquor equal to the Magnitude above the Gravity of the said Magnitude.

NIC. This sixth *Proposition* saith, that the Solids lighter than the Liquid demitted, thrust, or trodden by Force underneath the Liquids Surface, are returned or driven upwards with so much Force, by how much a quantity of the Liquid equal to the Solid shall exceed the said Solid in Gravity.

And to delucidate this *Proposition*, let the Solid A be lighter than the Liquid, and let us suppose that the Gravity of the said Solid A is B: and let the Gravity of a Liquid, equal in Mass to A, be B G. I say, that the Solid A depressed or demitted with Force into the said *Liquid*, shall be returned and repulsed upwards with a Force equal to the Gravity G. And to demonstrate this *Proposition*, take the Solid D, equal in Gravity to the said G. Now the Solid compounded of the two Solids A and D will be lighter than the *Liquid*: for the Gravity of the Solid compounded of them both is B G, and the Gravity of as much Liquor as equalleth in greatness the Solid A, is greater than the said Gravity B G,

for

for that BG is the Gravity of the Liquid equal in Maſs unto it: Therefore the Solid compounded of thoſe two Solids A and D being dimerged, it ſhall, by the precedent, ſo much of it ſubmerge, as that a quantity of the Liquid equal to the ſaid ſubmerged part ſhall have equal Gravity with the ſaid compounded Solid. And for an example of that *Propoſition* let the Superficies of any Liquid be that which proceedeth according to the Circumference ABGD: Becauſe now a Maſs or quantity of Liquor as big as the Maſs A hath equal Gravity with the whole compounded Solid AD: It is manifeſt that the ſubmerged part thereof ſhall be the Maſs A: and the remainder, namely, the part D, ſhall be wholly atop, that is, above the Surface of the Liquid.



It is therefore evident, that the part A hath ſo much virtue or Force to return upwards, that is, to riſe from below above the Liquid, as that which is upon it, to wit, the part D, hath to preſs it downwards, for that neither part is repulſed by the other: But D preſſeth downwards with a Gravity equal to G, it having been ſuppoſed that the Gravity of that part D was equal to G: Therefore that is manifeſt which was to be demonſtrated.

RIC. This was a fine Demonſtration, and from this I perceive that you collected your *Induſtrious Invention*; and eſpecially that part of it which you inſert in the firſt Book for the recovering of a Ship ſunk: and, indeed, I have many Queſtions to ask you about that, but I will not now interrupt the Diſcourſe in hand, but deſire that we may go on to the ſeventh *Propoſition*, the purport whereof is this.

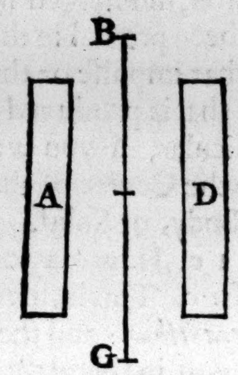
PROP. VII. THEOR. VII.

Solid Magnitudes heavier than the Liquid, being demitted into the [ſetled] Liquid, are boren downwards as far as they can deſcend: and ſhall be lighter in the Liquid by the Gravity of a Liquid Maſs of the ſame bigneſs with the Solid Magnitude.

NIC. This ſeventh *Propoſition* hath two parts to be demonſtrated.

The firſt is, That all Solids heavier than the Liquid, being demitted into the *Liquid*, are boren by their Gravities downwards as far as they can deſcend, that is untill they arrive at the Bottom. Which firſt part is manifeſt, becauſe the Parts of the *Liquid*, which ſtill lie under that Solid, are more preſſed than the others equijacent, becauſe that that Solid is ſuppoſed more grave than the Liquid. But

But now that that Solid is lighter in the Liquid than out of it, as is affirmed in the second part, shall be demonstrated in this manner. Take a Solid, as suppose A, that is more grave than the Liquid, and suppose the Gravity of that same Solid A to be B G. And of a Mass of Liquor of the same bigness with the Solid A, suppose the Gravity to be B: It is to be demonstrated that the Solid A, immerged in the Liquid, shall have a Gravity equal to G. And to demonstrate this, let us imagine another Solid, as suppose D, more light than the Liquid, but of such a quality as that its Gravity is equal to B: and let this D be of such a Magnitude, that a Mass of Liquor equal to it hath its Gravity equal to the Gravity B G. Now these two Solids D and A being compounded together, all that Solid compounded of these two shall be equally Grave with the Water: because the Gravity of these two Solids together shall be equal to these two Gravities, that is, to B G, and to B; and the Gravity of a Liquid that hath its Mass equal to these two Solids A and D, shall be equal to these two Gravities B G and B. Let these two Solids, therefore, be put in the *Liquid*, and they shall* remain in the Surface of that *Liquid*, (that is, they shall not be drawn or driven upwards, nor yet downwards:) For if the Solid A be more grave than the Liquid, it shall be drawn or born by its Gravity downwards towards the Bottom, with as much Force as by the Solid D it is thrust upwards: And because the Solid D is lighter than the *Liquid*, it shall raise it upward with a Force as great as the Gravity G: Because it hath been demonstrated, in the sixth *Proposition*, That Solid Magnitudes that are lighter than the Water, being demitted in the same, are repulsed or driven upwards with a Force so much the greater by how much a *Liquid* of equal Mass with the Solid is more Grave than the said Solid: But the *Liquid* which is equal in Mass with the Solid D, is more grave than the said Solid D, by the Gravity G: Therefore it is manifest, that the Solid A is pressed or born downwards towards the Centre of the World, with a Force as great as the Gravity G: Which was to be demonstrated.



* Or, according to *Commandine*, shall be equall in Gravity to the Liquid, neither moving upwards or downwards.

RIC. This hath been an ingenuous Demonstration; and in regard I do sufficiently understand it, that we may lose no time, we will proceed to the second *Supposition*, which, as I need not tell you, speaks thus.

SVPPOSITION II.

It is supposed that those Solids which are moved upwards, do all ascend according to the Perpendicular which is produced thorow their Centre of Gravity.

COMMANDINE.

And those which are moved downwards, descend, likewise, according to the Perpendicular that is produced thorow their Centre of Gravity, which he pretermitted either as known, or as to be collected from what went before.

NIC. For understanding of this second *Supposition*, it is requisite to take notice that every Solid that is lighter than the Liquid being by violence, or by some other occasion, submerged in the Liquid, and then left at liberty, it shall, by that which hath been proved in the sixth *Proposition*, be thrust or born upwards by the Liquid, and that impulse or thrusting is supposed to be directly according to the Perpendicular that is produced thorow the Centre of Gravity of that Solid; which Perpendicular, if you well remember, is that which is drawn in the Imagination from the Centre of the World, or of the Earth, unto the Centre of Gravity of that Body, or Solid.

RIC. How may one find the Centre of Gravity of a Solid?

NIC. This he sheweth in that Book, intituled *De Centris Gravium, vel de Æquiponderantibus*; and therefore repair thither and you shall be satisfied, for to declare it to you in this place would cause very great confusion.

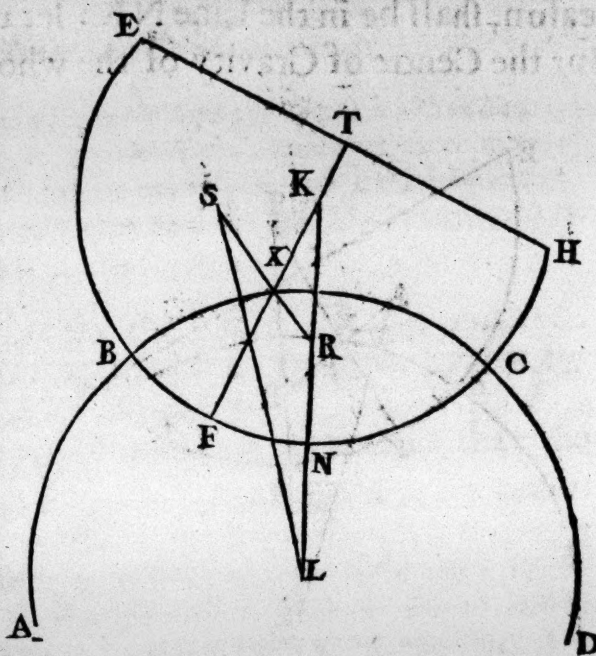
RIC. I understand you: some other time we will talk of this, because I have a mind at present to proceed to the last *Proposition*, the Exposition of which seemeth to me very confused, and, as I conceive, the Author hath not therein shewn all the Subject of that *Proposition* in general, but only a part: which Proposition speaketh, as you know, in this form.

PROP. VIII. THEOR. VIII.

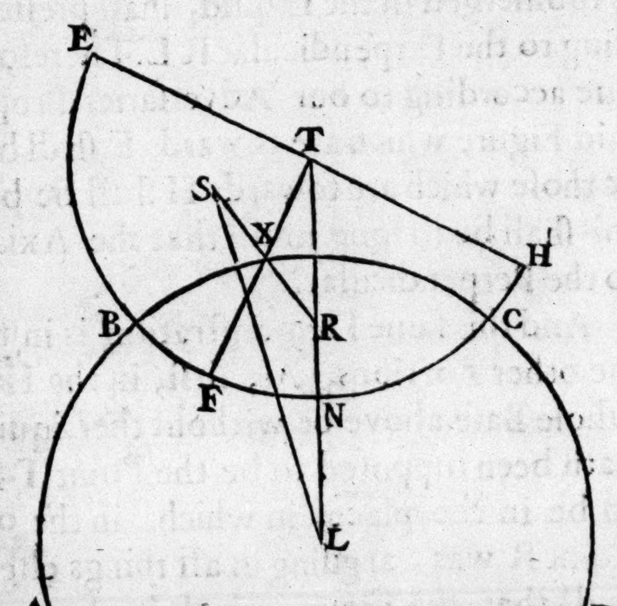
A *If any Solid Magnitude, lighter than the Liquid, that*
B *hath the Figure of a Portion of a Sphære, shall be demitted into the Liquid in such a manner as that the Base of the Portion touch not the Liquid, the Figure shall stand erectly, so, as that the Axis of the said Portion shall be according to the Perpendicular. And if the Figure shall be inclined to any side, so, as that the Base of the Portion touch the Liquid, it shall not continue so inclined as it was demitted, but shall return to its uprightness.*

For

FOr the declaration of this *Propoſition*, let a Solid Magnitude that hath the Figure of a portion of a Sphære, as hath been ſaid, be imagined to be demitted into the Liquid; and alſo, let a Plain be ſuppoſed to be produced thorow the Axis of that portion, and thorow the Center of the Earth: and let the Section of the Surface of the Liquid be the Circumference A B C D, and of the Figure, the Circumference E F H, & let E H be a right line, and F T the Axis of the Portion. If now it were poſſible, for ſatisfaction of the Adverſary, Let



it be ſuppoſed that the ſaid Axis were not according to the (*a*) Perpendicular; we are then to demonſtrate, that the Figure will not continue as it was conſtituted by the Adverſary, but that it will return, as hath been ſaid, unto its former poſition, that is, that the Axis F T ſhall be according to the Perpendicular. It is manifeſt, by the *Corollary* of the 1. of 3. *Euclide*, that the Center of the Sphære is in the Line F T, foraſmuch as that is the Axis of that Figure. And in regard that the Portion of a Sphære, may be greater or leſſer than an Hemiſphære; and may alſo be an Hemiſphære, let the Centre of the Sphære, in the Hemiſphære, be the Point T, and in the leſſer Portion the Point P, and in the greater, the Point K, and let the Centre of the Earth be the Point L. And ſpeaking, firſt, of that greater Portion which hath its Baſe out of, or above, the Liquid, thorow the Points K and L, draw the Line K L cutting the Circumference E F H in the Point N, Now, becauſe every Portion of a Sphære, hath its Axis in the Line, that from the Centre of the Sphære is drawn perpendicular unto its Baſe, and hath its Centre of Gravity in the Axis; therefore that Portion of the Figure which is within the Liquid, which is compounded of two Portions



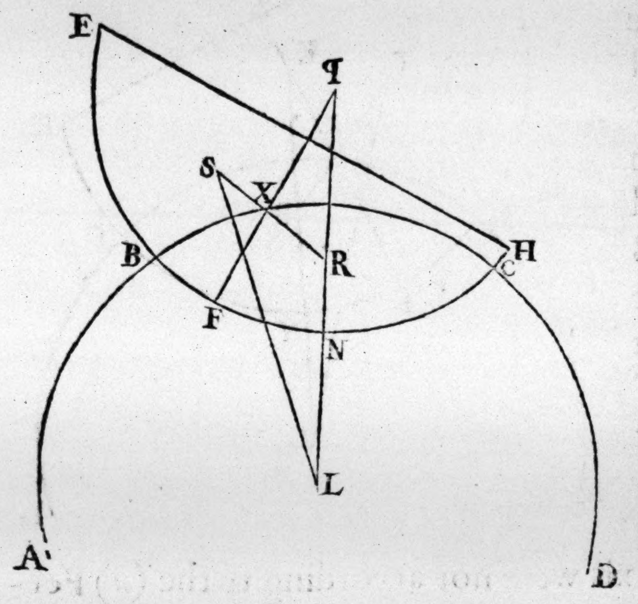
(a) *Perpendicular is taken here, as in all other places, by this Author for the Line K L drawn thorow the Centre and Circumference of the Earth.*

C

tions of a Sphære, shall have its Axis in the Perpendicular, that is drawn through the point K; and its Centre of Gravity, for the same reason, shall be in the Line N K: let us suppose it to be the Point R:

D But the Centre of Gravity of the whole Portion is in the Line F T, betwixt the Point R and the Point F; let us suppose it to be the Point X: The remainder, therefore, of that Figure elivated above the Surface of the Liquid, hath its Centre of Gravity in the Line R X produced or continued right out in the Part towards X, taken so, that the part prolonged may have the same proportion to X R, that the Gravity of that Portion that is demerged in the Liquid hath to the Gravity of that Figure which is above the Liquid; let us suppose that * that Centre of the said Figure be the Point S: and thorow that same Centre S draw the Perpendicular L S. Now the Gravity of the Figure that is above the Liquid shall presse from above downwards according to the Perpendicular S L; & the Gravity of the Portion that is submerged in the Liquid, shall presse from below upwards, according to the Perpendicular R L. Therefore that Figure will not continue according to our Adversaries Proposall, but those parts of the said Figure which are towards E, shall be born or drawn downwards, & those which are towards H shall be born or driven upwards, and this shall be so long untill that the Axis F T comes to be according to the Perpendicular.

E



* i. e. The Center of Gravity.

F

And this same Demonstration is in the same manner verified in the other Portions. As, first, in the Hæmisphere that lieth with its whole Base above or without the *L*iquid, the Centre of the Sphære hath been supposed to be the *P*oint T; and therefore, imagining T to be in the place, in which, in the other above mentioned, the *P*oint R was, arguing in all things else as you did in that, you shall find that the Figure which is above the *L*iquid shall press from above downwards according to the *P*erpendicular S L; and the *P*ortion that is submerged in the *L*iquid shall press from below upwards according to the *P*erpendicular R *L*. And therefore it shall follow, as in the other, namely, that the parts of the whole Figure which are towards E, shall be born or pressed downwards, and those that are towards H, shall be born or driven upwards: and this shall be so long untill that the Axis F T come to stand * *P*erpendicularly

* Or according to the Perpendicular.

ly. The like shall also hold true in the Portion of the Sphære less than an Hemisphere that lieth with its whole Base above the Liquid.

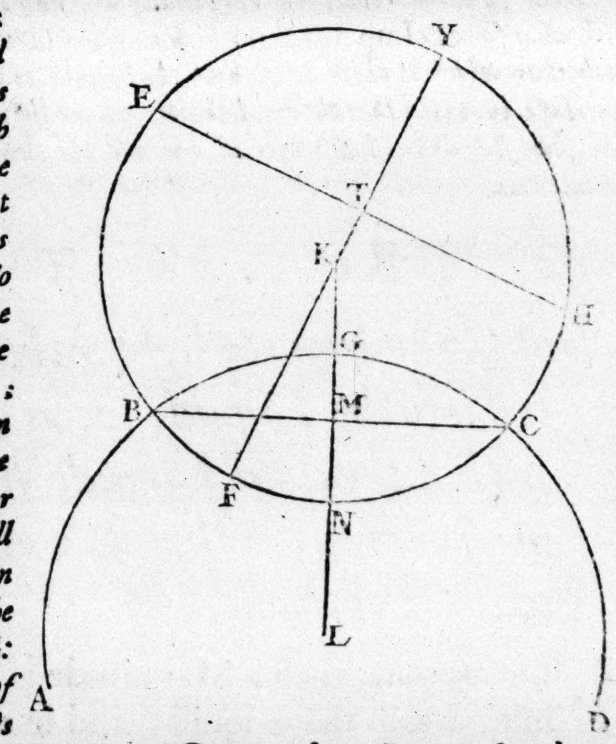
COMMANDINE.

The Demonstration of this Proposition is defaced by the Injury of Time, which we have restored, so far as by the Figures that remain, one may collect the Meaning of Archimedes, *for we thought it not good to alter them: and what was wanting to their declaration and explanation we have supplyed in our Commentaries, as we have also determined to do in the second* Proposition *of the second Book.*

If any Solid Magnitude lighter than the Liquid.] *These words, lighter than the Liquid, are added by us, and are not to be found in the Translation; for of these kind of Magnitudes doth* Archimedes *speak in this Proposition.* A

Shall be demitted into the Liquid in such a manner as that the Base of the Portion touch not the Liquid.] *That is, shall be so demitted into the Liquid as that the Base shall be upwards, and the* Vertex *downwards, which he opposeth to that which he saith in the Proposition following;* Be demitted into the Liquid, so, as that its Base be wholly within the Liquid; *For these words signifie the Portion demitted the contrary way, as namely, with the* Vertex *upwards and the Base downwards. The same manner of Speech is frequently used in the second Book; which treateth of the Portions of Rectangle Conoids.* B

Now because every Portion of a Sphære hath its Axis in the Line that from the Center of the Sphære is drawn perpendicular to its Base.] *For draw a Line from B to C, and let K L cut the Circumference A B C D in the Point G, and the Right Line B C in M: and because the two Circles A B C D, and E F H do cut one another in the Points B and C, the Right Line that conjoyneth their Centers, namely, K L, doth cut the Line B C in two equall parts, and at Right Angles; as in our Commentaries upon* Ptolomeys *Planisphære we do prove: But of the Portion of the Circle B N C the Diameter is M N; and of the Portion B G C the Diameter is M G; for the* (a) *Right Lines which are drawn on both sides parallel to B C do make Right Angles with N G; and* (b) *for that cause are thereby cut in two equall parts: Therefore the Axis of the Portion of the Sphære B N C is N M; and the Axis of the Portion B G C is M G: from whence it followeth that the Axis of the Portion demerged in the Liquid is in the Line K L, namely N G. And since the Center of Gravity of any Portion of a Sphære is in the Axis, as we have demonstrated in our Book* De Centro Gravitatis Solidorum, *the Centre of Gravity of the Magnitude compounded of both the Portions B N C & B G C, that is, of the Portion demerged in the Water, is in the Line N G that doth conjoyn the Centers of Gravity of those Portions of Sphæres. For suppose, if possible, that it be out of the Line N G, as in Q, and let the Center of the Gravity of the Portion B N C, be V, and draw V Q. Because therefore from the Portion demerged in the Liquid the Portion of the Sphære B N C, not having the same Center of Gravity, is cut off, the Center of Gravity of the Remainder of the Portion B G C shall, by the 8 of the first Book of* Archimedes, De Centro Gravitatis C



(a) By 29. of the first of *Eucl.*

(b) By 3. of the third.

Plano-

Planorum, *be in the Line V Q prolonged: But that is impossible; for it is in the Axis M G: It followeth, therefore, that the Center of Gravity or the Portion demerged in the Liquid be in the Line N K: which we propounded to be proved.*

D But the Centre of Gravity of the whole Portion is in the Line F T, betwixt the Point R and the Point F; let us suppose it to be the Point X.] *Let the Sphere be compleated, so as that there be added of that Portion the Axis T Y, and the Center of Gravity Z. And because that from the whole Sphere, whose Centre of Gravity is K, as we have also demonstrated in the (c) Book before named, there is cut off the Portion E T H, having the Centre of Gravity Z, the Centre of the remainder of the Portion E F H shall be in the Line Z K prolonged: And therefore it must of necessity fall betwixt K and F.*

(c) *By* 8 *of the first of* Archimedes.

E The remainder, therefore, of the Figure, elevated above the Surface of the Liquid, hath its Center of Gravity in the Line R X prolonged.] *By the same* 8 *of the first Book of* Archimedes, de Centro Gravitatis Planorum.

F Now the Gravity of the Figure that is above the Liquid shall press from above downwards according to S L; and the Gravity of the Portion that is submerged in the Liquid shall press from below upwards, according to the Perpendicular R L.] *By the second Supposition of this. For the Magnitude that is demerged in the Liquid is moved upwards with as much Force along R L, as that which is above the Liquid is moved downwards along S L: as may be shewn by* Proposition *6. of this. And because they are moved along severall other Lines, neither causeth the others being less moved; the which it continually doth when the Portion is set according to the Perpendicular: For then the Centers of Gravity of both the Magnitudes do concur in one and the same Perpendicular, namely, in the Axis of the* Portion*: and look with what force or* Impetus *that which is in the Liquid tendeth upwards, and with the like doth that which is above or without the Liquid tend downwards along the same Line: And therefore, in regard that the one doth not* * *exceed the other, the Portion shall no longer move, but shall stay and rest allwayes in one and the same Position, unless some extrinsick Cause chance to intervene.*

* *Or overcome.*

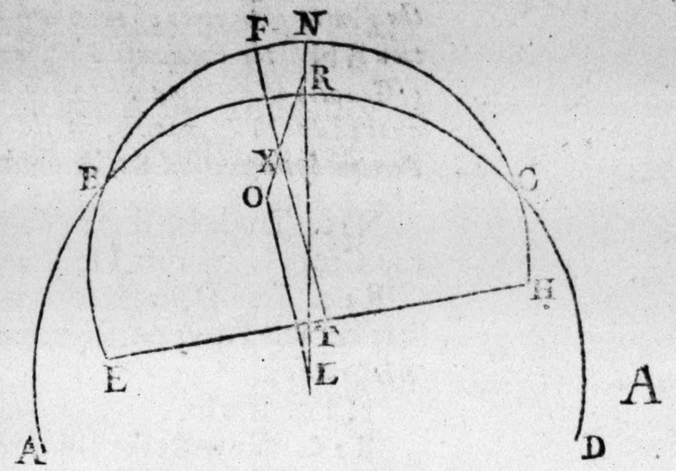
PROP. IX. THEOR. IX.

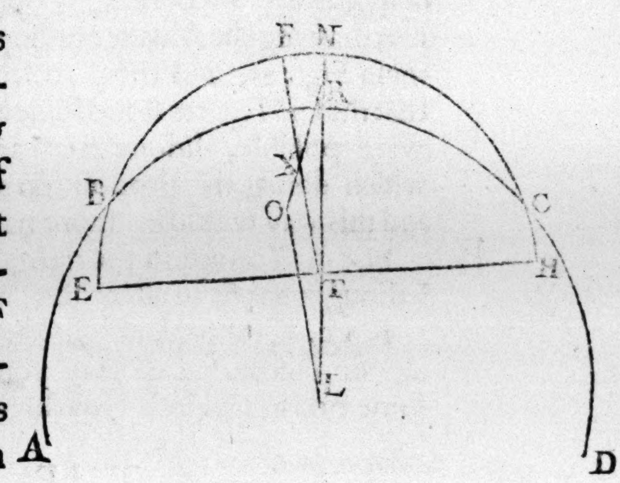
* *But if the Figure, lighter than the Liquid, be demitted into the Liquid, so, as that its Base be wholly within the said Liquid, it shall continue in such manner erect, as that its Axis shall stand according to the Perpendicular.*

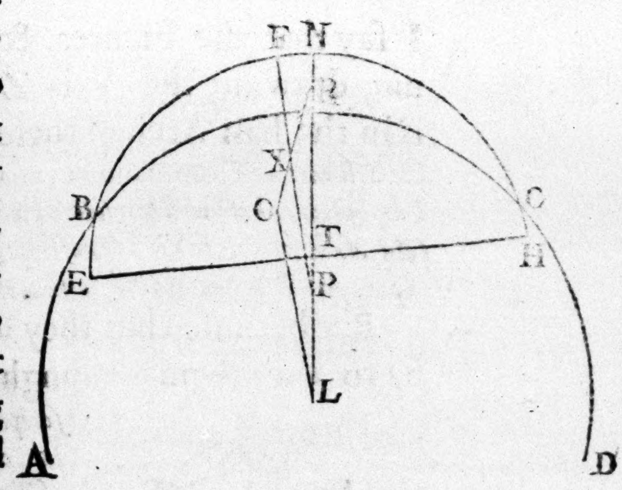
* In some Greek Coppies this is no distinct Proposition, but all Commentators, do divide it from the Precedent, as having a distinct demonstration in the Originall.

For suppose, such a Magnitude as that aforenamed to be demitted into the Liquid; and imagine a Plane to be produced thorow the Axis of the Portion, and thorow the Center of the Earth: And let the *Section* of the Surface of the Liquid, be the Circumference A B C D, and of the Figure the Circumference E F H: And let E H be a Right Line, and F T the Axis of the Portion. If now it were possible, for satisfaction of the Adversary, let it be supposed that the said Axis were not according to the Perpendicular: we are now to demonstrate that the Figure will not so continue,

nue,but will return to be according to the Perpendicular. It is manifest that the Centre of the Sphære is in the Line F T. And again, forasmuch as the Portion of a Sphære may be greater or lesser than an Hemisphære, and may also be an Hemisphære, let the Centre of the Sphære in the Hemisphære be the Point T, & in the lesser Portion the Point P, and in the Greater the Point R. And speaking first of that greater Portion which hath its Base within the Liquid, thorow R and L, the Earths Centre, draw the line RL. The Portion that is above the Liquid, hath its Axis in the Perpendicular passing thorow R; and by what hath been said before, its Centre of Gravity shall be in the Line N R; let it be the Point R: But the Centre of Gravity of the whole Portion is in the line F T, betwixt R and F; let it be X: The remainder therefore of that Figure, which is within the Liquid shall have its Centre in the Right Line R X prolonged in the part towards X, taken so, that the part prolonged may have the same Proportion to X R, that the Gravity of the Portion that is above the Liquid hath to the Gravity of the Figure that is within the Liquid. Let O be the Centre of that same Figure: and thorow O draw the Perpendicular L O. Now the Gravity of the Portion that is above the Liquid shall press according to the Right Line R L downwards; and the Gravity of the Figure that is in the Liquid according to the Right Line O L upwards: There the Figure shall not continue; but the parts of it towards H shall move downwards, and those towards E upwards: & this shall ever be, so long as F T is according to the Perpendicular.



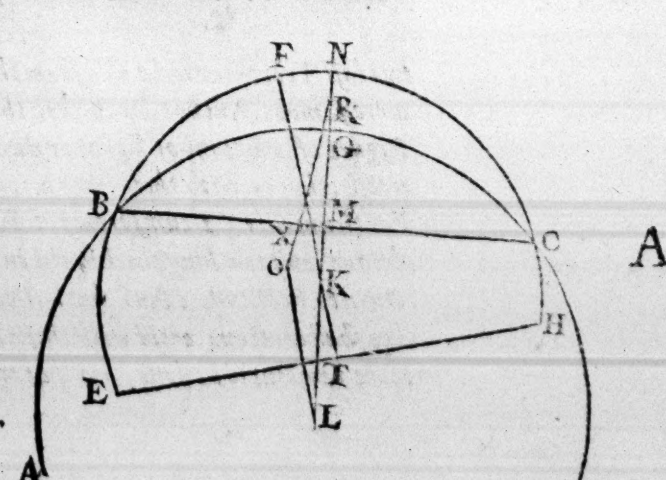




COMMANDINE.

The Portion that is above the Liquid hath its Axis in the Perpendicular passing thorow K.] *For draw B C cutting the Line N K in M; and let N K cut the Circumference* A B C D *in G. In the same manner as before: we will demonstrate, that the Axis*

of



of the Portion *of the Sphære is* N *M; and of the Portion* B *G C the Axis is G M: Wherefore the Centre of Gravity of them both shall be in the Line* N *M: And because that from the Portion* B N *C the Portion* B *G C, not having the same Centre of Gravity, is cut off, the Centre of Gravity of the remainder of the Magnitude that is above the Surface of the* Liquid *shall be in the* Line N *K; namely, in the Line which conjoyneth the Centres of Gravity of the said Portions by the foresaid* 8 *of* Archimedis de Centro Gravitatis Planorum.

NIC. Truth is, that in some of these Figures C is put for X, and so it was in the Greek Copy that I followed.

RIC. This Demostration is very difficult, to my thinking; but I believe that it is because I have not in memory the Propositions of that Book entituled *De Centris Gravium.*

NIC. It is so.

RIC. We will take a more convenient time to discourse of that, and now return A to speak of the two last Propositions. And I say that the Figures incerted in the demonstration would in my opinion, have been better and more intelligble unto me, drawing the Axis according to its proper Position; that is in the half Arch of these Figures, and then, to second the Objection of the Adversary, to suppose that the said Figures stood somewhat Obliquely, to the end that the said Axis, if it were possible, did not stand according to the Perpendicular so often mentioned; which doing, the Proposition would be proved in the same manner as before: and this way would be more naturall and clear.

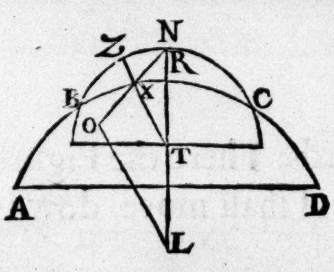
B NIC. You are in the right, but because thus they were in the Greek Copy, I thought not fit to alter them, although unto the better.

RIC. Companion, you have thorowly satisfied me in all that in the beginning of our Discourse I asked of you, to morrow, God permitting, we will treat of some other ingenious Novelties.

THE TRANSLATOR.

A I say that the Figures, &c. would have been more intelligible to me, drawing the Axis Z T according to its proper Position, that is in the half Arch of these Figures.] *And in this consideration I have followed the Schemes of* Commandine, *who being the Restorer of the Demonstrations of these two last Propositions, hath well considered what* Ricardo *here proposeth, and therefore hath drawn the said Axis (which in the Manuscripts that he had by him is lettered F T, and not as in that of* Tartaylia *Z T,) according to that its proper Position.*

B But because thus they were in the Greek Copy, I thought not fit to alter them although unto the better.] *The Schemes of those Manuscripts that* Tartaylia *had seen were more imperfect then those in Commandines Copies; but for variety sake, take here one of* Tartaylia, *it being that of the Portion of a Sphære, equall to an Hemisphære, with its Axis oblique, and its Base dimitted into the Liquid, and Lettered as in this Edition.*



Now Courteous Readers, I hope that you may, amidst the great Obscurity of the Originall in the Demonstrations of these two last Propositions, be able from the joynt light of these two Famous Commentators of our more famous Author, *to discern the truth of the Doctrine affirmed, namely, That Solids of the Figure of Portions of Sphæres demitted into the Liquid with their Bases upwards shall stand erectly, that is, with their Axis according to the Perpendicular drawn from the Centre of the Earth unto its Circumference: And that if the said* Portions *be demitted with their Bases oblique and touching the Liquid in one* Point, *they shall not rest in that Obliquity, but shall return to Rectitude: And that lastly, if these* Portions *be demitted with their Bases downwards, they shall continue erect with their Axis according to the* Perpendicular *aforesaid: so that no more remains to be done, but that we set before you the 2 Books of this our* Admirable Author.

AR-

ARCHIMEDES, HIS TRACT DE INSIDENTIBUS HUMIDO, OR, Of the NATATION of BODIES Upon, or Submersion In the WATER, or other LIQUIDS.

BOOK II.

PROP. I. THEOR. I.

If any Magnitude lighter than the Liquid be demitted into the said Liquid, it shall have the same proportion in Gravity to a Liquid of equal Masse, that the part of the Magnitude demerged hath unto the whole Magnitude.

FOR let any Solid Magnitude, as for instance F A, lighter than the Liquid, be demerged in the Liquid, which let be F A: And let the part thereof immerged be A, and the part above the Liquid F, It is to be demonstrated that the Magnitude F A hath the same proportion in Gravity to a Liquid of Equall Masse that A hath to F A. Take any Liquid Magnitude, as suppose N I, of equall Masse with F A; and let F be equall to N, and A to I: and let the Gravity of the whole Magnitude F A be B, and let that of the Magnitude N I be O, and let that of I be R. Now the Magnitude F A hath the same proportion unto N I that the Gravity B hath to the Gravity O R: But for asmuch as the Magnitude F A demitted into the Liquid is lighter than the said Liquid, it is manifest that a Masse of the Liquid, I, equall to the part of the Magnitude demerged, A, hath equall Gravity with the whole Magnitude, F A: For this was (*a*) above demonstrated: But B is the Gravity of the Magnitude F A, and R of I: Therefore

(a) *By 5. of the first of this.*

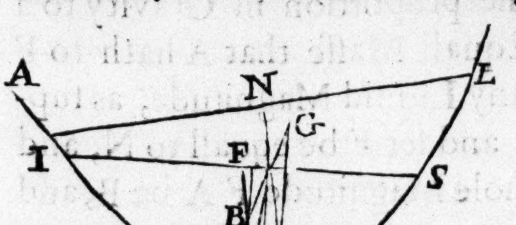
Therefore B and R are equall. And because that of the Magnitude F A the Gravity is B : Therefore of the Liquid Body N I the Gravity is O R. As F A is to N I, so is B to O R, or, so is R to O R : But as R is to O R, so is I to N I, and A to F A : Therefore I is to N I, as F A to N I : And as I to N I so is (b) A to F A. Therefore F A is to N I, as A is to F A : Which was to be demonstrated.

(b) *By* 11. *of the fifth of* Eucl.

PROP. II. THEOR. II.

A *The Right Portion of a Rightangled Conoide, when it shall have its Axis lesse than* sesquialter ejus quæ ad Axem (*or of its* Semi-parameter) *having any whatever proportion to the Liquid in Gravity, being demitied into the Liquid so as that its Base touch not the said Liquid, and being set stooping, it shall not remain stooping, but shall be restored to uprightnesse. I say that the said Portion shall stand upright when the Plane that cuts it shall be parallel unto the Surface of the Liquid.*

Let there be a Portion of a Rightangled Conoid, as hath been said; and let it lye stooping or inclining : It is to be demonstrated that it will not so continue but shall be restored to rectitude. For let it be cut through the Axis by a plane erect upon the Surface of the Liquid, and let the Section of the Portion be A P O L, the Section of a Rightangled Cone, and let the Axis of the Portion and Diameter of the Section be N O : And let the Section of the Surface of the Liquid be I S. If now the Portion be not erect, then neither shall A L be Parallel to I S : Wherefore N O will not be at Right Angles with I S.



* *Supplied by* Federico Commandino.

Draw therefore K ω, touching the Section of the Cone I, in the Point P [that is parallel to I S : and from the Point P unto I S
B draw P F parallel unto O N, * which shall be the Diameter of the Section I P O S, and the Axis of the Portion demerged in the Li-
C quid. In the next place take the Centres of Gravity : * and of the Solid Magnitude A P O L, let the Centre of Gravity be R ; and
D of I P O S let the Centre be B : * and draw a Line from B to R prolonged unto G ; which let be the Centre of Gravity of the remaining

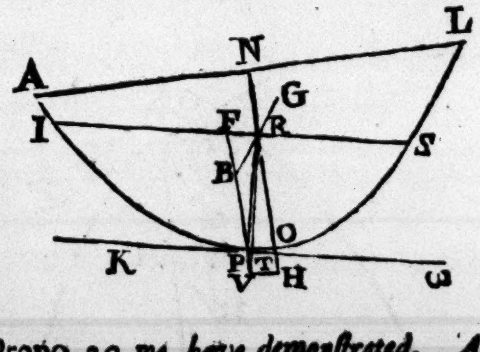
remaining Figure I S L A. Because now that N O is *Sesquialter* of R O, but less than *Sesquialter ejus quæ usque ad Axem* (or of its *Semi-parameter*;) * R O shall be lesse than *quæ usque ad Axem* (or than the *Semi-parameter*;) * whereupon the Angle R P ω shall be acute. For since the Line *quæ usque ad Axem* (or *Semi-parameter*) is greater than R O, that Line which is drawn from the Point R, and perpendicular to K ω, namely R T, meeteth with the line F P without the Section, and for that cause must of necessity fall between the Points *P* and ω: Therefore if Lines be drawn through B and G, parallel unto R T, they shall contain Right Angles with the Surface of the Liquid: * and the part that is within the Liquid shall move upwards according to the Perpendicular that is drawn thorow B, parallel to R T; and the part that is above the Liquid shall move downwards according to that which is drawn thorow G; and the Solid A P O L shall not abide in this Position; for that the parts towards A will move upwards, and those towards B downwards; Wherefore N O shall be constituted according to the Perpendicular.] E F G

COMMANDINE.

The Demonstration of this proposition hath been much desired; which we have (in like manner as the 8 Prop. of the first Book) restored according to Archimedes *his own Schemes, and illustrated it with Commentaries.*

The Right Portion of a Rightangled Conoid, when it shall have its Axis lesse than *Sesquialter ejus quæ usque ad Axem* (or of its *Semi-parameter*] *In the Translation of* Nicolo Tartaglia *it is falsly read* greater then Sesquialter, *and so its rendered in the following Proposition; but it is the Right Portion of a Conoid cut by a Plane at Right Angles, or erect, unto the Axis: and we say that Conoids are then constituted erect when the cutting Plane, that is to say, the Plane of the Base, shall be parallel to the Surface of the Liquid.* A

Which shall be the Diameter of the Section I P O S, and the Axis of the Portion demerged in the Liquid.] *By the 46 of the first of the Conicks of* Apollonious, *or by the Corollary of the 51 of the same.* B



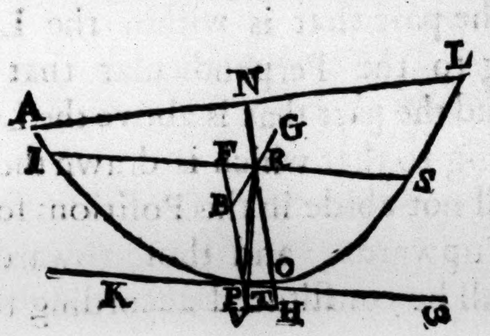
And of the Solid Magnitude A P O L, let the Centre of Gravity be R; and of I P O S let the Centre be B.] *For the Centre of Gravity of the Portion of a Rightangled Conoid is in its Axis, which it so divideth as that the part thereof terminating in the vertex, be double to the other part terminating in the Base; as in our Book* De Centro Gravitatis Solidorum Propo. 29. *we have demonstrated. And since the Centre of Gravity of the Portion A P O L is R, O R shall be double to R N and therefore N O shall be Sesquialter of O R. And for the same reason, B the Centre of Gravity of the Portion I P O S is in the Axis P F, so dividing it as that P B is double to B F;* C

And draw a Line from B to R prolonged unto G; which let be the Centre of Gravity of the remaining Eigure I S L A.] D

For if, the Line B R being prolonged unto G, G R hath the same proportion to R B as the Portion of the Conoid I P O S hath to the remaining Figure that lyeth above the Surface of the Liquid, the Point G shall be its Centre of Gravity; by the 8 of the second of Archimedes de Centro Gravitatis Planorum, vel de Æquiponderantibus.

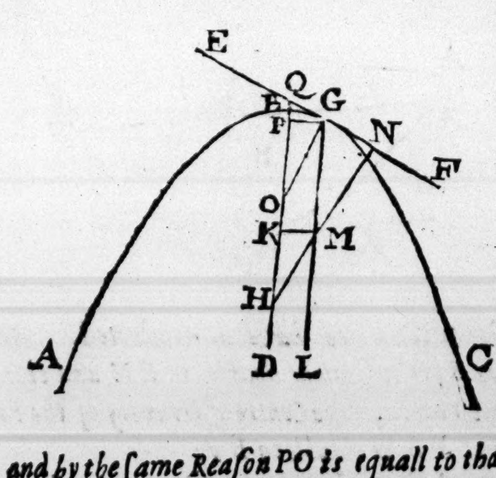
E R O shall be less than *quæ usque ad Axem* (or than the Semi-parameter.] *By the 10 Proposit. of* Euclids *fifth Book of Elements. The Line* quæ usque ad Axem, *(or the Semi-parameter) according to* Archimedes, *is the half of that* juxta quam possunt, quæ á Sectione ducuntur, *(or of the Parameter;) as appeareth by the 4 Proposit. of his Book* De Conoidibus & Shpæroidibus *: and for what reason it is so called, we have declared in the Commentaries upon him by us published.*

F Whereupon the Angle R P ω shall be acute.] *Let the Line N O be continued out to H, that so R H may be equall to the Semi-parameter. If now from the Point H a Line be drawn at Right Angles to N H, it shall meet with F P without the Section; for being drawn thorow O parallel to A L, it shall fall without the Section, by the 17 of our first Book of* Conicks; *Therefore let it meet in V: and because F P is parallel to the Diameter, and H V perpendicular to the same Diameter, and R H equall to the Semi-parameter, the Line drawn from the Point R to V shall make Right Angles with that Line which the Section toucheth in the Point P: that is with K ω, as shall anon be demonstrated: Wherefore the Perpendiculat R T falleth betwixt A and ω; and the Angle R P ω shall be an Acute Angle.*

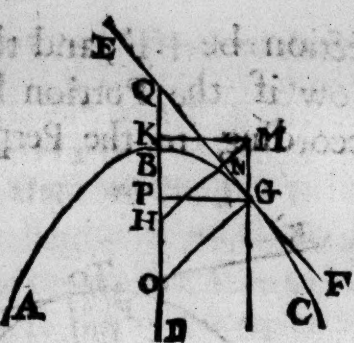


Let A B C be the Section of a Rightangled Cone, or a Parabola, and its Diameter B D; and let the Line E F touch the same in the Point G: and in the Diameter B D take the Line H K equall to the Semi-parameter: and thorow G, G L being drawn parallel to the Diameter, draw K M from the Point K at Right Angles to B D cutting G L in M: I say that the Line prolonged thorow H and M is perpendicular to E F, which it cutteth in N.

For from the Point G draw the Line G O at Right Angles to E F cutting the Diameter in O: and again from the same Point draw G P perpendicular to the Diameter: and let the said Diameter prolonged cut the Line E F in Q. P B shall be equall to B Q, by the 35 of our first Book of Conick Sections, (a) *and G P a Mean-proportionall betwixt Q P, and P O;* (b) *and therefore the Square of G P shall be equall to the Rectangle of O P Q: But it is also equall to the Rectangle comprehended under P B and the Line* juxta quam possunt, *or the Parameter, by the 11 of our first Book of* Conicks: (c) *Therefore, look what proportion Q P hath to P B, and the same hath the Parameter unto P O: But Q P is double unto P B, for that P B and B Q are equall, as hath been said: And therefore the Parameter shall be double to the said P O: and by the same Reason P O is equall to that which we call the Semi-parameter, that is, to K H: But* (d) *P G is equall to K M, and* (e) *the Angle O P G to the Angle H K M; for they are both Right Angles: And therefore O G also is equall to H M, and the Angle P O G unto the Angle*



(a) *By Cor. of 8. of 6. of* Euclide.
(b) *By 17. of the 6.*
(c) *By 14. of the 6.*
(d) *By 33. of the 1.*
(e) *By 4. of the 1.*



Angle K H M : *Therefore* (f) O G *and* H N *are parallel, and the* (g) *Angle* H N F *equall to the Angle* O G F ; *for that* G O *being Perpendicular to* E F, H N *shall also be perpendicular to the same* : *Which was to be demonstrated.*

(f) *By* 28. *of the* 1.

(g) *By* 29. *of th* 1

G

And the part which is within the Liquid doth move upwards according to the Perpendicular that is drawn thorow B parallel to R T.] *The reason why this moveth upwards, and that other downwards, along the Perpendicular Line, hath been shewn above in the* 8 *of the first Book of this; so that we have judged it needlesse to repeat it either in this, or in the rest that follow.*

THE TRANSLATOR.

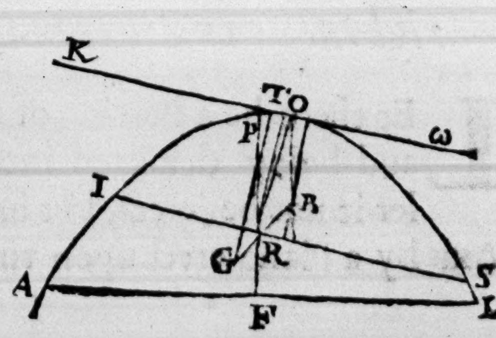
In the Antient *Parabola (namely that assumed in a Rightangled Cone) the* Line juxta quam Possunt quæ in Sectione ordinatim ducuntur *(which I, following* Mydorgius, *do call the* Parameter*) is* (a) *double to that* quæ ducta est à Vertice Sectionis usque ad Axem, *or in* Archimedes *phrase,* τῆς μέχρι τοῦ ἄξονος ; *which I for that cause, and for want of a better word, name the* Semiparameter : *but in* Modern *Parabola's it is greater or lesser then double. Now that throughout this Book* Archimedes *speaketh of the Parabola in a Rectangled Cone, is manifest both by the first words of each Proposition, & by this that no Parabola hath its Parameter double to the Line* quæ est a Sectione ad Axem, *save that which is taken in a Rightangled Cone. And in any other Parabola, for the Line* τῆς μέχρι τοῦ ἄξονος *or* quæ usque ad Axem *to usurpe the Word* Semiparameter *would be neither proper nor true: but in this case it may pass*

(a) Rivalt. *in* Archimed. *de Conoid & Sphæroid.* Prop. 3. Lem. 1.

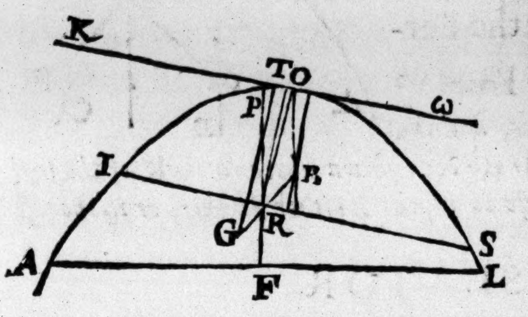
PROP. III. THEOR. III.

The Right Portion of a Rightangled Conoid, when it shall have its Axis lesse than sesquialter of the Semi-parameter, the Axis having any what ever proportion to the Liquid in Gravity, being demitted into the Liquid so as that its Base be wholly within the said Liquid, and being set inclining, it shall not remain inclined, but shall be so restored, as that its Axis do stand upright, or according to the Perpendicular.

Let any Portion be demitted into the Liquid, as was said; and let its Base be in the Liquid; and let it be cut thorow the Axis, by a Plain erect upon the Surface of the Liquid, and let the Section be A P O L, the Section of a Right angled Cone: and let the Axis of the Portion and Diameter of the Section



Section be P F, and the Section of the Surface of the Liquid I S. Now if the Portion lye inclined or ſtooping its Axis ſhall not be according to the Perpendicular : Wherefore P F ſhall not be at Right angles with I S. Draw K ω parallel unto I S touching the Section A P O L in O : and of the Solid Magnitude A P O L let the Center of Gravity be R ; and of the Solid I P O S let the Centre be B ; and draw a Line from B to R prolonging it to G, that ſo G may be the Centre of Gravity of the remaining Figure I S L A. It ſhall be demonſtrated that the Angle under R O K is acute, and that the Perpendicular continued out from R unto ω K ſhall fall betwixt O and K ; let it be R T. And if from the Points G and B Parallels be drawn to R T, that part of the Solid Magnitude which is within the Liquid ſhall move upwards according to the Perpendicular drawn thorow G ; and that part which is without, or above, the Liquid downwards according to the Perpendicular drawn thorow B : and the Solid A P O L ſhall not remain or continue in that Poſition in the Liquid, but the part towards A ſhall move upwards, and the part towards L downwards : And therefore P F ſhall ſtand according to the Perpendicular.

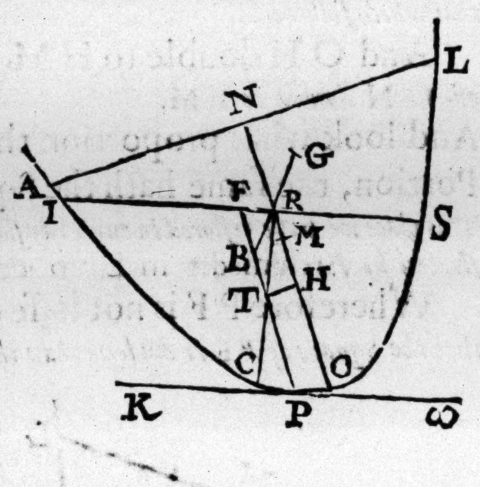


PROP. IV. THEOR. IV.

The Right Portion of a Rightangled Conoid lighter than the Liquid, when it ſhall have its Axis greater than Seſquialter of the Semi-parameter, if it have not leſſer proportion in Gravity to the Liquid of equall Maſſe than the Square made of the Exceſſe by which the Axis is greater than Seſquialter of the Semi-parameter hath to the Square made of the Axis, being demitted into the Liquid ſo as that its Baſe touch not the Liquid, and being ſet inclining, it ſhall not continue inclined, but ſhall return to uprightneſſe or Perpendicularity.

LEt there be a Portion of a Rightangled Conoid, as was ſaid; and being demitted into the Liquid, if it be poſſible, let it not be erect, but inclining : and let it be cut thorow the Axis by a Plane erect upon the Surface of the Liquid, and let the Section

Section of the Portion be A P O L, the Section of a Rightangled Cone; and let the Axis of the Portion and Diameter of the Section be N O, and the Section of the Surface of the Liquid I S. If now the Portion be not erect, then N O shall not be at equall Angles with I S. Draw R ω touching the Section of the Rightangled Conoid in P, and parallel to I S: and from the Point P and parall to O N draw P F: and take the Centers of Gravity; and of the Solid A P O L let the Centre be R; and of that which lyeth within the Liquid let the Centre be B; and draw a Line from B to R prolonging it to G, that G may be the Centre of Gravity of the Solid that is above the Liquid. And because N O is sesquialter of R O, and is greater than sesquialter of the Semi-Parameter; it is manifest that (a) R O is greater than the Semi-parameter. *Let therefore R H be equall to the Semi-Parameter, *and O H double to H M. Forasmuch therefore as N O is sesquialter of R O, and M O of O H, (b) the Remainder N M shall be sesquialter of the Remainder R H: Therefore the Axis is greater than sesquialter of the Semi-parameter by the quantity of the Line M O. And let it be supposed that the Portion hath not lesse proportion in Gravity unto the Liquid of equall Masse, than the Square that is made of the Excesse by which the Axis is greater than sesquialter of the Semi-parameter hath to the Square made of the Axis: It is therefore manifest that the Portion hath not lesse proportion in Gravity to the Liquid than the Square of the Line M O hath to the Square of N O: But look what proportion the Portion hath to the Liquid in Gravity, the same hath the Portion submerged to the whole Solid: for this hath been demonstrated (c) above: *And look what proportion the submerged Portion hath to the whole Portion, the same hath the Square of P F unto the Square of N O: For it hath been demonstrated in (d) *Lib. de Conoidibus*, that if from a Rightangled Conoid two Portions be cut by Planes in any fashion produced, these Portions shall have the same Proportion to each other as the Squares of their Axes: The Square of P F, therefore, hath not lesse proportion to the Square of N O than the Square of M O hath to the Square of N O: *Wherefore P F is not lesse than M O, *nor B P than H O. *If therefore, a Right Line be drawn from H at Right Angles unto N O, it shall meet with B P, and shall fall betwixt B and P; let it fall in T: (e) And because P F is parallel to the Diameter, and H T is perpendicular unto the same Diameter, and R H equall to the Semi-parameter; a Line drawn from R to T and prolonged, maketh Right Angles with the Line contingent



(a) *By* 10. *of the fifth.*

A

B

(b) *By* 19. *of the fifth.*

C

(c) *By* 1. *of this second Book.*

(d) *By* 6. *De Conoidibus & Sphæroidibus of Archimedes.*

D

E

F

(e) *By* 2. *of this second Book.*

contingent unto the Section in the Point P: Wherefore it also maketh Right Angles with the Surface of the Liquid: and that part of the Conoidall Solid which is within the Liquid shall move upwards according to the Perpendicular drawn thorow B parallel to R T; and that part which is above the Liquid shall move downwards according to that drawn thorow G, parallel to the said R T: And thus it shall continue to do so long untill that the Conoid be restored to uprightnesse, or to stand according to the Perpendicular.

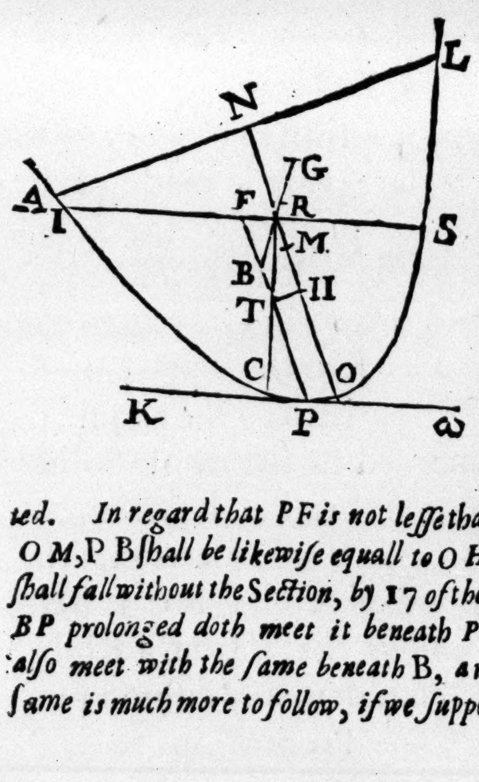
COMMANDINE.

A Let therefore R H be equall to the Semi-parameter.] *So it is to be read, and not R M, as* Tartaglia's *Translation hath it; which may be made appear from that which followeth.*

B And O H double to H M.] *In the Translation aforenamed it is falsly rendered,* O N *double to* R M.

C And look what proportion the Submerged Portion hath to the whole Portion, the same hath the Square of P F unto the Square of N O.] *This place we have restored in our Translation, at the request of some friends: But it is demonstrated by* Archimedes in Libro de Conoidibus & Sphæroidibus, Propo. 26.

D Wherefore P F is not lesse than M O.] *For by* 10 *of the fifth it followeth that the Square of P F is not lesse than the Square of M O: and therefore neither shall the Line P F be lesse than the Line M O, by* 22 *of the sixth.*



E Nor B P than H O,] *For as P F is to P B, so is M O to H O: and, by Permutation, as P F is to M O, so is B P to H O; But P F is not lesse than M O as hath bin proved;* (a) *Therefore neither shall B P be lesse than H O.*

(a) *By* 14. *of the sixth.*

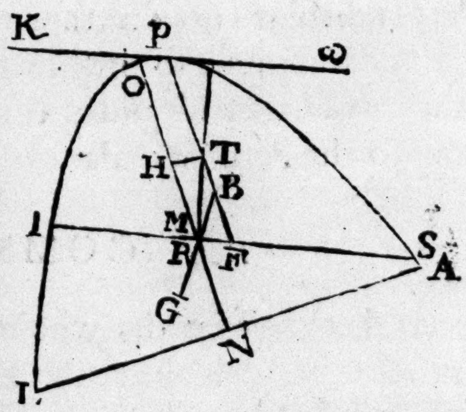
F If therefore a Right Line be drawn from H at Right Angles unto N O, it shall meet with B P, and shall fall betwixt B and P.] *This Place was corrupt in the Translation of* Tartaglia: *But it is thus demonstrated. In regard that P F is not lesse than O M, nor P B than O H, if we suppose P F equall to O M, P B shall be likewise equall to O H: Wherefore the Line drawn thorow O, parallel to A L shall fall without the Section, by* 17 *of the first of our Treatise of Conicks; And in regard that B P prolonged doth meet it beneath P, Therefore the Perpendicular drawn thorow H doth also meet with the same beneath B, and it doth of necessity fall betwixt B and P: But the same is much more to follow, if we suppose P F to be greater than O M.*

PROP.

PROP. V. THEOR. V.

The Right Portion of a Right-Angled Conoid lighter than the Liquid, *when it shall have its Axis greater than* Sesquialter *of the Semi-parameter, if it have not greater proportion in Gravity to the* Liquid *[of equal Mass] than the Excesse by which the Square made of the Axis is greater than the Square made of the Excesse by which the Axis is greater than* sesquialter *of the Semi-Parameter hath to the Square made of the Axis being demitted into the Liquid, so as that its Base be wholly within the Liquid, and being set inclining, it shall not remain so inclined, but shall turn about till that its Axis shall be according to the Perpendicular.*

For let any Portion be demitted into the Liquid, as hath been said; and let its Base be wholly within the Liquid, And being cut thorow its Axis by a Plain erect upon the Surface of the Liquid; its Section shall be the Section of a Rightangled Cone: Let it be A P O L, and let the Axis of the Portion and Diameter of the Section be N O; and the Section of the Surface of the Liquid I S. And because the Axis is not according to the Perpendicular, N O will not be at equall angles with I S. Draw K ω touching the Section A P O L in P, and parallel unto I S: and thorow P, draw P F parallel unto N O: and take the Centres of Gravity; and of the Solid A P O L let the Centre be R; and of that which lyeth above the Liquid let the Centre be B; and draw a Line from B to R, prolonging it to G; which let be the Centre of Gravity of the Solid demerged within the Liquid: and moreover, take R H equall to the Semi-parameter, and let O H be double to H M; and do in the rest as hath been said (a) above. Now forasmuch as it was supposed that the Portion hath not greater proportion in Gravity to the Liquid, than the Excesse by which the Square N O is greater than the Square M O, hath to the said Square N O: And in regard that whatever proportion in Gravity



(a) In 4. Prop. of this.

the Portion hath to the Liquid of equall Masse, the same hath the Magnitude of the Portion submerged unto the whole Portion; as hath been demonstrated in the first Proposition; The Magnitude submerged, therefore, shall not have greater proportion to the whole (b) Portion, than that which hath been mentioned: *And therefore the whole Portion hath not greater proportion unto that which is above the Liquid, than the Square N O hath to the Square M O: But the (c) whole Portion hath the same proportion unto that which is above the Liquid that the Square N O hath to the Square P F: Therefore the Square N O hath not greater proportion unto the Square P F, than it hath unto the Square M O: *And hence it followeth that P F is not lesse than O M, nor P B than O H: * A Line, therefore, drawn from H at Right Angles unto N O shall meet with B P betwixt P and B: Let it be in T: And because that in the Section of the Rectangled Cone P F is parallel unto the Diameter N O; and H T perpendicular unto the said Diameter; and R H equall to the Semi-parameter: It is manifest that R T prolonged doth make Right Angles with K P ω: And therefore doth also make Right Angles with I S: Therefore R T is perpendicular unto the Surface of the Liquid; And if thorow the Points B and G Lines be drawn parallel unto R T, they shall be perpendicular unto the Liquids Surface. The Portion, therefore, which is above the Liquid shall move downwards in the Liquid according to the Perpendicular drawn thorow B; and that part which is within the Liquid shall move upwards according to the Perpendicular drawn thorow G; and the Solid Portion A P O L shall not continue so inclined, [*as it was at its demersion*], but shall move within the Liquid untill such time that N O do stand according to the Perpendicular.

(a) *By* 11. *of the fifth*.

A

(b) *By* 26. *of the Book* De Conoid. & Sphæroid.

B

C

COMMANDINE.

A And therefore the whole Portion hath not greater proportion unto that which is above the Liquid, than the Square N O hath to the Square M O.] *For in regard that the Magnitude of the Portion demerged within the Liquid hath not greater proportion unto the whole Portion than the Excesse by which the Square N O is greater than the Square M O hath to the said Square N O; Converting of the Proportion, by the* 26. *of the fifth of* Euclid, *of* Campanus *his Translation, the whole Portion shall not have lesser proportion unto the Magnitude submerged, than the Square N O hath unto the Excesse by which N O is greater than the Square M O. Let a Portion be taken; and let that part of it which is above the Liquid be the first Magnitude; the part of it which is submerged the second: and let the third Magnitude be the Square M O; and let the Excesse by which the Square N O is greater than the Square M O be the fourth. Now of these Magnitudes, the proportion of the first and second, unto the second, is not lesse than that of the third & fourth unto the fourth: For the Square M O together with the Excesse by which the Square N O exceedeth the Square M O is equall unto the said Square N O: Wherefore, by Conversion of Proportion, by* 30 *of the said fifth Book, the proportion of the first and second unto the first, shall not be greater than that of the third and fourth unto the third: And, for the same cause,*

the proportion of the whole Portion unto that part thereof which is above the Liquid shall not be greater than that of the Square NO unto the Square M O: which was to be demonstrated.

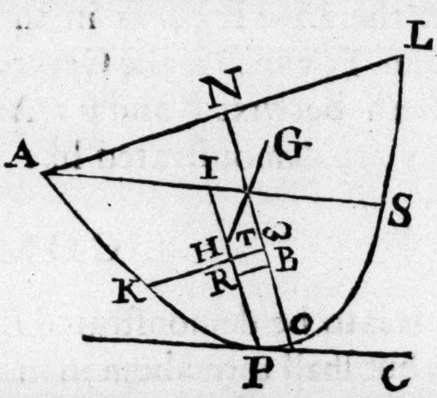
And hence it followeth that P F is not lesse than O M, nor P B than O H.] *This followeth by the* 10 *and* 14 *of the fifth, and by the* 22 *of the sixth of* Euclid, *as hath been said above.* B

A Line, therefore, drawn from H at Right Angles unto N O shall meet with P B betwixt P and B.] *Why this so falleth out, we will shew in the next.* C

PROP. VI. THEOR. VI.

The Right Portion of a Rightangled Conoid lighter than the Liquid, when it shall have its Axis greater than sesquialter of the Semi-parameter, but lesse than to be unto the Semi-parameter in proportion as fifteen to fower, being demitted into the Liquid so as that its Base do touch the Liquid, it shall never stand so enclined as that its Base toucheth the Liquid in one Point only.

Let there be a Portion, as was said; and demit it into the Liquid in such fashion as that its Base do touch the Liquid in one only Point: It is to be demonstrated that the said Portion shall not continue so, but shall turn about in such manner as that its Base do in no wise touch the Surface of the Liquid. For let it be cut thorow its Axis by a Plane erect upon the Liquids Surface: and let the Section of the Superficies of the Portion be A P O L, the Section of a Rightangled Cone; and the Section of the Surface of the Liquid be A S; and the Axis of the Portion and Diameter of the Section N O: and let it be cut in F, so as that O F be double to F N; and in ω so, as that N O may be to F ω in the same proportion as fifteen to four; and at Right Angles to N O draw ω K. Now because N O hath greater proportion unto F ω than unto the Semi-parameter, let the Semi-parameter be equall to F B: and draw P C parallel unto A S, and touching the Section A P O L in P; and P I parallel unto *N O*; and first let P I cut K ω in H. Forasmuch, therefore, as in the Portion A P O L, contained betwixt the Right Line and the Section of the Rightangled Cone, K ω is parallel to A L, and P I parallel unto the Diameter, and cut by the A

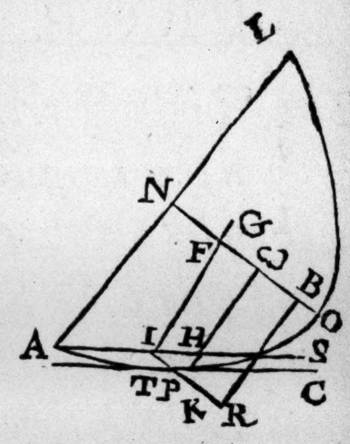


B

C

said $K\omega$ in H, and A S is parallel unto the *Line* that toucheth in P; It is necessary that P I hath unto P H either the same proportion that $N\omega$ hath to ω O, or greater; for this hath already been demonstrated: But $N\omega$ is sesquialter of ω O; and P I, therefore, is either Sesquialter of H P, or more than sesquialter: Wherefore

D P H is to H I either double, or lesse than double. *Let* P T be double to T I: the Centre of Gravity of the part which is within the *Liquid* shall be the Point T. Therefore draw a *Line* from T to F prolonging it; and let the Centre of Gravity of the part which is above the *Liquid* be G: and from the Point B at Right Angles unto *N O* draw B R. And seeing that P I is parallel unto the Diameter *N O*, and B R perpendicular unto the said Diameter, and F B equall to the Semi-parameter; It is manifest that the *Line* drawn thorow the Points F and R being prolonged, maketh equall Angles with that which toucheth the Section A P O L in the Point P: and therefore doth also make Right Angles with A S, and with the Surface of the *Liquid*: and the *Lines* drawn thorow T and G parallel unto F R shall be also perpendicular to the Surface of the *Liquid*: and of the Solid Magnitude A P O L, the part which is within the *Liquid* moveth upwards according to the Perpendicular drawn thorow T; and the part which is above the *Liquid* moveth downwards according to that drawn thorow G:



E The Solid A *P* O L, therefore, shall turn about, and its Base shall not in the least touch the Surface of the *Liquid*, And if *P* I do not cut the *Line* $K\omega$, as in the second Figure, it is manifest that the *Point* T, which is the Centre of Gravity of the submerged *Portion*, falleth betwixt *P* and I: And for the other particulars remaining, they are demonstrated like as before.

COMMANDINE.

A It is to be demonstrated that the said *Portion* shall not continue so, but shall turn about in such manner as that its Base do in no wise touch the Surface of the Liquid.] *These words are added by us, as having been omitted by* Tartaglia.

B Now because N O hath greater proportion to $F\omega$ than unto the Semi-parameter.] *For the Diameter of the Portion* N O *hath unto* $F\omega$ *the same proportion as fifteen to fower: But it was supposed to have lesse proportion unto the Semi-parameter than fifteen to fower: wherefore* N O *hath greater proportion unto* $F\omega$ *than unto the Semi-parameter: And therefore* (a) *the Semi-parameter shall be greater than the said* $F\omega$.

(a) *By* 10. *of the fifth.*

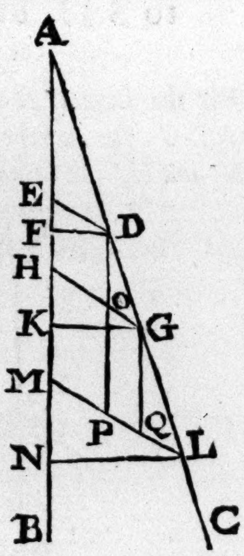
C Forasmuch, therefore, as in the *Portion A P O L*, contained, betwixt the Right Line and the Section of the Rightangled Cone $K\omega$ is parallel to A L, and *PI* parallel unto the Diameter, and cut by the

the said K in H, and A S is parallel unto the Line that toucheth in P; It is necessary that P I hath unto PH either the same proportion that N hath to O, or greater; for this hath already been demonstrated] *Where this is demonstrated either by* Archimedes *himself, or by any other, doth not appear; touching which we will here insert a Demonstration, after that we have explained some things that pertaine thereto.*

LEMMA I.

Let the Lines A B and A C contain the Angle B A C; and from the point D, taken in the Line A C, draw D E and D F at pleasure unto A B: and in the same Line any Points G and L being taken, draw G H & L M parallel to D E, & G K and L N parallel unto F D: Then from the Points D & G as farre as to the Line M L draw D O P, cutting G H in O, and G Q parallel unto B A. I say that the Lines that lye betwixt the Parallels unto F D have unto those that lye betwixt the Parallels unto D E (namely K N to G Q or to O P; F K to D O; and F N to D P) the same mutuall proportion: that is to say, the same that A F hath to A E.

For in regard that the Triangles A F D, A K G, and A N L are alike, and E F D, H K G, and M N L are also alike: Therefore, (a) as A F is to F D, so shall A K be to K G; and as F D is to F E, so shall K G be to K H: Wherefore, ex equali, *as A F is to F E, so shall A K be to K H: And, by Conversion of proportion, as A F is to A E, so shall A K be to K H. It is in the same manner proved that, as* A F *is to* A E, *so shall* A N *be to* A M. *Now* A N *being to* A M, *as* A K *is to* A H; *The (b) Remainder K N shall be unto the Remainder H M, that is unto G Q, or unto O P, as* A N *is to* A M: *that is, as* A F *is to* A E: *Again,* A K *is to* A H, *as* A F *is to* A E: *Therefore the Remainder F K shall be to the Remainder E H, namely to D O, as* A F *is to* A E. *We might in like manner demonstrate that so is F N to D P: Which is that that was required to be demonstrated.*



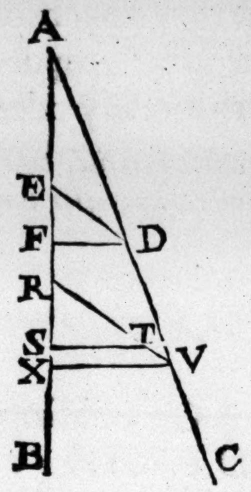
(a) *By 4. of the sixth.*

(b) *By 5. of the fifth.*

LEMMA II.

In the same Line A B let there be two Points R and S, so disposed, that A S may have the same Proportion to A R that A F hath to A E; and thorow R draw R T parallel to E D, and thorow S draw S T parallel to F D, so, as that it may meet with R T in the Point T. I say that the Point T falleth in the Line A C.

For



(a) By 9. of the fifth.

For if it be possible, let it fall short of it: and let RT be prolonged as farre as to AC in V; and then thorow V draw VX parallel to FD. Now, by the things we have last demonstrated, AX *shall have the same proportion unto* AR, *as* AF *hath to* AE. *But* AS *hath also the same proportion to* AR: *Wherefore* (a) AS *is equall to* AX, *the part to the whole, which is impossible. The same absurdity will follow if we suppose the Point T to fall beyond the Line AC: It is therefore necessary that it do fall in the said AC. Which we propounded to be demonstrated.*

LEMMA III.

* Or touch it.

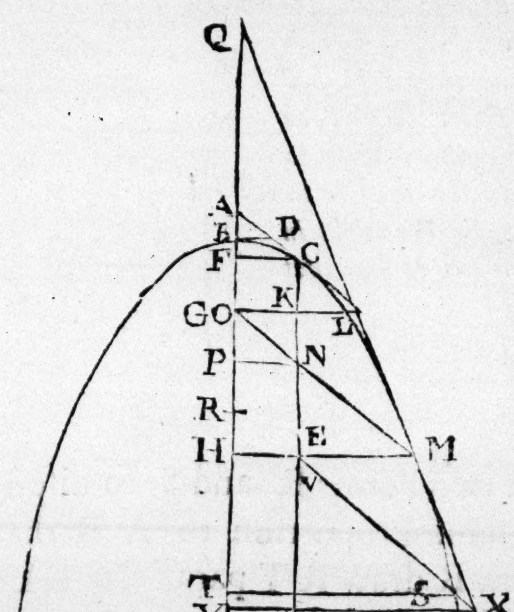
Let there be a Parabola, whose Diameter let be AB; and let the Right Lines AC and BD be * contingent to it, AC in the Point C, and BD in B: And two Lines being drawn thorow C, the one CE, parallel unto the Diameter; the other CF, parallel to BD; take any Point in the Diameter, as G; and as FB is to BG, so let BG be to BH: and thorow G and H draw GKL, and HEM, parallel unto BD; and thorow M draw MNO parallel to *AC*, and cutting the Diameter in O: and the Line NP being drawn thorow N unto the Diameter let it be parallel to BD. I say that HO is double to GB.

For the Line MNO cutteth the Diameter either in G, or in other Points: *and if it do cut it in G, one and the same Point shall be noted by the two letters G and O. Therfore FC, PN, and HEM being* Parallels, *and* AC *being* Parallels *to MNO, they shall make the Triangles* AFC, OPN *and* OHM *like to each other: Wherefore* (a) *OH shall be to HM, as* AF *to FC: and* * Permutando, *OH shall be to* AF, *as HM to FC: But the Square HM is to the Square GL as the Line* HB *is to the Line BG, by 20. of our first Book of* Conicks; *and the Square GL is unto the Square FC, as the Line GB is to the Line BF: and the Lines* HB, BG *and* BF *are thereupon* Proportionals: *Therefore the* (b) *Squares* HM, *GL and FC and there Sides, shall also be* Proportionals: *And, therefore, as the* (c) *Square* HM *is to the Square GL, so is the Line* HM *to the Line FC: But as* HM *is to FC, so is* OH *to* AF; *and as the Square* HM *is to the Square GL, so is the Line* HB *to BG; that is, BG to BF: From whence it followeth that OH is to* AF, *as BG to BF: And* Permutando, *OH is to BG, as* AF *to FB; But* AF *is double to FB: Therefore* AB *and BF are equall, by 35. of our first Book of* Conicks: And *therefore* NO *is double to GB: Which was to be demonstrated.*

(a) By 4. of the sixth.
* Or permitting.

(b) By 22. of the sixth.

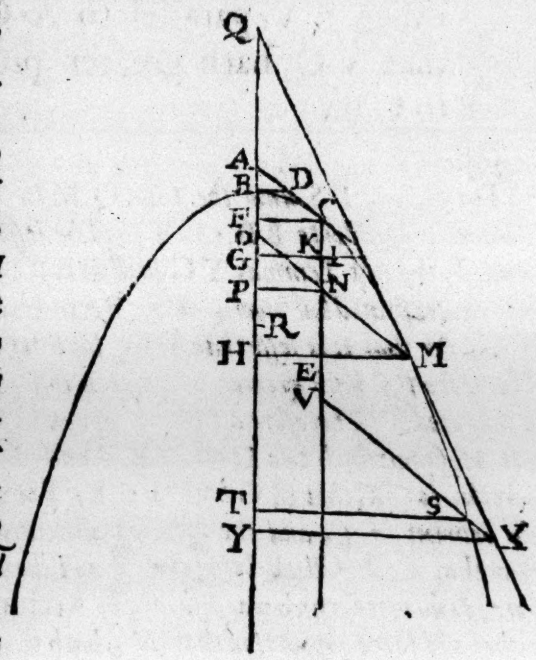
(c) By Cor. of 20. of the sixth.



LEMMA

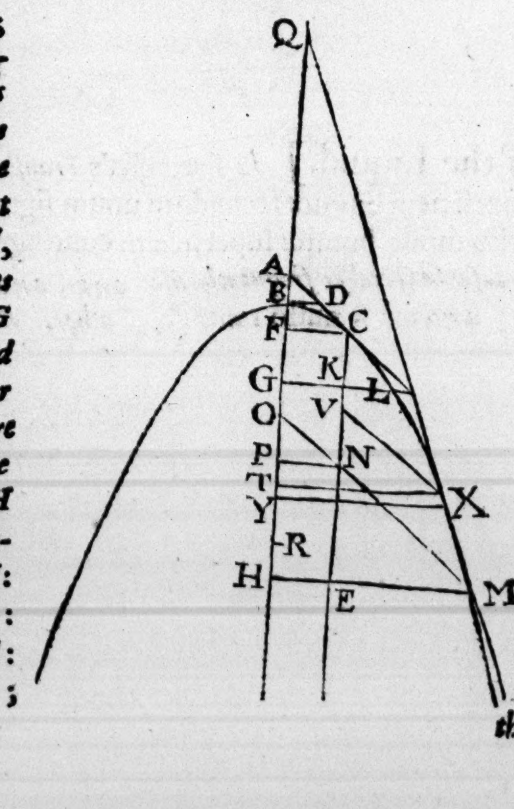
LEMMA IV.

The same things assumed again, and M Q being drawn from the Point M unto the Diameter, let it touch the Section in the Point M. I say that H Q hath to Q O, the same proportion that G H hath to C N.



For make H R equall to G F; and seeing that the Triangles A F C and O P N are alike, and P N equall to F C, we might in like manner demonstrate P O and F A to be equall to each other: Wherefore P O shall be double to F B: But H O is double to G B: Therefore the Remainder P H is also double to the Remainder F G; that is, to R H: And therefore it followeth that P R, R H and F G are equall to one another; as also that R G and P F are equall: For P G is common to both R P and G F. Since therefore, that H B is to B G, as G B is to B F, by Conversion of Proportion, B H shall be to H G, as B G is to G F: But Q H is to H B, as H O to B G. For by 35 *of our first Book of* Conicks, *in regard that Q M toucheth the Section in the Point M,* H B *and* B Q *shall be equall, and* Q H *double to* H B: *Therefore,* ex æquali, *Q H shall be to* H G, *as H O to* G F: *that is, to* H R: *and,* Permutando, *Q H shall be to* H O, *as* H G *to* H R: *again, by Conversion, H Q shall be to Q O, as H G to G R; that is, to* P F; *and, by the same reason, to* C N: *Which was to be demonstrated.*

These things therefore being explained, we come now to that which was propounded. I say, therefore, first that N C hath to C K the same proportion that H G hath to G B.



For since that H Q is to Q O, as H G to C N; that is, to A O, equall to the said C N: The Remainder G Q shall be to the Remainder Q A, as H Q to Q O: and, for the same cause, the Lines A C and G L prolonged, by the things that wee have above demonstrated, shall intersect or meet in the Line Q M. Again, G Q is to Q A, as H Q to Q O; that is, as H G to F P; as (a) *was but now demonstrated, But unto* (b) *G Q two Lines taken together, Q B that is H B, and B G are equall: and to Q A H F is equall; for if from the equall Magnitudes H B and B Q there be taken the equall Magnitudes* F B *and* B A, *the Remainder shall be equall; Therefore taking H G from the two Lines H B and B G, there shall remain a Magnitude double to* B G; *that is,* O H: *and* P F *taken from* F H, *the Remainder is* H P: *Wherefore* (c) O H *is to* H P, *as* G Q *to Q A: But as* G Q *is to Q A, so is* H Q *to* Q O;

(a) *By* 2. Lemma.
(b) *By* 4. Lemma.
(b) *By* 19. *of the fifth.*

that

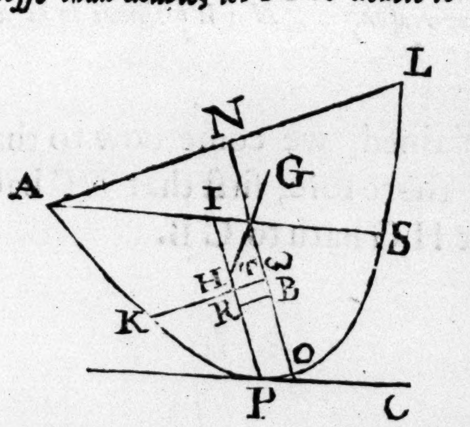
(d) By 15. of the fifth.

that is, HG *to* NC *: and as* (d) OH *is to* HP*, so is* GB *to* CK*; For* OH *is double to* GB*, and* HP *also double to* GF*; that is, to* CK*: Therefore* HG *hath the same proportion to* NC*, that* GB *hath to* CK*: And* Permutando*,* NC *hath to* CK *the same proportion that* HG *hath to* GB.

Then take some other Point at pleasure in the Section, which let be S : and thorow S draw two Lines, the one S T parallel to D B, and cutting the Diameter in the Point T; the other S V parallel to A C, and cutting C E in V. I say that V C hath greater proportion to C K, than T G hath to G B.

For prolong VS *unto the Line* QM *in* X*; and from the Point* X *draw* XY *unto the Diameter parallel to* BD *:* GT *shall be lesse than* GY*, in regard that* VS *is lesse than* VX *: And, by the first Lemma,* YG *shall be to* VC*, as* HG *to* NC*; that is, as* GB *to* CK*, which was demonstrated but now; And,* Permutando*,* YG *shall be to* GB*, as* VC *to* CK*: But* TG*, for that it is lesse than* YG*, hath lesse proportion to* GB*, than* YG *hath to the same; Therefore* VC *hath greater proportion to* CK*. than* TG *hath to* GB *: Which was to be demonstrated. Therefore a Position given* GK*, there shall be in the Section one only Point, to wit* M*, from which two Lines* MEH *and* MNO *being drawn,* NC *shall have the same proportion to* CK*, that* HG *hath to* GB*; For if they be drawn from any other, that which falleth betwixt* AC*, and the Line parallel unto it shall alwayes have greater proportion to* CK*, than that which falleth betwixt* GK *and the Line parallel unto it hath to* GB*. That, therefore, is manifest which was affirmed by* Archimedes*, to wit, that the Line* PI *hath unto* PH*, either the same proportion that* Nω *hath to* ωO*, or greater.*

D Wherefore P H is to H I either double, or lesse than double.] *If lesse than double, let* PT *be double to* TI *: The Centre of Gravity of that part of the Portion that is within the Liquid shall be the Point* T *: But if* PH *be double to* HI*,* H *shall be the Centre of Gravity; And draw* HF*, and prolong it unto the Centre of that part of the Portion which is above the Liquid, namely, unto* G*, and the rest is demonstrated as before. And the same is to be understood in the* Proposition *that followeth.*



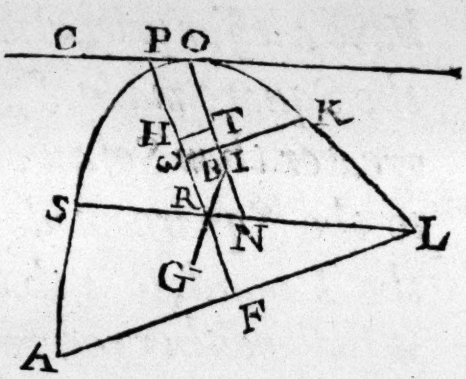
The Solid A P O L, therefore, shall turn about, and its Base shall not in the least touch the Surface of the Liquid.] *In* Tartaglia's *Translation it is rendered* ut Basis ipsius non tangent superficiem humidi secundum unum signum; *but we have chosen to read* ut Basis ipsius nullo modo humidi superficiem contingent, *both here, and in the following* Propositions, *because the Greekes frequently use* οὐδὲ εἷς, οὐδὲ ἕν *pro* οὐδεὶς & οὐδέν *: so that* οὐδὲ εἷς ἐστιν οὐδείς, nullus est; οὐδ' ὑφ' ἑνὸς à nullo, *and so of others of the like nature.*

PROP.

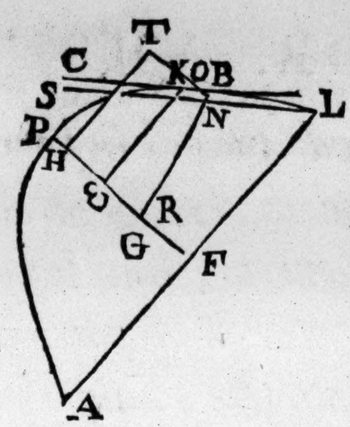
PROP. VII. THEOR. VII.

The Right Portion of a Rightangled Conoid lighter than the Liquid, when it ſhall have its Axis greater than Seſquialter of the Semi-parameter, but leſſe than to be unto the ſaid Semi-parameter in proportion as fifteen to fower, being demitted into the Liquid ſo as that its Baſe be wholly within the Liquid, it ſhall never ſtand ſo as that its Baſe do touch the Surface of the Liquid, but ſo, that it be wholly within the Liquid, and ſhall not in the leaſt touch its Surface.

Let there be a Portion as hath been ſaid; and let it be demitted into the Liquid, as we have ſuppoſed, ſo as that its Baſe do touch the Surface in one Point only: It is to be demonſtrated that the ſame ſhall not ſo continue, but ſhall turn about in ſuch manner as that its Baſe do in no wiſe touch the Surface of the Liquid. For let it be cut thorow its Axis by a Plane erect upon the Liquids Surface: and let the Section be A P O L, the Section of a Rightangled Cone; the Section of the Liquids Surface S L; and the Axis of the Portion and Diameter of the Section P F: and let P F be cut in R, ſo, as that R P may be double to R F, and in • ſo as that P F may be to R • as fifteen to fower: and draw • K at Right Angles to P F: (*a*) R • ſhall be leſſe than the Semi-parameter. Therefore let R H be ſuppoſed equall to the Semi-parameter: and draw C O touching the Section in O and parallel unto S L; and let N O be parallel unto P F; and firſt let N O cut K • in the Point I, as in the former Schemes: It ſhall be demonſtrated that N O is to O I either ſeſquialter, or greater than ſeſquialter. Let O I be leſſe than double to I N; and let O B be double to B N: and let them be diſpoſed like as before. We might likewiſe demonſtrate that if a Line be drawn thorow R and T it will make Right Angles with the Line C O, and with the Surface of the Liquid: Wherefore Lines being drawn from the Points B and G parallels unto R T, they alſo ſhall be Perpendiculars to the Surface of the Liquid: The Portion therefore which is above the Liquid ſhall move downwards



(*a*) By 10 of the fifth.



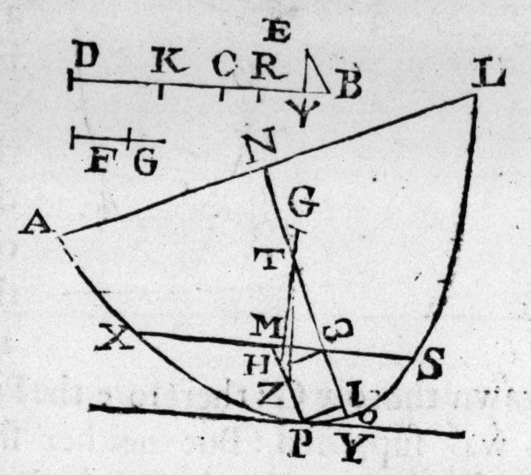
wards according to that ſame Perpendicular which paſſeth thorow B; and the Portion which is within the Liquid ſhall move upwards acording to that paſſing thorow G: From whence it is manifeſt that the Solid ſhall turn about in ſuch manner, as that its Baſe ſhall in no wiſe touch the Surface of the Liquid; for that now when it toucheth but in one Point only, it moveth downwards on the part towards L. And though N O ſhould not cut *K, yet ſhall the ſame hold true.

PROP. VIII. THEOR. VIII.

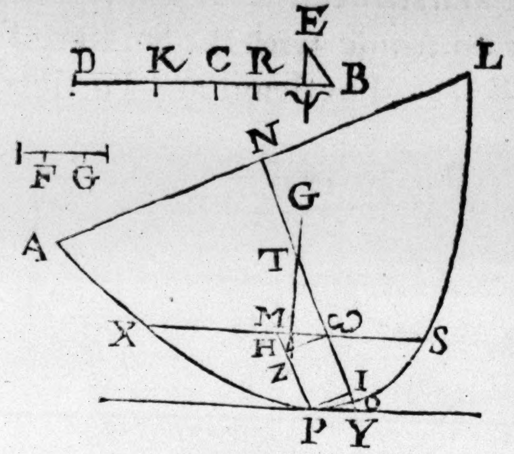
The Right Portion of a Rightangled Conoid, when it ſhall have its Axis greater than ſeſquialter of the Semi-parameter, but leſſe than to be unto the ſaid Semi-parameter, in proportion as fifteen to fower, if it have a leſſer proportion in Gravity to the Liquid, than the Square made of the Exceſſe by which the Axis is greater than Seſquialter of the Semi-parameter hath to the Square made of the Axis, being demitted into the Liquid, ſo as that its Baſe touch not the Liquid, it ſhall neither return to Perpendicularity, nor continue inclined, ſave only when the Axis makes an Angle with the Surface of the Liquid, equall to that which we ſhall preſently ſpeak of.

Let there be a Portion as hath been ſaid; and let B D be equall to the Axis: and let B K be double to K D; and R K equall
A to the Semi-parameter: and let C B be Seſquialter of B R: C D ſhall be alſo Seſquialter of K R. And as the Portion is to the Liquid in Gravity, ſo let the Square F Q be to the Square D B; and let F be double to Q: It is manifeſt, therefore, that F Q hath to D B, leſs proportion than C B hath to B D; For C B is the Exceſs by which the Axis is greater than Seſquialter of the Semi-
B parameter: And, therefore, F Q is leſs than B C; and, for the
C ſame reaſon, F is leſs than B R. Let R ✝ be equall to F; and draw ✝ E perpendicular to B D; which let be in power or contence the half of that which the Lines K R and ✝ B containeth; and draw a Line from B to E: It is to be demonſtrated, that the Portion

Portion demitted into the Liquid, like as hath been said, shall stand enclined so as that its Axis do make an Angle with the Surface of the Liquid equall unto the Angle E B ψ: For demit any Portion into the Liquid so as that its Base touch not the Liquids Surface; and, if it can be done, let the Axis not make an Angle with the Liquids Surface equall to the Angle E B ψ; but first, let it be greater: and the Portion being cut thorow the Axis by a Plane erect unto [*or upon*] the Surface of the Liquid, let the Section be A P O L the Section of a Rightangled

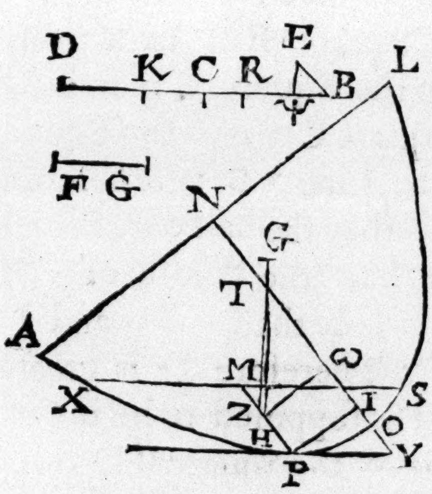


Cone; the Section of the Surface of the Liquid X S; and let the Axis of the Portion and Diameter of the Section be N O: and draw P Y parallel to X S, and touching the Section A P O L in P; and P M parallel to N O; and P I perpendicular to N O: and moreover, let B R be equall to O ω, and R K to T ω; and let ω H be perpendicular to the Axis. Now because it hath been supposed D
that the Axis of the Portion doth make an Angle with the Surface of the Liquid greater than the Angle B, the Angle P Y I shall be greater than the Angle B: Therefore the Square P I hath greater E
proportion to the Square Y I, than the Square E ψ hath to the Square ψ B: But as the Square P I is to the Square Y I, so is the F
Line K R unto the Line I Y; and as the Square E ψ is to the Square G
ψ B, so is half of the Line K R unto the Line ψ B: Wherefore (*a*) K R hath greater proportion to I Y, than the half of K R hath (a) *By* 13. *of the*
to ψ B: And, consequently, I Y is lesse than the double of ψ B, *fifth.*
and is the double of O I: Therefore O I is lesse than ψ B; and I ω H
greater than ψ R: but ψ R is equall to F: Therefore I ω is greater K
than F. And because that the Portion is supposed to be in Gravity unto the Liquid, as the Square F Q is to the Square B D; and since that as the Portion is to the Liquid in Gravity, so is the part thereof submerged unto the whole Portion; and in regard that as the part thereof submerged is to the whole, so is the Square P M to the Square O N; It followeth, that the Square P M is to the Square N O, as the Square F Q is to the Square B D: And therefore F L
Q is equall to P M: But it hath been demonstrated that P H is M
greater than F: It is manifest, therefore, that P M is lesse than sesquialter of P H: And consequently that P H is greater than the double of H M. Let P Z be double to Z M: T shall be the Centre of Gravity of the whole Solid; the Centre of that part of it which is within the Liquid, the Point Z; and of the remaining N
part the Centre shall be in the Line Z T prolonged unto G. In the



the ſame manner we might demonſtrate the *Line* T H to be perpendicular unto the Surface of the *Liquid*: and that the Portion demerged within the *Liquid* moveth or aſcendeth out of the *Liquid* according to the Perpendicular that ſhall be drawn thorow Z unto the Surface of the Liquid; and that the part that is above the Liquid deſcendeth into the Liquid according to that drawn thorow G: therefore the Portion will not continue ſo inclined as was ſuppoſed: But neither ſhall it return to Rectitude or Perpendicularity; For that of the Perpendiculars drawn thorow Z and G, that paſſing thorow Z doth fall on thoſe parts which are towards L; and that that paſſeth thorow G on thoſe towards A: Wherefore it followeth that the Centre Z do move upwards, and G downwards: Therefore the parts of the whole Solid which are towards A ſhall move downwards, and thoſe towards L upwards. Again let the Propoſition run in other termes; and let the Axis of the Portion make an Angle with the Surface of the

O Liquid leſſe than that which is at B. Therefore the *S*quare P I hath leſſer Proportion unto the Square I Y, than the Square E ψ hath to the Square ψ B: Wherefore K R hath leſſer proportion to I Y, than the half of K R hath to ψ B: And, for the ſame reaſon, I Y is greater than double of ψ B: but it is double of O I: Therefore O I ſhall be greater than ψ B: *B*ut the Totall O ω is equall to R B, and the Remainder ω I leſſe than ψ R: Wherefore P H ſhall alſo be leſſe than F. And, in regard that

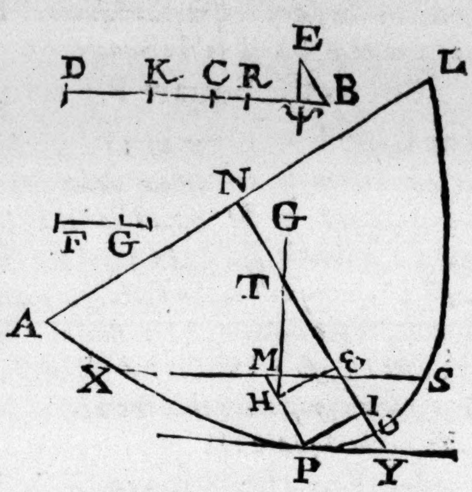


M P is equall to F Q, it is manifeſt that P M is greater than ſeſquialter of P H; and that P H is leſſe than double of *H* M. Let P Z be double to Z M. The Centre of Gravity of the whole Solid ſhall again be T; that of the part which is within the Liquid Z; and drawing a *L*ine from Z to T, the Centre of Gravity of that which is above the Liquid ſhall be found in that Line portracted, that is

P in G: Therefore, Perpendiculars being drawn thorow Z and G unto the Surface of the Liquid that are parallel to T H, it followeth that the ſaid Portion ſhall not ſtay, but ſhall turn about till that its Axis do make an Angle with the Waters Surface greater than that which it now maketh. And becauſe that when before we

did

did suppose that it made an Angle greater than the Angle B, the Poriton did not rest then neither; It is manifest that it shall stay or rest when it shall make an Angle eqnall to B. For so shall I O be equall to ♆ B; and ω I equall to ♆ R; and P H equall to F: Therefore M P shall be sesquialter of P H, and P H double of H M: And therefore since H is the Centre of Gravity of that part of it which is within the Liquid, it shall move upwards along the same Perpendicular according to which the whole Portion moveth; and along the same also shall the part which is above move downwards: The Portion therefore shall rest; forasmuch as the parts are not repulsed by each other. Q



COMMANDINE.

A And let C B be sesquialter of B R: C D shall also be sesquialter of K R.] *In the Translation it is read thus:* Sit autem & C B quidem hemeolia ipsius B R: C D autem ipsius K R. *But we at the reading of this passage have thought fit thus to correct it; for it is not supposed so to be, but from the things supposed is proved to be so. For if B ♆ be double of ♆ D, D B shall be sesquialter of B ♆. And because E B is sesquialter of B R, it followeth that the (a) Remainder C D is sesquialter of ♆ R; that is, of the Semi-parameter: Wherefore B C shall be the Excesse by which the Axis is greater than sesquialter of the Semi-parameter.*

(a) *By 19. of the fifth.*

B And therefore F Q is lesse than B C.] *For in regard that the Portion hath the same proportion in Gravity unto the Liquid, as the Square F Q hath to the Square D B; and hath lesser proportion than the Square made of the Excesse by which the Axis is greater than Sesquialter of the Semi-parameter, hath to the Square made of the Axis; that is, lesser than the Square C B hath to the Square B D; for the Line B D was supposed to be equall unto the Axis: Therefore the Square F Q shall have to the Square D B lesser proportion than the Sqnare C B to the same Square B D: And therefore the Square (b) F Q shall be lesse than the Square C B: And, for that reason, the Line F Q shall be lesse than B C.*

(b) *By 8 of the fifth.*

C And, for the same reason, F is lesse than B R.] *For C B being sesquialter of B R, and F Q sesquialter of F: (c) F Q shall be likewise lesse than B C; and F lesse than B R.*

(c) *By 14 of the fifth.*

D Now because it hath been supposed that the Axis of the Portion doth make an Angle with the Surface of the Liquid greater than the Angle B, the Angle P Y I shall be greater than the Angle B.] *For the Line P Y being parallel to the Surface of the Liquid, that is, to X S; (d) the Angle P Y I shall be equall to the Angle contained betwixt the Diameter of the Portion N O, and the Line X S: And therefore shall be greater than the Angle B.*

(d) *By 29 of the first.*

E Therefore the Square P I hath greater proportion to the Square Y I, than the Square E ♆ hath to the Square ♆ B.] *Let the Triangles P I Y and E ♆ B, be described apart: And seeing that the Angle P Y I is greater than the Angle E B ♆, unto the Line I Y, and at the Point Y assigned in the same, make the Angle V Y I equall to the Angle E B ♆; But the Right Angle at I, is equall unto the Right Angle at ♆; therefore the*

Remaining

Remaining Angle Y V I is equall to the Remaining Angle B E ✠: And therefore the (e) *Line V I hath to the Line I Y the same proportion that the Line E ✠ hath to ✠ B: But the* (f) *Line P I, which is greater than V I, hath unto I Y greater proportion than V I hath unto the same: Therefore* (g) *T I shall have greater proportion unto I Y, than E ✠ hath to ✠ B: And, by the same reason, the Square T I shall have greater proportion to the Square I Y, than the Square E ✠ hath to the Square ✠ B.*

(e) By 4. of the sixth.
(f) By 8. of the fifth.
(g) By 13 of the fifth.

F But as the Square P I is to the Square Y I, so is the Line K R unto the Line I Y.] *For by* 11. *of the first of our* Conicks, *the Square P I is equall to the Rectangle contained under the Line I O, and under the Parameter; which we supposed to be equall to the Semi-parameter; that is, the double of K R: But I Y is double of I O, by* 33 *of the same: And, therefore, the* (h) *Rectangle made of K R and I Y, is equall to the Rectangle contained under the Line I O, and under the Parameter; that is, to the Square P I: But as the* (i) *Rectangle compounded of K R and I Y is to the Square I Y, so is the Line K R unto the Line I Y: Therefore the Line K R shall have unto I Y, the same proportion that the Rectangle compounded of K R and I Y; that is, the Square P I hath to the Square I Y.*

(h) By 26. of the sixth.
(i) By Lem. 22 of the tenth.

G And as the Square E ✠ is to the Square ✠ B, so is half of the Line K R unto the Line ✠ B.] *For the Square E ✠ having been supposed equall to half the Rectangle contained under the Line K R and ✠ B: that is, to that contained under the half of K R and the Line ✠ B; and seeing that as the* (k) *Rectangle made of half K R and of B ✠ is to the Square ✠ B, so is half K R unto the Line ✠ B; the half of K R shall have the same proportion to ✠ B, as the Square E ✠ hath to the Square ✠ B.*

(k) By Lem. 22 of the tenth.

H And, consequently, I Y is lesse than the double of ✠ B.] *For, as half K R is to ✠ B, so is K R to another Line: it shall be* (l) *greater than I Y; that is, than that to which K R hath lesser proportion; and it shall be double of ✠ B: Therefore I Y is lesse than the double of ✠ B.*

(l) By 10 of the fifth.

K And I ω greater than ✠ R.] *For O ω having been supposed equall to B R, if from B R, ✠ B be taken, and from O ω, O I, which is lesser than ✠ B, be taken; the Remainder I ω shall be greater than the Remainder ✠ R.*

L And, therefore, F Q is equall to P M.] *By the fourteenth of the fifth of* Euclids *Elements: For the Line O N is equall to B D.*

M But it hath been demonstrated that P H is greater than F.] *For it was demonstrated that I ω is greater than F: And P H is equall to I ω.*

N In the same manner we might demonstrate the Line T H to be Perpendicular unto the Surface of the Liquid.] *For T ω is equall to K R; that is, to the Semi-parameter: And, therefore, by the things above demonstrated, the Line T H shall be drawn Perpendicular unto the Liquids Surface.*

O Therefore, the Square P I hath lesser proportion unto the Square I Y, than the Square E ✠ hath to the Square ✠ B.] *These, and other particulars of the like nature, that follow both in this and the following Propositions, shall be demonstrated by us no otherwise than we have done above.*

P Therefore Perpendiculars being drawn thorow Z and G, unto the Surface of the Liquid, that are parallel to T H, it followeth that the said Portion shall not stay, but shall turn about till that its Axis do make an Angle with the Waters Surface greater than that which it now maketh.] *For in that the Line drawn thorow G, doth fall perpendicularly towards those parts which are next to L; but that thorow Z, towards those next to A; It is necessary that the Centre G do move downwards, and Z upwards: and, therefore, the parts of the Solid next to L shall move downwards, and those towards A upwards, that the Axis may make a greater Angle with the Surface of the Liquid.*

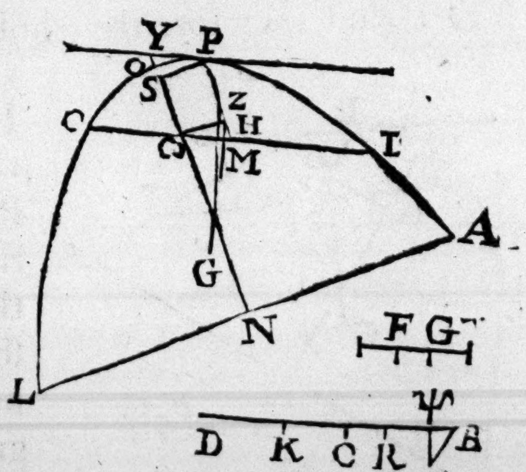
Q For so shall I O be equall to ✠ B; and ω I equall to I R; and P H equall to F.] *This plainly appeareth in the third Figure, which is added by us.*

PROP.

PROP. IX. THEOR. IX.

The Right Portion of a Rightangled Conoid, when it shall have its Axis greater than Sesquialter of the Semi-parameter, but lesser than to be unto the said Semi-parameter in proportion as fifteen to four, and hath greater proportion in Gravity to the Liquid, than the excess by which the Square made of the Axis is greater than the Square made of the Excess, by which the Axis is greater than Sesquialter of the Semi-parameter, hath to the Square made of the Axis, being demitted into the Liquid, so as that its Base be wholly within the Liquid, and being set inclining, it shall neither turn about, so as that its Axis stand according to the Perpendicular, nor remain inclined, save only when the Axis makes an Angle with the Surface of the Liquid, equall to that assigned as before.

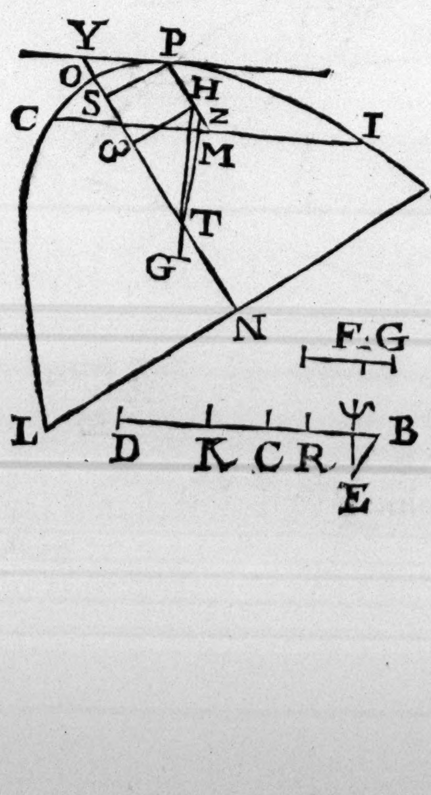
Let there be a Portion as was said; and suppose D B equall to the Axis of the Portion: and let B K be double to K D; and K R equall to the Semi-parameter: and C B Sesquialter of B R. And as the Portion is to the Liquid in Gravity, so let the Excesse by which the Square B D exceeds the Square F Q be to the Square B D: and let F be double to Q: It is manifest, therefore, that the Excesse by which the Square B D is greater than the Square B C hath lesser proportion to the Square B D, than the Excess by which the Square B D is greater than the Square F Q hath to the Square B D; for B C is the Excess by which the Axis of the Portion is greater than Sesquialter of the Semi-parameter: And, therefore, the Square B D doth more exceed the Square F Q, than doth the Square B C: And, consequently, the Line F Q is less than B C;

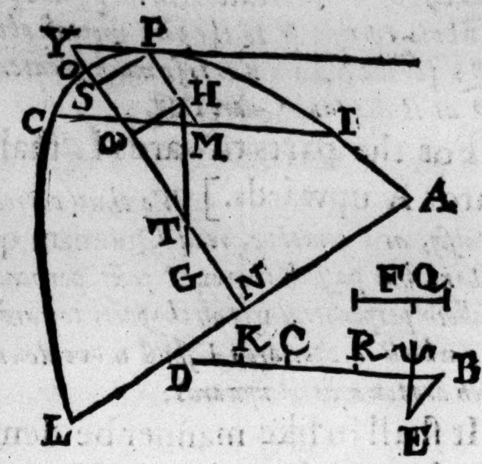


A

and

and F less than B R. Let R ✝ be equall to F; and draw ✝ E perpendicular to B D; which let be in power the half of that which the Lines K R and ✝ B containeth; and draw a Line from B to E: I say that the Portion demitted into the Liquid, so as that its Base be wholly within the Liquid, shall so stand, as that its Axis do make an Angle with the Liquids Surface, equall to the Angle B. For let the Portion be demitted into the Liquid, as hath been said; and let the Axis not make an Angle with the Liquids Surface, equall to B, but first a greater: and the same being cut thorow the Axis by a Plane erect unto the Surface of the Liquid, let the Section of the Portion be A P O L, the Section of a Rightangled Cone; the Section of the Surface of the Liquid ʳ I; and the Axis of the Portion and Diameter of the Section N O; which let be cut in the Points ω and T, as before: and draw Y P, parallel to ʳ I, and touching the Section in P, and M P parallel to N O, and P S perpendicular to the Axis. And because now that the Axis of the Portion maketh an Angle with the Liquids Surface greater than the Angle B, the Angle S Y P shall also be greater than the Angle B: And, therefore, the Square P S hath greater proportion to the Square
B S Y, than the Square ✝ E hath to the Square ✝ B: And, for that cause, K R hath greater proportion to S Y, than the half of K R
C hath to ✝ B: Therefore, S Y is less than the double of ✝ B; and S O less than ✝ B: And, therefore, S ω is greater than R ✝; and
D P H greater than F. And, because that the Portion hath the same proportion in Gravity unto the Liquid, that the Excess by which the Square B D, is greater than the Square F Q, hath unto the Square B D; and that as the Portion is in proportion to the Liquid in Gravity, so is the part thereof submerged unto the whole Portion; It followeth that the part submerged, hath the same proportion to the whole Portion, that the Excess by which the Square B D is greater than the Square F Q hath unto the Square
E B D: And, therefore, the whole Portion shall have the same proportion to that part which is above the Liquid, that the Square B D hath to the Square F Q: But as the whole Portion is to that part which is above the Liquid, so is the Square N O unto the Square P M: Therefore, P M shall be equall to F Q: But it hath been demonstrated, that P H is greater than F. And, therefore, M H shall be less than Q; and P H greater than double of H M. Let therefore, P Z be double to Z M: and





and drawing a Line from Z to T prolong it unto G. The Centre of Gravity of the whole Portion shall be T; of that part which is above the Liquid Z; and of the Remainder which is within the Liquid, the Centre shall be in the Line Z T prolonged; let it be in G: It shall be demonstrated, as before, that T H is perpendicular to the Surface of the Liquid, and that the Lines drawn thorow Z and G parallel to the said T H, are also perpendiculars unto the same: Therefore, the Part which is above the Liquid shall move downwards, along that which passeth thorow Z; and that which is within it, shall move upwards, along that which passeth thorow G: And, therefore, the Portion shall not remain so inclined, nor shall so turn about, as that its Axis be perpendicular unto the Surface of the Liquid; for the parts towards L shall move F
downwards, and those towards *A* upwards; as may appear by the things already demonstrated. And, if the Axis should make an Angle with the Surface of the Liquid, less than the Angle B; it shall in like manner be demonstrated, that the Portion will not G
rest, but incline untill that its Axis do make an Angle with the Surface of the Liquid, equall to the Angle B.

COMMANDINE.

And, therefore, the Square B D doth more exceed the Square A
F Q, than doth the Square B C: And, consequently, the Line F Q, is less than B C; and F less than B R.] *Because the Excess by which the Square B D exceedeth the Square B C; having less proportion unto the Square B D, than the Excess by which the Square B D exceedeth the Square F Q, hath to the said Square; (a) the Excess by which the Square B D exceedeth the Square B C shall be less than the Excess by which it exceedeth the Square F Q: Therefore, the Square F Q is less than the Square B C: and, consequently, the Line F Q less than the Line B C: But F Q hath the same proportion to F, that B C hath to B R; for the Antecedents are each Sesquialter of their consequents: And (b) F Q being less than B C, F shall also be less than B R.*

(a) *By 8. of the fifth.*

(b) *By 14. of the fifth.*

And, for that cause, K R hath greater proportion to S Y, than the half of K R hath to ψ B.] *For K R is to S Y, as the Square P S is to the Square S Y: and the half of the Line K R is to the Line ψ B, as the Square E ψ is to the Square ψ B.* B

And S O less than ψ B.] *For S Y is double of S O.* C

And P H greater than F.] *For P H is equall to S ω, and R ψ equall to F.* D

And, therefore, the whole Portion shall have the same proportion to that part which is above the Liquid, that the Square B D hath to the Square F Q.] *Because that the part submerged, being to the whole Portion as the Excess by which the Square B D is greater than the Square F Q, is to the Square B D; the whole Portion, Converting, shall be to the part thereof submerged, as the Square B D is to* E

the Excess by which it exceedeth the Square F Q: And, therefore, by Conversion of Proportion the whole Portion is to the part thereof above the Liquid, as the Square B D is to the Square F Q: for the Square B D is so much greater than the Excess by which it exceedeth the Square F Q as is the said Square F Q.

F For the parts towards L shall move downwards, and those towards A upwards.] *We thus correct these words, for in* Tartaglia's *Translation it is falsly, as I conceive, read* Quoniam quæ ex parte L ad superiora ferentur, *because the Line that passeth thorow Z falls perpendicularly on the parts towards L, and that thorow G falleth perpendicularly on the parts towards A: Whereupon the Centre Z, together with those parts which are towards L shall move downwards; and the Centre G, together with the parts which are towards A upwards.*

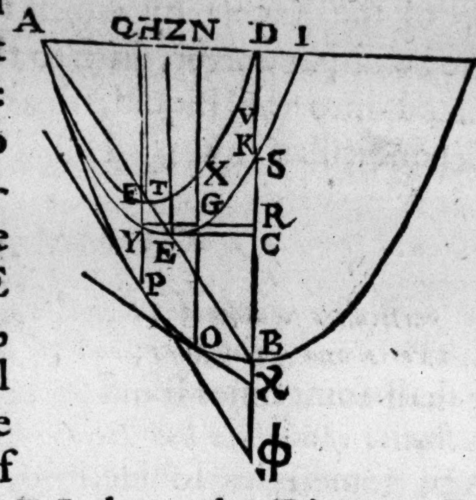
G It shall in like manner be demonstrated that the Portion shall not rest, but incline untill that its Axis do make an Angle with the Surface of the Liquid, equall to the Angle B.] *This may be easily demonstrated, as well from what hath been said in the precedent Proposition, as also from the two latter Figures, by us inserted*

PROP. X. THEOR. X.

The Right Portion of a Rightangled Conoid, lighter than the Liquid, when it shall have its Axis greater than to be unto the Semiparameter, in proportion as fifteen to four, being demitted into the Liquid, so as
A *that its Base touch not the same, it shall sometimes*
B *stand perpendicular; sometimes inclined; and sometimes so inclined, as that its Base touch the Surface of the Liquid in one Point only, and that in two Po-*
C *sitions; sometimes so that its Base be more submer-*
D *ged in the Liquid; and sometimes so as that it doth not in the least touch the Surface of the Liquid;*
E *according to the proportion that it hath to the Liquid in Gravity. Every one of which Cases shall be anon demonstrated.*

Let there be a Portion, as hath been said; and it being cut thorow its Axis, by a Plane erect unto the Superficies of the Liquid, let the Section be A P O L, the Section of a Rightangled Cone; and the Axis of the Portion and Diameter of the Section B D: and let B D be cut in the Point K, so as that B K
F be double of K D; and in C, so as that B D may have the same proportion to K C, as fifteen to four: It is manifest, therefore,
G that K C is greater than the Semi-parameter: Let the Semi-parameter

parameter be equall to K R : and let D S be Sesquialter of K R : but S B is also Sesquialter of B R : Therefore, draw a Line from A to B ; and thorow C draw C E Perpendicular to B D, cutting the Line A B in the Point E ; and thorow E draw E Z parallel unto B D. Again, A B being divided into two equall parts in T, draw T H parallel to the same B D : and let Sections of Rightangled Cones be described, A E I about the Diameter E Z ; and A T D about the Diameter T H ; and let them be like to the Portion A B L : Now the Section of the Cone A E I, shall pass thorow K ; and the Line drawn from R perpendicular unto B D, shall cut the said A E I ; let it cut it in the Points Y G : and thorow Y and G draw P Y Q and O G N parallels unto B D, and cutting A T D in the Points F and X : lastly, draw P ✱ and O X touching the Section A P O L in the Points P and O. In regard, therefore, that the three Portions A P O L, A E I, and A T D are contained betwixt Right Lines, and the Sections of Rightangled Cones, and are right alike and unequall, touching one another, upon one and the same Base ; and N X G O being drawn from the Point N upwards, and Q F Y P from Q : O G shall have to G X a proportion compounded of the proportion, that I L hath to L A, and of the proportion that A D hath to D I : But I L is to L A, as two to five : And C B is to B D, as six to fifteen ; that is, as two to five : And as C B is to B D, so is *E B* to *B A* ; and D Z to D A : And of D Z and D A, L I and L A are double : and A D is to D I, as five to one : But the proportion compounded of the proportion of two to five, and of the proportion of five to one, is the same with that of two to one : and two is to one, in double proportion : Therefore, O G is double of G X : and, in the same manner is P Y proved to be double of Y F : Therefore, since that D S is Sesquialter of K R ; *B S* shall be the Excess by which the Axis is greater than Sesquialter of the Semi-parameter. If therefore, the Portion have the same proportion in Gravity unto the Liquid, as the Square made of the Line *B S*, hath to the Square made of *B D*, or greater, being demitted into the Liquid, so as hat its Base touch not the Liquid, it shall stand erect, or perpendicular: For it hath been demonstrated above, that the Portion whose Axis is greater than Sesquialter of the Semi-parameter, if it have not lesser proportion in Gravity unto the Liquid, than the Square made



H K L M N O P Q R

made of the Exceſs by which the Axis is greater than Seſquialter of the Semi-parameter, hath to the Square made of the Axis, being demitted into the Liquid, ſo as hath been ſaid, it ſhall ſtand erect, or Perpendicular.

COMMANDINE.

The particulars contained in this Tenth Propoſition, are divided by Archimedes *into five Parts and Concluſions, each of which he proveth by a diſtinct Demonſtration.*

A It ſhall ſometimes ſtand perpendicular.] *This is the firſt Concluſion, the Demonſtration of which he hath ſubjoyned to the Propoſition.*

B And ſometimes ſo inclined, as that its Baſe touch the Surface of the Liquid, in one Point only.] *This is demonſtrated in the third Concluſion.*

C Sometimes, ſo that its Baſe be moſt ſubmerged in the Liquid.] *This pertaineth unto the fourth Concluſion.*

D And, ſometimes, ſo as that it doth not in the leaſt touch the Surface of the Liquid.] *This it doth hold true two wayes, one of which is explained in the ſecond, and the other in the fifth Concluſion.*

E According to the proportion, that it hath to the Liquid in Gravity. Every one of which Caſes ſhall be anon demonſtrated.] *In* Tartaglia's *Verſion it is rendered, to the confuſion of the ſence,* Quam autem proportionem habeant ad humidum in Gravitate ſingula horum demonſtrabuntur.

F It is manifeſt, therefore, that K C is greater than the Semiparameter] *For, ſince B D hath to K C the ſame proportion, as fifteen to four, and hath unto the Semi-parameter greater proportion;* (a) *the Semi-parameter ſhall be leſs than K C.*

(a) *By* 10. *of the fifth.*

G Let the Semi-parameter be equall to K R.] *We have added theſe words, which are not to be found in* Tartaglia.

H But S B is alſo Seſquialter of B R.] *For, D B is ſuppoſed* Seſquialter *of B K; and* D S *alſo is Seſquialter of K R: Wherefore as* (b) *the whole D B, is to the whole B K, ſo is the part D S to the part K R: Therefore, the Remainder* S B, *is alſo to the Remainder B R, as D B is to B K.*

(b) *By* 19 *of the fifth.*

K And let them be like to the Portion A B L.] Apollonius *thus defineth like Portions of the Sections of a Cone, in* Lib. 6. Conicorum, *as* Eutocius *writeth* *; ἐν οἷς ἀχθεισῶν ἐν ἑκάστῳ παραλλήλων τῇ βάσει, ἴσων τὸ πλῆθος, αἱ παράλληλοι, καὶ αἱ βάσεις πρὸς τὰς ἀποτεμνομένας ἀπὸ τῶν διαμέτρων ταῖς κορυφαῖς ἐν τοῖς αὐτοῖς λόγοις εἰσί, καὶ αἱ ἀποτεμνόμεναι πρὸς τὰς ἀποτεμνομένας· *that is,* In both of which an equall number of Lines being drawn parallel to the Baſe; the parallel and the Baſes have to the parts of the Diameters, cut off from the Vertex, the ſame proportion: as alſo, the parts cut off, to the parts cut off. *Now the Lines parallel to the Baſes are drawn, as I ſuppoſe, by making a Rectilineall Figure (called)* Signally inſcribed [σχῆμα γνωρίμως ἐγγραφόμενον] *in both portions, having an equall number of Sides in both. Therefore, like Portions are cut off from like Sections of a Cone; and their Diameters, whether they be perpendicular to their Baſes, or making equall Angles with their Baſes, have the ſame proportion unto their Baſes.*

* *Upon prop.* 3 *lib.* 2 Archim. *Æquipond.*

Vide Archim. *ante prop.* 2. *lib.* 2. *Æquipond.*

L Now the Section of the Cone *A E I* ſhall paſs thorow K.] *For, if it be poſſible, let it not paſs thorow K, but thorow ſome other Point of the Line D B, as thorow V. In regard, therefore, that in the Section of the Right-angled Cone A E I, whoſe Diameter is E Z, A E is drawn and prolonged; and* D B *parallel unto the Diameter, cutteth both A E and A I; A E in* B, *and A I in* D; D B *ſhall have to* B V, *the ſame proportion that*

that A Z *hath to* Z D: *by the fourth Proposition of* Archimedes, De quadratura Parabolæ: *But* A Z *is Sesquialter of* Z D; *for it is as three to two, as we shall anon demonstrate: Therefore* D B *is Sesquialter of* B V; *but* D B *and* B K *are Sesquialter: And, therefore, the Lines* (c) B V *and* B K *are equall: Which is impossible: Therefore the Section of the Right-angled Cone* A E I, *shall pass thorow the Point* K; *which we would demonstrate.*

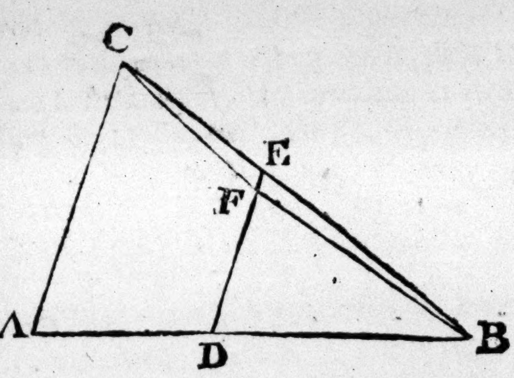
(c) *By 9. of the fifth.*

In regard, therefore, that the three Portions A P O L, A E I M and A T D are contained betwixt Right Lines and the Sections of Right-angled Cones, and are Right, alike and unequall, touching one another, upon one and the same Base.] *After these words,* upon one and the same Base, *we may see that something is obliterated, that is to be desired: and for the Demonstration of these particulars, it is requisite in this place to premise some things: which will also be necessary unto the things that follow.*

LEMMA. I.

Let there be a Right Line A B; and let it be cut by two Lines, parallel to one another, A C and D E, so, that as *A B* is to B D. so *A C* may be to D E. I say that the Line that conjoyneth the Points C and B shall likewise pass by E.

For, if possible, let it not pass by E, but either above or below it. Let it first pass below it, as by F. The Triangles A B C and D B F shall be alike: And, therefore, as (a) *A B is to B D, so is A C to D F: But as A B is to B D, so was A C to D E: Therefore* (b) *D F shall be equall to D E; that is, the part to the whole: Which is absurd. The same absurditie will follow, if the Line C B be supposed to pass above the Point E: And, therefore, C B must of necessity pass thorow E: Which was required to be demonstrated.*



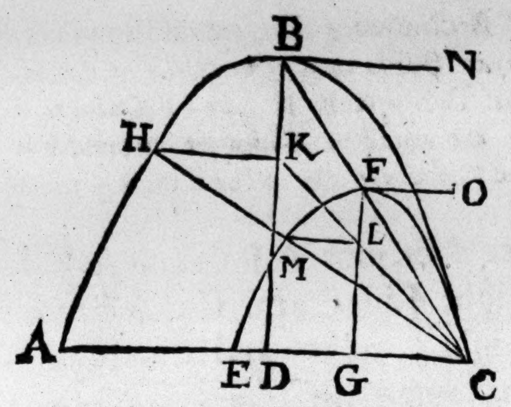
(a) *By 4. of the sixth.*
(b) *By 9. of the fifth.*

LEMMA. II.

Let there be two like Portions, contained betwixt Right Lines, and the Sections of Right-angled Cones; A B C the greater, whose Diameter let be B D; and E F C the lesser, whose Diameter let be F G: and, let them be so applyed to one another, that the greater include the lesser; and let their Bases A C and E C be in the same Right Line, that the same Point C, may be the term or bound of them both: And, then in the Section A B C, take any Point, as H; and draw a Line from H to C. I say, that the Line H C, hath to that part of it self, that lyeth betwixt C and the Section E F C, the same proportion that A C hath to C E.

Draw B C, which shall pass thorow F. For, in regard, that the Portions are alike, the Diameters with the Bases contain equall Angles: And, therefore, B D and F G are parallel to one another: and B D is to A C, as F G is to E C: and, Permutando, *B D is to F G, as A C is to C E; that is,* (a) *as their halfes* D C *to* C G; *therefore, it followeth, by the preceding Lemma, that the Line B C shall pass by the Point F. Moreover, from the Point H unto the Diameter B D, draw the Line H K, parallel to the Base* A C: *and, draw a Line from*

(a) *By 15. of the fifth.*



from K *to* C, *cutting the Diameter* F G *in* L; *and, thorow* L, *unto the Section* E F G, *on the part* E, *draw the Line* L M, *parallel unto the same Base* A C. *And, of the Section* A B C, *let the Line* B N *be the Parameter; and, of the Section* E F C, *let* F O *be the Parameter. And, because the Triangles* C B D *and* C F G *are alike;* (b) *therefore, as* B C *is to* C F, *so shall* D C *be to* C G, *and* B D *to* F G. *Again, because the Triangles* C K B *and* C L F, *are also alike to one another; therefore, as* B C *is to* C F, *that is, as* B D *is to* F G, *so shall* K C *be to* C L, *and* B K *to* F L: *Wherefore,* K C *to* C L, *and,* B K *to* F L, *are as* D C *to* C G; *that is,* (c) *as their duplicates* A C *and* C E: *But as* B D *is to* F G, *so is* D C *to* C G; *that is,* A D *to* E G: *And,* Permutando, *as* B D *is to* A D, *so is* F G *to* E G: *But the Square* A D, *is equall to the Rectangle* D B N, *by the* 11 *of our first of* Conicks: *Therefore, the* (d) *three Lines* B D, A D *and* B N *are Proportionalls. By the same reason, likewise, the Square* E G *being equall to the Rectangle* G F O, *the three other Lines* F G, E G *and* F O, *shall be also Proportionals: And, as* B D *is to* A D, *so is* F G *to* E G: *And, therefore, as* A D *is to* B N, *so is* E G *to* F O: Ex equali, *therefore, as* D B *is to* B N, *so is* G F *to* F O: *And,* Permutando, *as* D B *is to* G F, *so is* B N *to* F O: *But as* D B *is to* G F, *so is* B K *to* F L: *Therefore,* B K *is to* F L, *as* B N *is to* F O: *And,* Permutando, *as* B K *is to* B N, *so is* F L *to* F O. *Again, because the* (e) *Square* H K *is equall to the Rectangle* B N; *and the Square* M L, *equall to the Rectangle* L F O, *therefore, the three Lines* B K, K H *and* B N *shall be Proportionals; and* F L, L M, *and* F O *shall also be Proportionals: And, therefore,* (f) *as the Line* B K *is to the Line* B N, *so shall the Square* B K, *be to the Square* H K: *And, as the Line* F L *is to the Line* F O, *so shall the Square* F L *be to the Square* L M: *Therefore, because that as* B K *is to* B N, *so is* F L *to* F O; *as the Square* B K *is to the Square* K H, *so shall the Square* F L *be to the Square* L M: *Therefore,* (g) *as the Line* B K *is to the Line* K H, *so is the Line* F L *to* L M: *And,* Permutando, *as* B K *is to* F L, *so is* K H *to* L M: *But* B K *was to* F L, *as* K C *to* C L: *Therefore,* K H *is to* L M, *as* K C *to* C L: *And, therefore, by the preceding Lemma, it is manifest that the Line* H C *also shall pass thorow the Point* M: *As* K C, *therefore, is to* C L, *that is, as* A C *to* C E, *so is* H C *to* C M; *that is, to the same part of it self, that lyeth betwixt* C *and the Section* E F C. *And, in like manner might we demonstrate, that the same happeneth in other Lines, that are produced from the Point* C, *and the Sections* E B C. *And, that* B C *hath the same proportion to* C F, *plainly appeareth; for* B C *is to* C F, *as* D C *to* C G; *that is, as their Duplicates* A C *to* C E.

(b) *By* 4. *of the sixth.*

(c) *By* 15. *of the fifth.*

(d) *By* 17. *of the sixth.*

(e) *By* 11 *of our first of* Conicks.

(f) *By* Cor. *of* 20. *of the sixth.*

(g) *By* 22. *of the sixth.*

From whence it is manifest, that all Lines so drawn, shall be cut by the said Section in the same proportion. For, by Division and Conversion, C M is to M H, and C F to F B, as C E to E A.

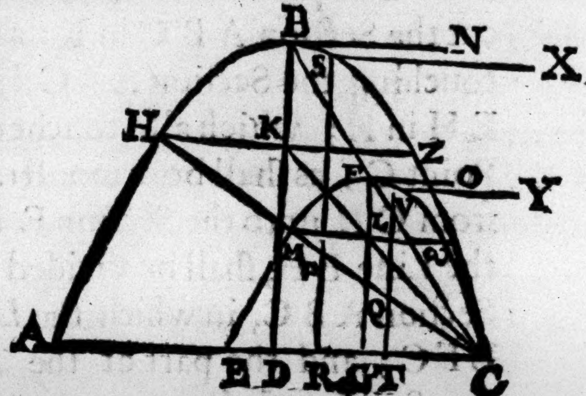
LEMMA. III.

And, hence it may also be proved, that the Lines which are drawn in like Portions, so, as that with the Bases, they contain equall Angles, shall also cut off like Portions; that is, as in the foregoing Figure, the Portions H B C and M F C, which the Lines C H and C M do cut off, are also alike to each other.

For let C H *and* C M *be divided in the midst in the Points* P *and* Q; *and thorow those Points draw the Lines* R P S *and* T Q V *parallel to the Diameters. Of the Portion* H S C *the Diameter shall be* P S, *and of the Portion* M V C *the Diameter shall be*

QV. And, suppose that as the Square CR is to the Square CP, so is the Line BN unto another Line; which let be SX: And, as the Square CT is to the Square CQ, so let FO be to VT. Now it is manifest, by the things which we have demonstrated, in our Commentaries, upon the fourth Proposition of Archimedes, De Conoidibus & Sphearoidibus, that the Square CP is equall to the Rectangle PSX; and also, that the Square CQ is equall to the Rectangle QVT; that is, the Lines SX and VT, are the Parameters of the Sections HSC and MVC: But since the Triangles CPR and CQT are alike; CR shall have to CP, the same Proportion that CT hath to CQ; And, therefore, the (a) Square CR shall have to the Square CP, the same proportion that the Square CT hath to the Square CQ: Therefore, also, the Line BN shall be to the Line SX, as the Line FO is to VT: But HC was to CM, as AC to CE: And, therefore, also, their halves CP and CQ, are also to one another, as AD and EG: And. Permutando, CP is to AD, as CQ is to EG: But it hath been proved, that AD is to BN, as EG to FO: and BN to SX, as FO to VT: Therefore, ex æquali, CP shall be to SX, as CQ is to VT. And, since the Square CP is equall to the Rectangle PSX, and the Square CQ to the Rectangle QVT; the three Lines SP, PC and SX shall be proportionalls, and VQ, QC and VT shal be Proportionalls also: And therefore also SP shall be to PC as VQ to QC And as PC is to CH, so shall QC be to CM: Therefore, ex æquali, as SP the Diameter of the Portion HSC is to its Base CH, so is VQ the Diameter of the Portion MVS the Base CM; and the Angles which the Diameter with the Bases do contain, are equall; and the Lines SP and VQ are parallel: Therefore the Portions, also, HSC and MVC shall be alike: Which was proposed to be demonstrated.

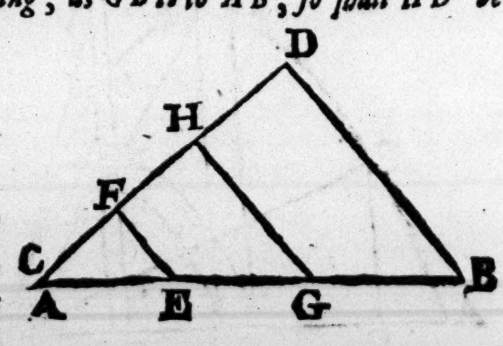
(a) By 22. of the sixth.



LEMMA. IV.

Let there be two Lines AB and CD; and let them be cut in the Points E and F, so that as AE is to EB, CF may be to FD: and let them be cut again in two other Points G and H; and let CH be to HD, as AG is to GB. I say that CF shall be to FH as AE is EG.

For in regard that as AE is to EB, so is CF to FD; it followeth that, by Compounding, as AB is to EB, so shall CD be to FD. Again, since that as AG is to GB, so is CH, to HD; it followeth that, by Compounding and Converting, as GB is to AB, so shall HD be CD: Therefore, ex æquali, and Converting as EB is to GB, so shall FD be to HD: And, by Conversion of Proposition, as EB is to EG, so shall FD be to FH: But as AE is to EB, so is CF to FD: Ex æquali, therefore, as AE is to EG, so shall CF be to FH. Again, another way. Let the Lines AB and CD be applyed to one another, so as that they doe make an Angle at the parts A and C: and let A and C be in one and the same Point: then draw Lines from D to B, from H to G, and from F to E. And since that as AE is to EB, so is CF, that is AF to FD: therefore FE shall be parallel to DB; (a) and likewise HG shall be parallel to DB; for that AH is to HD, as AG to GB: (b) Therefore FE and HG are parallel to each other: And consequently, as AE is to EG, so is AH, that is, CF to FH: Which was to be demonstrated.



(a) By 2. of the sixth.
(b) By 30 of the first.

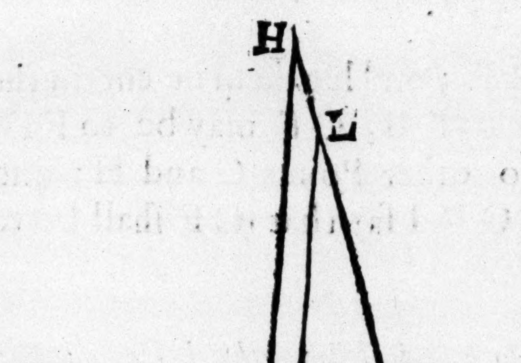
LEMMA

LEMMA. V.

Again, let there be two like Portions, contained betwixt Right Lines and the Sections of Right-angled Cones, as in the foregoing figure, ABC, whose Diameter is BD; and EFC, whose Diameter is FG; and from the Point E, draw the Line EH parallel to the Diameters BD and FG; and let it cut the Section ABC in K: and from the Point C draw CH touching the Section ABC in C, and meeting with the Line EH in H; which also toucheth the Section EFC in the same Point C, as shall be demonstrated: I say that the Line drawn from CH unto the Section EFC so as that it be parallel to the Line EH, shall be divided in the same proportion by the Section ABC, in which the Line CA is divided by the Section EFC; and the part of the Line CA which is betwixt the two Sections, shall answer in proportion to the part of the Line drawn, which also falleth betwixt the same Sections: that is, as in the foregoing Figure, if DB be produced untill it meet with CH in L, that it may intersect the Section EFC in the Point M, the Line LB shall have to BM the same proportion that CE hath to EA.

For let GF be prolonged untill it meet the same Line CH in N, cutting the Section ABC in O; and drawing a Line from B to C, which shall passe by F, as hath been shewn, the Triangles CGF and CDB shall be alike; as also the Triangles CFN and CBL: Wherefore (a) as GF is to DB, so shall CF be to CB: And as (b) CF is to CB, so shall FN be to BL: Therefore GF shall be to DB, as FN to BL: And, Permutando, *GF shall be to FN, as DB to BL: But DB is equall to BL, by 35 of our First Book of* Conicks: *Therefore (c) GF also shall be equall to FN: And by 33 of the same, the Line CH toucheth the Section EFC in the same Point. Therefore, drawing a Line from C to M, prolong it untill it meet with the Section ABC in P; and from P unto AC draw PQ parallel to BD. Because, now, that the Line CH toucheth the Section EFC in the Point C; LM shall have the same proportion to MD that CD hath to DE, by the Fifth Proposition of* Archimedes *in his Book* De Quadratura Parabolæ: *And by reason of the Similitude of the Triangles CMD and CPQ, as CM is to CD, so shall CP be to CQ: And,* Permutando, *as CM is to CP, so shall CD be to CQ: But as CM is to CP, so is CE to CA; as we have but even now demonstrated: And therefore, as CE is to CA, so is CD to CQ; that is as the whole is to the whole, so is the part to the part: The remainder, therefore, DE is to the Remainder QA, as CE is to CA; that is, as CD is to CQ: And,* Permutando, *CD is to DE, as CQ is to QA: And LM is also to MD, as CD to DE: Therefore LM is to*

(a) *By 4. of the sixth.*
(b) *By 11 of the fifth.*
(c) *By 14 of the fifth.*



to M D, as C Q to Q A: But L B is to B D, by 5 of Archimedes, *before recited, as C D to D A: It is manifest therefore, by the precedent Lemma, that C D is to D Q, as L B is to B M: But as C D is to D Q, so is C M to M P: Therefore L B is to B M, as C M to M P: And it haveing been demonstrated, that C M is to M P, as C E to E A; L B shall be to B M, as C E to E A. And in like manner it shall be demonstrated that so is N O to O F, as also the Remainders. And that also H K is to K E, as C E to E A, doth plainly appeare by the same 5. of* Archimedes *: Which is that that we propounded to be demonstrated.* By 2. of the sixth

LEMMA. VI.

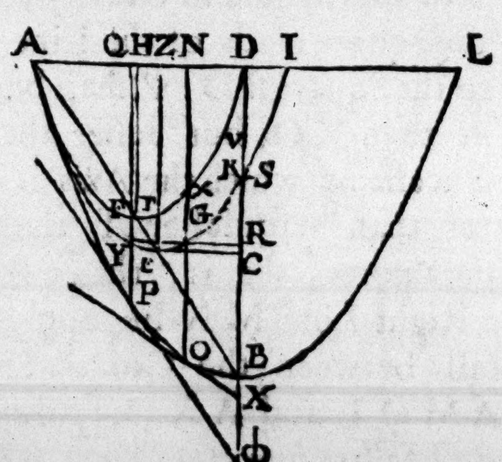
And, therefore, let the things stand as above; and describe yet another like Portion, contained betwixt a Right Line, and the Section of the Rightangled Cone D R C, whose Diameter is R S, that it may cut the Line F G in T; and prolong S R unto the Line C H in V, which meeteth the Section A B C in X, and E F C in Y. I say, that B M hath to M D, a proportion compounded of the proportion that E A hath to A C; and of that which C D hath to D E.

For, we shall first demonstrate, that the Line C H toucheth the Section D R C in the Point C; and that L M is to M D, as also N F to F T, and V Y to Y R, as C D is to E D. And, because now that L B is to B M, as C E is to E A; therefore, Compounding and Converting, B M shall be to L M, as E A *to* A C : And, *as L M is to M D, so shall C D be to D E : The proportion, therefore, of B M to M D, is compounded of the proportion that B M hath to L M, and of the proportion that L M hath to M D : Therefore, the proportion of B M to M D, shall also be compounded of the proportion that* E A *hath to* A C, *and of that which C D hath to D E. In the same manner it shal be demonstrated, that O F hath to F T, and also X Y to Y R, a proportion compounded of those same proportions; and so in the rest : Which was to be demonstrated.*

By which it appeareth that the Lines so drawn; which fall betwixt the Sections A B C and D R C, shall be divided by the Section E F C in the same Proportion.

N

And C B is to B D, as six to fifteen.] *For we have supposed that B K is double of K D: Wherefore, by* Composition *B D shall be to K D as three to one; that is, as fifteen to five: But B D was to K C as fifteen to four; Therefore B D is to D C as fifteen to nine: And, by Conversion of proportion and Converting, C B is to B D, as six to fifteen.*



O

And as C B is to B D, so is E B to B A; and D Z to D A.] *For the Triangles C B E and D B A being alike; As C B is to B E, so shall D B be to B A: And,* Permutando, *as C B is to B D, so shall E B be to B A: Againe, as B C is to* C E *so shall B D be to* D A, *And,* Permutando, *as* C B *is to B D, so shall* C E, *that is,* D Z *equall to it, be to* D A.

P

And of D Z and D A, L I and L A are double.] *That the Line* L A *is double of* D A, *is manifest, for that* B D *is the Diameter of the Portion. And that L I is double to D Z shall be thus demonstrated. Forasmuch as* Z D *is to* D A, *as two to five: therefore,* Converting *and* Dividing, A Z, *that is, I Z, shall be to Z D, as three to two:*

Again, by dividing, I D shall be to D Z, as one to two: But Z D was to D A, that is, to D L, as two to five: Therefore, ex equali, *and Converting, L D is to D I, as five to one: and, by Conversion of Proportion, D L is to D I, as five to four: But D Z was to D L, as two to five: Therefore, again,* ex equali, *D Z is to L I, as two to four: Therefore L I is double of D Z: Which was to be demonstrated.*

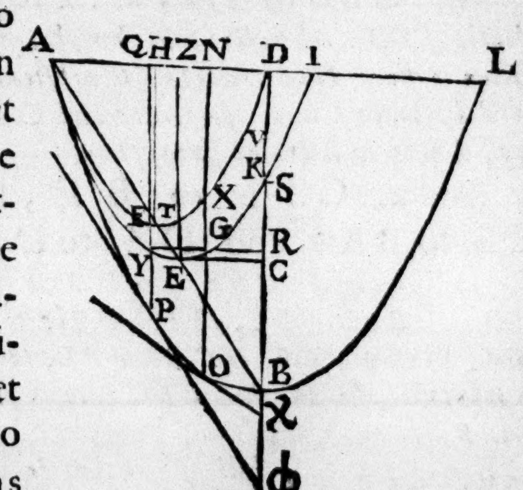
Q And, A D is to D I, as five to one.] *This we have but just now demonstrated.*

R For it hath been demonstrated, above, that the Portion whose Axis is greater than Sesquialter of the Semi-parameter, if it have not lesser proportion in Gravity to the Liquid, &c.] *He hath demonstrated this in the fourth Proposition of this Book.*

CONCLUSION II.

A *If the Portion have lesser proportion in Gravity to the Liquid, than the Square S B hath to the Square B D, but greater than the Square X O hath to the Square B D, being demitted into the Liquid, so inclined, as that its Base touch not the Liquid, it shall continue inclined, so, as that its Base shall not in the least touch the Surface of the Liquid, and its Axis shall make an Angle with the Liquids Surface, greater than the Angle X.*

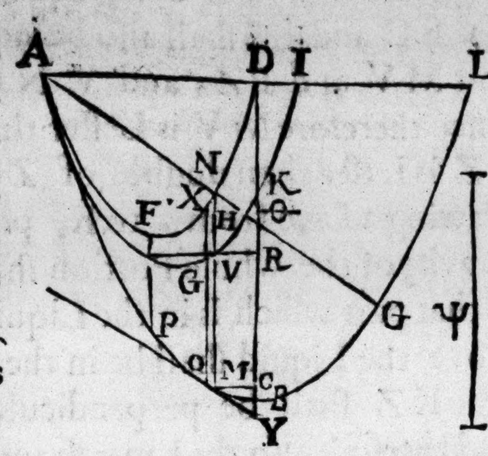
THerfore repeating the first figure, let the Portion have unto the Liquid in Gravitie a proportion greater than the Square X O hath to the square B D, but lesser than the Square made of the Excesse by which the Axis is greater than Sesquialter of the Semi-Parameter, that is, of S B, hath to the Square B D: and as the Portion is to the Liquid in Gravity, so let the Square made of the Line Ψ be



B to the Square B D: Ψ shall be greater than X O, but lesser than the Excesse by which the Axis is greater than Sesquialter of the Semi-parameter, that is, than S B. Let a Right Line M N be applyed to fall between the Conick-Sections A M Q L and A X D, [*parallel to B D falling betwixt O X and B D,*] and equall to the Line Ψ: and let it cut the remaining Conick Section A H I in the point H, and the

C Right Line R G in V. It shall be demonstrated that M H is double to H N, like as it was demonstrated that O G is double to G X. And,

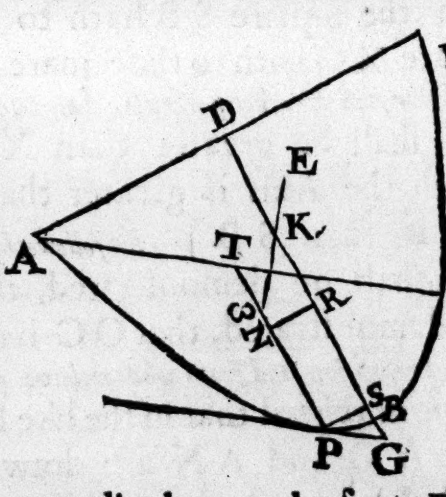
And from the Point M draw M Y touching the Section A M Q L in M; and M C perpendicular to B D: and lastly, having drawn A N & prolonged it to Q, the Lines A N & N Q shall be equall to each other. For in regard that in the Like Portions A M Q L and A X D the Lines A Q and A N are drawn from the Bases unto the Portions, which Lines contain equall Angles with the said



D

Bases, Q A shall have the same proportion to A M that L A hath to A D: Therefore A N is equall to N Q, and A Q parallel to M Y. E F

It is to be demonstrated that the Portion being demitted into the Liquid, and so inclined as that its Base touch not the Liquid, it shall continue inclined so as that its Base shall not in the least touch the Surface of the Liquid, and its Axis shall make an Angle with the Liquids Surface greater than the Angle X. Let it be demitted into the Liquid, and let it stand, so, as that its Base do touch the Surface of the Liquid in one Point only; and let the Portion be cut thorow the Axis by a Plane erect unto the Surface of the Liquid, and Let the Section of the Superficies of the Portion be A P O L, the Section of a Rightangled Cone, and let the Section of the Liquids Surface be A O; And let the Axis of the Portion and Diameter of the Section be B D: and let B D be cut in the Points K and R as hath been said; also draw P G Parallel to A O and touching the Section A P O L in P; and from that Point



G

draw P T Parallel to B D, and P S perpendicular to the same B D. Now, forasmuch as the Portion is unto the Liquid in Gravity, as the Square made of the Line Ψ is to the Square B D; and since that as the portion is unto the Liquid in Gravitie, so is the part thereof submerged unto the whole Portion; and that as the part submerged is to the whole, so is the Square T P to the Square B D; It followeth that the Line Ψ shall be equall to T P: And therefore the Lines M N and P T, as also the Portions A M Q and A P O shall likewise be equall to each other. And seeing that in the Equall and Like Portions A P O L and A M Q L the Lines A O and A Q are drawn from the extremites of their Bases, so, as that the Portions cut off do make Equall Angles with their Diameters; as also the H

 Angle

Angles at Y and G being equall; therefore the Lines Y B and G B, and B C and B S shall also be equall: And therefore C R and S R, and M V and P Z, and V N and Z T, shall be equall likewise.

K Since therefore M V is Lesser than double of V N, it is manifest that P Z is lesser than double of Z T. Let P ω be double of ω T; and drawing a *Line* from ω to K, prolong it to E. Now the Centre of Gravity of the whole Portion shall be the point K; and the Centre of that part which is in the Liquid shall be ω, and of that which is above the Liquid shall be in the *Line* K E, which let be E: But the *Line* K Z shall be perpendicular unto the Surface of the *Liquid*: And therfore also the Lines drawn thorow the Points E and ω parallell unto K Z, shall be perpendiculars unto the same:

L Therefore the Portion shall not abide, but shall turn about so, as that its *Base* do not in the least touch the Surface of the *Liquid*; in regard that now when it toucheth in but one Point only, it moveth upwards, on

M the part towards A: It is therefore perspicuous, that the Portion shall consist so, as that its Axis shall make an Angle with the *Liquids* Surface greater than the Angle X.

COMMANDINE.

A If the Portion have lesser proportion in Gravity to the Liquid, than the Square S B hath to the Square B D, but greater than the Square X O hath to the Square B D.] *This is the second Part of the Tenth proposition; and the other parts with their Demonstrations, shall hereafter follow in the same Order.*

B ψ shall be greater than X O, but lesser than the Excess by which the Axis is greater than Sesquialter of the Semi-parameter, that is than S B.] *This followeth from the* 10 *of the fifth Book of* Euclids *Elements.*

C It shall be demonstrated, that M H is double to H N, like as it was demonstrated, that O G is double to G X.] *As in the first Conclusion of this Proposition, and from what we have but even now written, thereupon appeareth:*

D For in regard that in the like Portions A M Q L and A X D, the Lines A Q and A N are drawn from the Bases unto the Portions, which Lines contain equall Angles with the said Bases, Q A shall have the same proportion to A N, that L A hath to A D.] *This we have demonstrated above.*

E Therefore A N is equall to N Q.] *For since that Q A is to A N, as L A to A D; Dividing and Converting, A N shall be to N Q, as* A D *to* D L: *But* A D *is equall to* D L; *for that* D B *is supposed to be the* Diameter *of the Portion: Therefore also* (a) *A N is equall to N Q.*

(a) *By* 14 *of the fifth.*

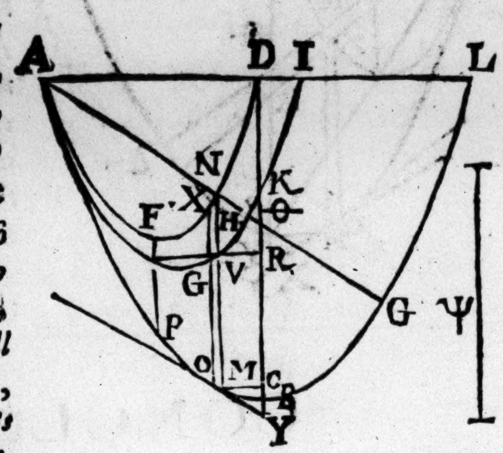
F And A Q parallel to M Y.] *By the fifth of the second Book of* Apollonius *his Conicks.*

G And let B D be cut in the Points K and R as hath been said.] *In the first Conclusion of this Proposition: And let it be cut in K, so, as that B K be double to K D, and in R, so, as that K R may be equall to the Semi-parameter.*

H And, seeing that in the Equall and Like Portions A P O L and A M Q L, the Lines A O and A Q are drawn from the Extremities of their Bases, so, as that the Portions cut off, do make equall Angles

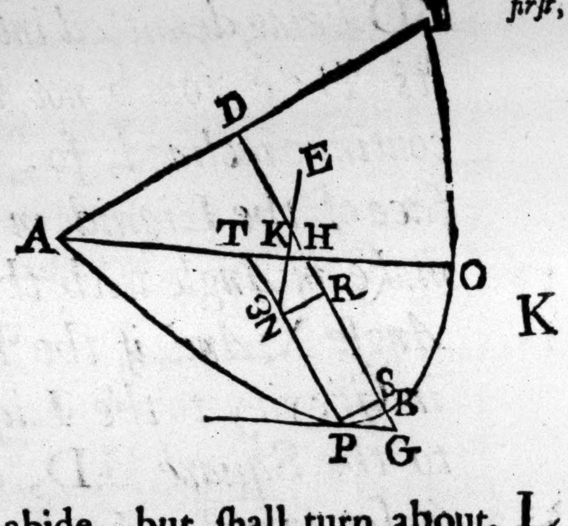
with

with their Diameters; as also, the Angles at Y and G being equall; Therefore, the Lines Y B and G B, & B C & B S, shall also be equall.] *Let the Line A Q cut the* Diameter D B *in* γ, *and let it cut A O in* δ. *Now because that in the equall and like Portions A P O L & A M Q L, from the Extremities of their Bases, A O and A Q are drawn, that contain equall Angles with those Bases; and since the Angles at D, are both Right: Therefore, the Remaining Angles A* δ *D and A* γ *D shall be equall to one another: But the Line P G is parallel unto the Line A O; also M T is parallel to A Q; and P S and M C to A D: Therefore, the Triangles P G S and M T C, as also the Triangles A* δ *D and A* γ *D, are all alike to each other:* (b) *And as A D is to A* δ, *so is A D to A* γ: *and,* Permutando, *the Lines A D and* A D *are equall to each other: Therefore, A* δ *and A* γ *are also equall: But A O and A Q are equall to each other: as also their halves A T and A N: Therefore the Remainders T* δ *and N* γ: *that is, T G and M T, are also equall. And, as* (c) *P G is to G S, so is M T to T C: and* Permutando, *as P G is to M T, so is G S to T C: And, therefore, G S and T C are equall; as also their halves B S and B C: From whence it followeth, that the Remainders S R and C R are also equall: And, consequently, that P Z and M V, and V N and Z T, are likewise equall to one another.*



(b) By 4. of the sixth.

(c) By 34 of the first.



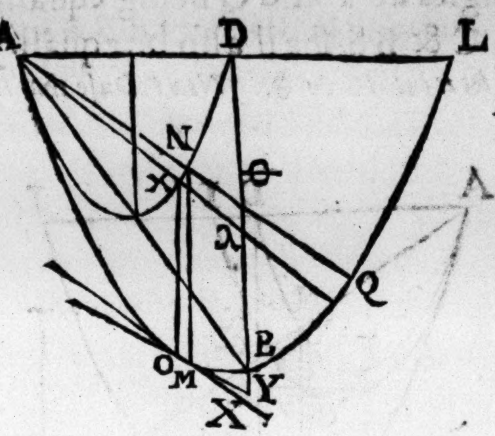
Since, therefore, that N V is lesser than double of V N.] *For M H is double of H N, and M V is lesser than M H: Therefore, M V is lesser than double of H N, and much lesser than double of V N.*

Therefore, the Portion shall not abide, but shall turn about, so, as that its Base do not in the least touch the Surface of the Liquid; in regard that now when it toucheth in but one Point only, it moveth upwards on the part towards A.] Tartaglia's *his Translation hath it thus,* Non ergo manet Portio sed inclinabitur ut Basis ipsius, nec secundum unum tangat Superficiem Humidi, quoniam nunc secundum unum tacta ipsa reclinatur: *Which we have thought fit in this manner to correct, from other Places of* Archimedes, *that the sense might be the more perspicuous. For in the sixth Proposition of this, he thus writeth (as we also have it in the Translation,)* The Solid A P O L, therefore, shall turn about, and its Base shall not in the least touch the Surface of the Liquid. *Again, in the seventh Proposition;* From whence it is manifest, that its Base shall turn about in such manner, as that its Base doth in no wise touch the Surface of the Liquid; For that now when it toucheth but in one Point only, it moveth downwards on the part towards L. *And that the Portion moveth upwards, on the part towards A, doth plainly appear: For since that the Perpendiculars unto the Surface of the Liquid, that pass thorow* ϕ, *do fall on the part towards A, and those that pass thorow E, on the part towards L; it is necessary that the Centre* ϕ *do move upwards, and the Centre E downwards.*

It is therefore perspicuous, that the Portion shall consist, so, as that its Axis shall make an Angle with the Liquids Surface greater than the Angle X.] *For drawing a Line from A to X, prolong it untill it do cut the Diameter*

(d) By 29 of the first.

(e) By 16. of the first.

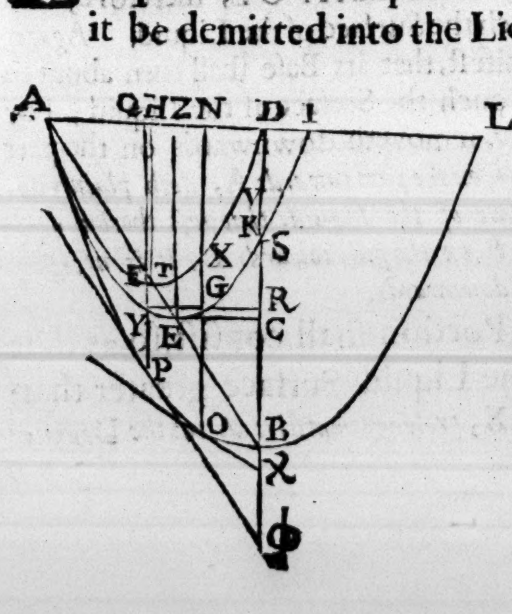


BD in λ; and from the Point O, and parallel to Aλ, draw OX; and let it touch the Section in O, as in the first Figure: And the (d) Angle at X, shall be equall also to the angle λ: But the angle at Y is equall to the Angle at γ; and the (e) Angle A Y D greater than the Angle A λ D, which falleth without it: Therefore the Angle at Y shall be greater than that at X. And because now the Portion turneth about, so, as that the Base doth not touch the Liquid, the Axis shall make an Angle with its Surface greater than the Angle G; that is, than the Angle Y: And, for that reason, much greater than the Angle X.

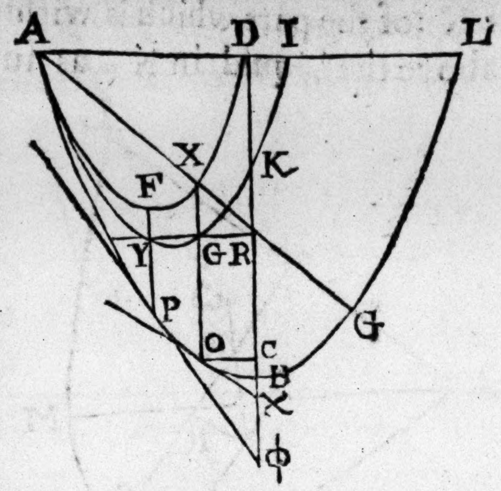
CONCLUSION III.

If the Portion have the same proportion in Gravity to the Liquid, that the Square X O hath to the Square BD, being demitted into the Liquid, so inclined, as that its Base touch not the Liquid, it shall stand and continue inclined, so, as that its Base touch the Surface of the Liquid, in one Point only, and its Axis shall make an Angle with the Liquids Surface equall to the Angle X. And, if the Portion have the same proportion in Gravity to the Liquid, that the Square P F hath to the Square B D, being demitted into the Liquid, & set so inclined, as that its Base touch not the Liquid, it shall stand inclined, so, as that its Base touch the Surface of the Liquid in one Point only, & its Axis shall make an Angle with it, equall to the Angle ☉.

LEt the Portion have the same proportion in Gravity to the Liquid that the Square X O hath to the Square B D; and let it be demitted into the Liquid so inclined, as that its Base touch not the Liquid. And cutting it by a Plane thorow the Axis, erect unto the Surface of the Liquid, let the Section of the Solid, be the Section of a Right-angled Cone, A P M L; let the Section of the Surface of the Liquid be I M; and the Axis of the Portion and Diameter of the Section B D; and let B D be divided as before; and draw PN parallel to I M and



and touching the Section in P, and T P parallel to B D, and P S perpendicular unto B D. It is to be demonstrated that the Portion shall not stand so, but shall encline until that the Base touch the Surface of the Liquid, in one Point only, for let the superior figure stand as it was, and draw O C, Perpendicular to B D; and drawing a Line from A to X, prolong it to Q: A X shalbe equall to X Q. Then draw O X parallel to A Q. And because the Portion is supposed to have the same proportion in Gravity to the Liquid that the square X O hath to the Square B D; the part thereof submerged shall also have the same proportion to the whole; that is, the Square T P to the Square B D; and so T P shall be equal to X O: And since that of the Portions I P M and A O Q the Diameters are equall, the portions shall also be equall. Again, because that in the Equall and Like Portions A O Q L and A P M L the Lines A Q and I M, which cut off equall Portions, are drawn, that, from the Extremity of the Base, and this not from the Extremity; it appeareth that that which is drawn from the end or Extremity of the Base, shall make the Acute Angle with the Diameter of the whole Portion lesser. And the Angle at X being lesse than the Angle at N, B C shall be greater than B S; and C R lesser than S R: And, therfore O G shall be lesser than P Z; and G X greater than Z T: Therfore P Z is greater than double of Z T; being that O G is double of G X. Let P H be double to H T; and drawing a Line from H to K, prolong it to ω. The Center of Gravity of the whole Portion shall be K; the Center of the part which is within the Liquid H, and that of the part which is above the Liquid in the Line K ω; which supposed to be ω. Therefore it shall be demonstrated, both, that K H is perpendicular to the Surface of the Liquid, and those Lines also that are drawn thorow the Points H and ω parallel to K H: And therfore the Portion shall not rest, but shall encline untill that its Base do touch the Surface of the Liquid in one Point; and so it shall continue. For in the Equall Portions A O Q L and A P M L, the Lines A Q and A M, that cut off equall Portions, shall be dawn from the Ends or Terms of the Bases; and A O Q and A P M shall be demonstrated, as in the former, to be equall: Therfore A Q and A M, do make equall Acute Angles with the Diameters of the Portions; and the

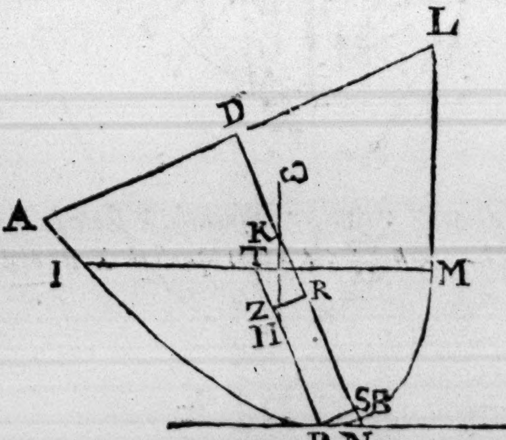


A

B

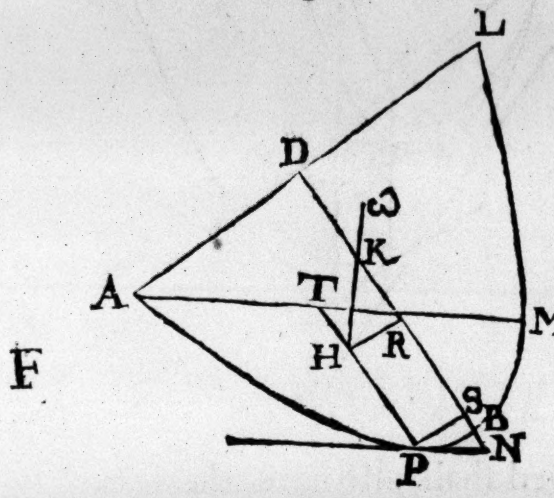
C

D



E

the Angles at X and N are equall. And, therefore, if drawing H K, it be prolonged to ω, the Centre of Gravity of the whole Portion shall be K; of the part which is within the Liquid H; and of the part which is above the Liquid in K ω as suppose in ω; and H K perpendicular to the Surface of the Liquid. Therfore along the same Right Lines shall the part which is within the Liquid move upwards, and the part above it downwards: And therfore the Portion shall rest with one of its Points touching the Surface of the Liquid, and its Axis shall make with the same an Angle equall to X. It is to be demonstrated in the same manner that the Portion that hath the same proportion in Gravity to the Liquid, that the Square P F hath to the Square B D, being demitted into the Liquid, so, as that its Base touch not the Liquid, it shall stand inclined, so, as that its Base touch the Surface of the Liquid in one Point only; and its Axis shall make therwith an Angle equall to the Angle ω.



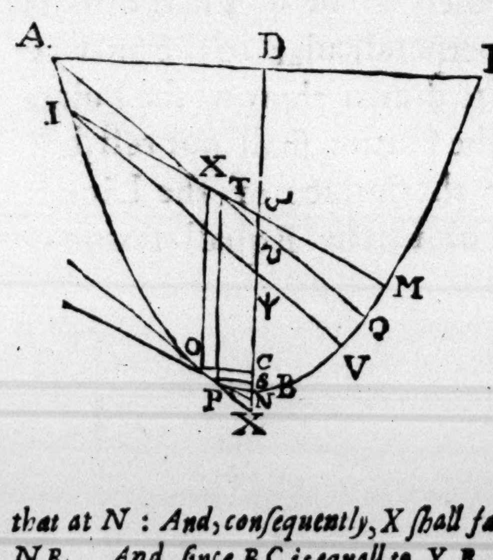
COMMANDINE.

A That is the Square T P to the Square B D.] *By the twenty sixth of the Book of* Archimedes, De Conoidibus & Sphæroidibus: *Therefore,* (a) *the Square T P shall be equall to the Square X O: And for that reason, the Line T P equall to the Line X O.*

(a) *By 9 of the fifth.*

B The Portions shall also be equall.] *By the twenty fifth of the same Book.*

C Again, because that in the Equall and Like Portions, A O Q L and A P M L.] *For, in the Portion A P M L, describe the Portion A O Q equall to the Portion I P M: The Point Q falleth beneath M; for otherwise, the Whole would be equall to the Part. Then draw I V parallel to A Q, and cutting the Diameter in ψ; and let I M cut the same σ; and A Q in σ. I say that the Angle A υ D, is lesser than the Angle I σ D. For the Angle I ψ D is equall to the Angle A υ D:* (b) *But the interiour Angle I ψ D is lesser than the exteriour I σ D: Therefore,* (c) *A υ D shall also be lesser than I σ D.*



(b) *By 29 of the first.*
(c) *By 16 of the first.*

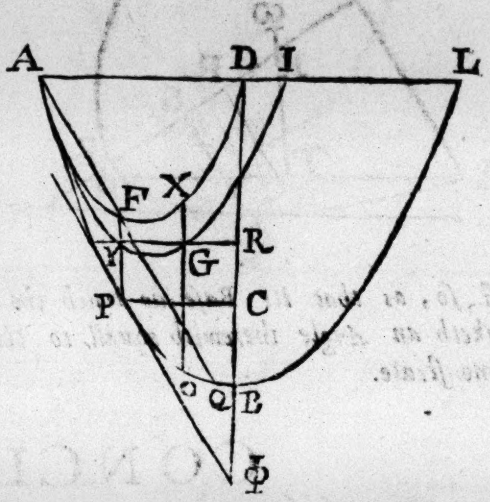
D And the Angle at X, being lesse than the Angle at N.] *Thorow O draw two Lines, O C perpendicular to the Diameter B D, and O X touching the Section in the Point O, and cutting the Diameter in X:* (d) *O X shall be parallel to A Q: and the* (e) *Angle at X, shall be equall to that at υ: Therefore, the* (f) *Angle at X, shall be lesser than the Angle at σ; that is, to that at N: And, consequently, X shall fall beneath N: Therefore, the Line X B is greater than N B. And, since B C is equall to X B, and B S equall to N B; B C shall be greater than B S.*

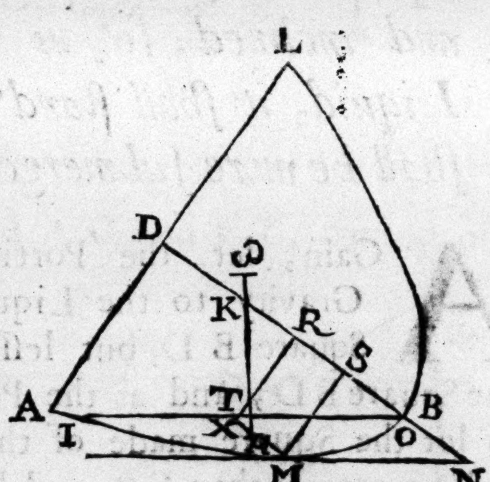
(d) *By 5 of our Second of* Conicks.
(e) *By 29 of the first.*
(f) *By 39 of our first of* Conicks.

Therefore,

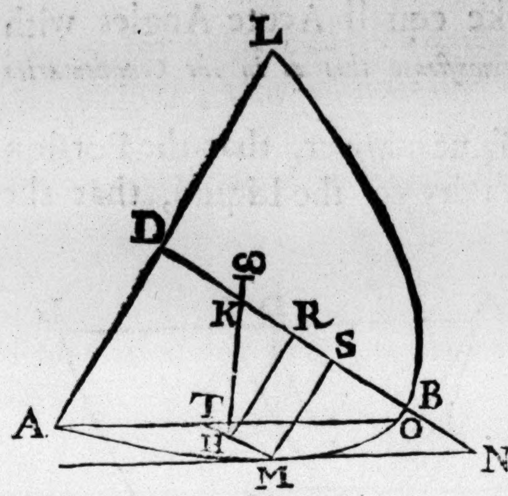
Therefore, A Q and A M do make equall Acute Angles with the Diameters of the Portions.] *We demonstrate this as in the Commentaries upon the second Conclusion.* E

It is to be demonstrated in the same manner, that the Portion that hath the same proportion in Gravity to the Liquid, that the Square P F hath to the Square B D, being demitted into the Liquid, so, as that its Base touch not the Liquid, it shall stand inclined, so, as that its Base touch the Surface of the Liquid in one point only; and its Axis shall make therewith an angle equall to the Angle φ.] *Let the Portion be to the Liquid in Gravity, as the Square P F to the Square B D: and being demitted into the Liquid, so inclined, as that its Base touch not the Liquid, let it be cut thorow the Axis by a Plane erect to the Surface of the Liquid, that that the Section may be A M O L, the Section of a Rightangled Cone; and, let the Section of the Liquids Surface be I O; and the Axis of the Portion and Diameter of the Section B D; which let be cut into the same parts as we said before, and draw M N parallel to I O, that it may touch the Section in the Point M; and M T parallel to B D, and P M S perpendicular to the same. It is to be demonstrated, that the Portion shall not rest, but shall incline, so, as that it touch the Liquids Surface, in one Point of its Base only. For, draw P C perpendicular to B D; and drawing a Line from A to F, prolong it till it meet with the Section in Q; and thorow P draw P φ parallel to A Q: Now, by the things allready demonstrated by us, A F and F Q shall be equall to one another. And being that the Portion hath the same proportion in Gravity unto the Liquid, that the Square P F hath to the Square B D; and seeing that the part submerged, hath the same proportion to the whole Portion; that is, the Square M T to the Square B D; (g) the Square M T shall be equall to the Square P F: and, by the same reason, the Line M T equall to the Line P F. So that there being drawn in the equall & like Portions A P Q L and A M O L, the Lines A Q and I O which cut off equall Portions, the first from the Extreme term of the Base, the last not from the Extremity; it followeth, that A Q drawn from the Extremity, containeth a lesser Acute Angle with the Diameter of the Portion, than I O: But the Line P φ is parallel to the Line A Q, and M N to I O: Therefore, the Angle at φ shall be lesser than the Angle at N; but the Line B C greater than B S; and S R, that is, M X, greater than C R, that is, than P Y: and, by the same reason, X T lesser than Y F. And, since P Y is double to Y F, M X shall be greater than double to Y F, and much greater than double of X T. Let M H be double to H T, and draw a Line from H to K, prolonging it. Now, the Centre of Gravity of the whole Portion shall be the Point K; of the part within the Liquid H; and of the Remaining part above the Liquid in the Line H K produced, as suppose in ω. It shall be demonstrated in the same manner, as before, that both the Line K H and those that are drawn thorow the Points H and ω parallel to the said K H, are perpendicular to the Surface of the Liquid: The Portion, therefore, shall not rest; but when it shall be enclined so far as to touch the Surface of the Liquid in one Point and no more, then it shall stay. For the Angle at N shall*





(g) B 39 of the fifth.

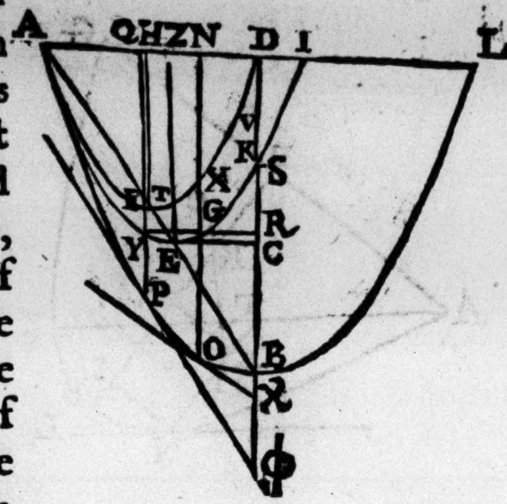


shall be equall to the Angle at φ; and the Line B S equall to the Line B C, and S R to C R: Wherefore, M H shall be likewise equall to P T. Therefore, having drawn H K and prolonged it; the Centre of Gravity of the whole Portion shall be K; of that which is in the Liquid H; and of that which is above it, the Centre shall be in the Line prolonged: let it be in ω. Therefore, along that same Line K H, which is perpendicular to the Surface of the Liquid, shall the part which is within the Liquid move upwards, and that which is above the Liquid downwards: And, for this cause, the Portion, shall be no longer moved, but shall stay, and rest, so, as that its Base do touch the Liquids Surface in but one Point; and its Axis maketh an Angle therewith equall to the Angle φ: And, this is that which we were to demonstrate.

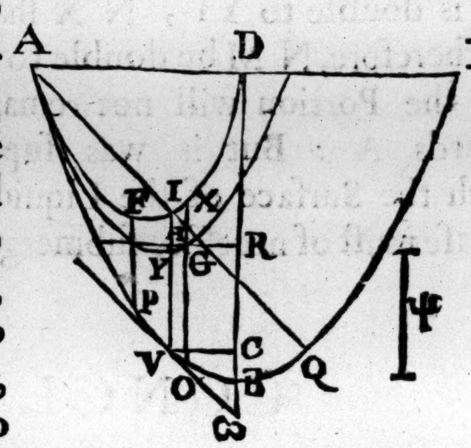
CONCLVSION IV.

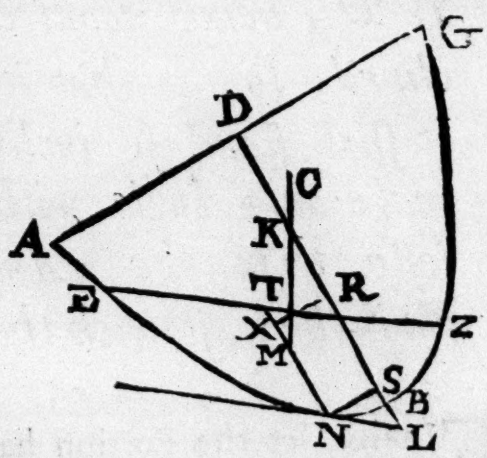
If the Portion have greater proportion in Gravity to the Liquid, than the Square F P to the Square B D, but lesser than that of the Square X O to the Square B D, being demitted into the Liquid, and inclined, so, as that its Base touch not the Liquid, it shall stand and rest, so, as that its Base shall be more submerged in the Liquid.

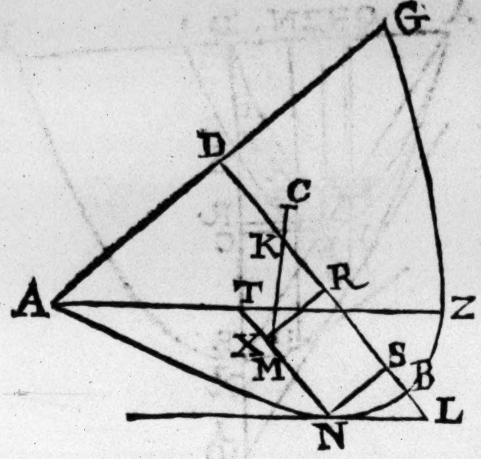
Again, let the Portion have greater proportion in Gravity to the Liquid, than the Square F P to the Square B D, but lesser than that of the Square X O to the Square B D; and as the Portion is in Gravity to the Liquid, so let the Square made of the Line ψ be to the Square B D. ψ shall be greater than F P, and lesser than X O. Apply, therefore, the right Line I V to fall betwixt the Portions A V Q L and A X D; and let it be equall to ψ, and parallel to B D; and let it meet the Remaining Section in Y: V Y shall also be proved double to Y I, like as it hath been demonstrated, that O G is double off G X. And, draw from V, the Line V ω, touching the Section A V Q L in V; and drawing a Line from A to I, prolong it unto Q. We prove in the same manner, that the Line A I is equall to I Q; and that A Q is parallel to V ω. It is to be demonstrated, that the Portion being demitted into the Liquid, and so inclined, as that its Base touch not the Liquid, shall stand, so, that its Base shall be more submerged in the Liquid, than to touch it Surface in but



but one Point only. For let it be demitted into the Liquid, as hath been said; and let it first be so inclined, as that its Base do not in the least touch the Surface of the Liquid. And then it being cut thorow the Axis, by a Plane erect unto the Surface of the Liquid, let the Section of the Portion be A N Z G; that of the Liquids Surface E Z; the Axis of the Portion and Diameter of the Section B D; and let B D be cut in the Points K and R, as before; and draw N L parallel to E Z, and touching the Section A N Z G in N, and N S perpendicular to B D. Now, seeing that the Portion is in Gravity unto the Liquid, as the Square made of the Line is to the Square B D; ψ shall be equall to N T: Which is to be demonstrated as above: And, therefore, N T is also equall to V I: The Portions, therefore, A V Q and E N Z are equall to one another. And, since that in the Equall and like Portions A V Q L and A N Z G, there are drawn A Q and E Z, cutting off equall Portions, that from the Extremity of the Base, this not from the Extreme, that which is drawn from the Extremity of the Base, shall make the Acute Angle with the Diameter of the Portion lesser: and in the Triangles N L S and V ∙ C, the Angle at L is greater than the Angle at ∙: Therefore, B S shall be lesser than B C; and S R lesser than C R: and, consequently, N X greater than V H; and X T lesser than H I. Seeing, therefore, that V Y is double to Y I; It is manifest, that N X is greater than double to X T. Let N M be double to M T: It is manifest, from what hath been said, that the Portion shall not rest, but will incline, untill that its Base do touch the Surface of the Liquid: and it toucheth it in one Point only, as appeareth in the Figure: And other things







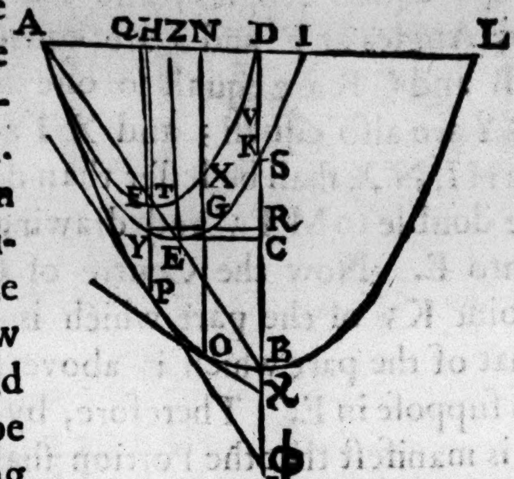
standing as before, we will again demonstrate, that N T is equall to V I; and that the Portions A V Q and A N Z are equall to each other. Therefore, in regard, that in the Equall and Like Portions A V Q L and A N Z G, there are drawn A Q and A Z cutting off equall Portions, they shall with the Diameters of the Portions, contain equall Angles. Therefore, in the Triangles N L S and V • C, the Angles at the Points L and • are equall; and the Right Line B S equall to B C; S R to C R; N X to V H; and X T to H I: And, since V Y is double to Y I, N X shall be greater than double of X T. Let therefore, N M be double to M T. It is hence again manifest, that the Portion will not remain, but shall incline on the part towards A: But it was supposed, that the said Portion did touch the Surface of the Liquid in one sole Point: Therefore, its Base must of necessity submerge farther into the Liquid.

CONCLVSION V.

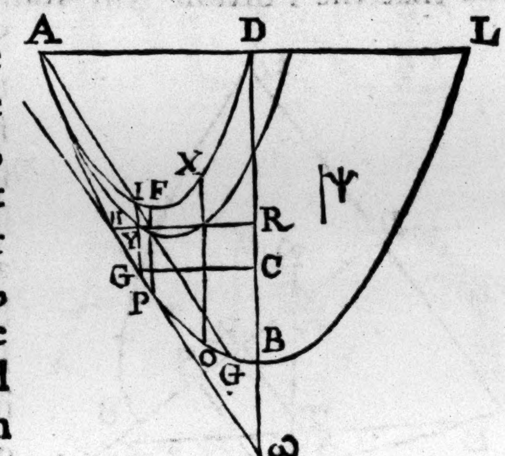
If the Portion have lesser proportion in Gravity to the Liquid, than the Square F P to the Square B D, being demitted into the Liquid, and inclined, so, as that its Base touch not the Liquid, it shall stand so inclined, as that its Axis shall make an Angle with the Surface of the Liquid, lesse than the Angle ψ; And its Base shall not in the least touch the Liquids Surface.

Finally, let the Portion have lesser proportion to the Liquid in Gravity, than the Square F P hath to the Square B D; and as the Portion is in Gravity to the Liquid, so let the Square made of the Line ψ be to the Square B D. ψ shall be lesser than P F. Again, apply any Right Line as G I, falling betwixt the Sections A G Q L and A X D, and parallel to B D; and let it cut the Middle Conick Section in the Point H, and the

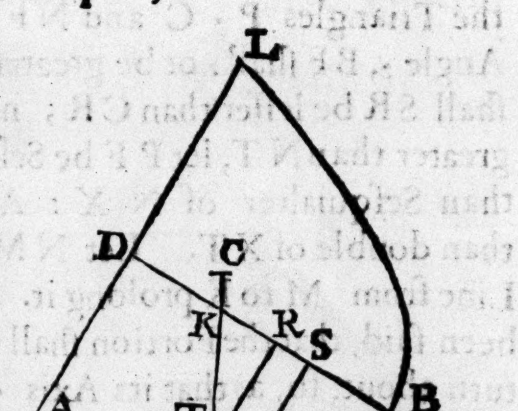
the Right Line R Y in Y. We shall demonstrate G H to be double to H I, as it hath been demonstrated, that O G is double to G X. Then draw G ω touching the Section A G Q L in G; and G C perpendicular to B D; and drawing a Line from A to I, prolong it to Q. Now A I shall be equall to I Q; and A Q parallel to G ω. It is to be demonstrated, that the Portion being



demitted into the Liquid, and inclined, so, as that its Base touch the Liquid, it shall stand so inclined, as that its Axis shall make an Angle with the Surface of the Liquid lesse than the Angle φ; and its Base shall not in the least touch the Liquids Surface. For let it be demitted into the Liquid, and let it stand, so, as that its Base do touch the Surface of the Liquid in one Point only: and the Portion being cut thorow the Axis by a

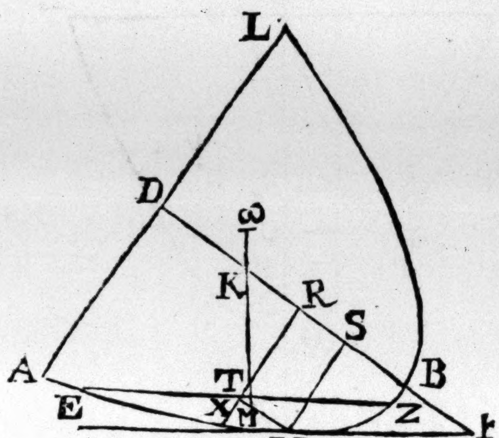


Plane erect unto the Surface of the Liquid, let the Section of the Portion be A N Z L, the Section of a Rightangled Cone; that of the Surface of the Liquid A Z; and the Axis of the Portion and Diameter of the Section B D; and let B D be cut in the Points K and R as hath been said above; and draw N F parallel to A Z, and touching the Section of the Cone in the Point N; and N T parallel to B D; and N S perpendicular to the same. Because, now, that the Portion is in Gravity to the Liquid, as



the Square made of ψ is to the Square B D; and since that as the Portion is to the Liquid in Gravity, so is the Square N T to the Square B D, by the things that have been said; it is plain, that N T is equall to the *Line* ψ: And, therefore, also, the Portions A N Z and A G Q are equall. And, seeing that in the Equall and Like Portions A G Q L and A N Z L; there are drawn from the Extremities of their Bases, A Q and A Z which cut off equall Portions: It is obvious, that with the Diameters of the Portions they make

make equall Angles; and that in the Triangles N F S and G φ C, the Angles at F and φ are equall; as also, that S B and B C, and S R and C R are equall to one another : And, therefore, N X and G Y are also equall; and X T and Y I. And since G H is double to H I, N X shall be lesser than double of X T. Let N M therefore be double to M T; and drawing a Line from M to K, prolong it unto E. Now the Centre of Gravity of the whole shall be the Point K; of the part which is in the Liquid the Point M; and that of the part which is above the Liquid in the Line prolonged, as suppose in E. Therefore, by what was even now demonstrated, it is manifest that the Portion shall not stay thus, but shall incline, so, as that its Base do in no wise touch the Surface of the Liquid. And that the Portion will stand, so, as to make an Angle with the Surface of the Liquid lesser than the Angle φ, shall thus be demonstrated. Let it, if possible, stand, so, as that it do not make an Angle lesser than the Angle φ; and dispose all things else in the same manner as before; as is done in the present Figure.



We are to demonstrate in the same method, that N T is equall to ψ; and by the same reason, equall also to G I. And since that in the Triangles P φ C and N F S, the Angle F is not lesser than the Angle φ, B F shall not be greater than B C : And, therefore, neither shall S R be lesser than C R; nor N X than P Y : But since P F is greater than N T, let P F be Sesquialter of P Y : N T shall be lesser than Sesquialter of N X : And, therefore, N X shall be greater than double of X T. Let N M be double of M T; and drawing a Line from M to K prolong it. It is manifest, now, by what hath been said, that the Portion shall not continue in this position, but shall turn about, so, as that its Axis do make an Angle with the Surface of the Liquid, lesser than the Angle φ.

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Presented to the Most Serene Don Cosimo II. Great Duke of *TUSCANY*:
CONCERNING
The Natation of BODIES Upon, or Submersion In, the WATER.

Considering (Most Serene Prince) that the publishing this present Treatise, of so different an Argument from that which many expect, and which according to the intentions I proposed in my * Astronomicall *Adviso*, I should before this time have put forth, might peradventure make some thinke, either that I had wholly relinquished my farther imployment about the new Celestiall Observations, or that, at least, I handled them very remissely; I have judged fit to render an account, aswell of my deferring that, as of my writing, and publishing this treatise.

*His Nuncio Siderio.

As to the first, the last discoveries of *Saturn* to be tricorporeall, and of the mutations of Figure in *Venus*, like to those that are seen in the Moon, together with the Consequents depending thereupon, have not so much occasioned the demur, as the investigation of the times of the Conversions of each of the Four Medicean Planets about *Jupiter*, which I lighted upon in *April* the year past, 1611, at my being in *Rome*; where, in the end, I assertained my selfe, that the first and neerest to *Jupiter*, moved about 8 *gr.* & 29 *m.* of its Sphere in an houre, makeing its whole revolution in one naturall day, and 18 hours, and almost an halfe. The second moves in its Orbe 14 *gr.* 13 *min.* or very neer, in an hour, and its compleat conversion is consummate in 3 dayes, 13 hours, and one third, or thereabouts. The third passeth in an hour, 2 *gr.* 6 *min.* little more or less of its Circle, and measures it all in 7 dayes, 4 hours, or very neer. The fourth, and more remote than the rest, goes in one houre, 0 *gr.* 54 *min.* and almost an halfe of its Sphere, and finisheth it all in 16 dayes, and very neer 18 hours. But because the excessive velocity of their returns or restitutions, requires a most scrupulous precisenesse to calculate their places, in times past

and future, especially if the time be for many Moneths or Years; I am therefore forced, with other Observations, and more exact than the former, and in times more remote from one another, to correct the Tables of such Motions, and limit them even to the shortest moment: for such exactnesse my first Observations suffice not; not only in regard of the short intervals of Time, but because I had not as then found out a way to measure the distances between the said Planets by any Instrument: I Observed such Intervals with simple relation to the Diameter of the Body of *Jupiter*; taken, as we have said, by the eye, the which, though they admit not errors of above a Minute, yet they suffice not for the determination of the exact greatness of the Spheres of those Stars. But now that I have hit upon a way of taking such measures without failing, scarce in a very few Seconds, I will continue the observation to the very occultation of JUPITER, which shall serve to bring us to the perfect knowledge of the Motions, and Magnitudes of the Orbes of the said Planets, together also with some other consequences thence arising. I adde to these things the observation of some obscure Spots, which are discovered in the Solar Body, which changing position in that, propounds to our consideration a great argument either that the Sun revolves in it selfe, or that perhaps other Starrs, in like manner as *Venus* and *Mercury*, revolve about it, invisible in other times, by reason of their small digressions, lesse than that of *Mercury*, and only visible when they interpose between the Sun and our eye, or else hint the truth of both this and that; the certainty of which things ought not to be contemned, nor omitted.

The Authors Observations of the Solar Spots.

Continuall observation hath at last assured me that these Spots are matters contiguous to the Body of the Sun, there continually produced in great number, and afterwards dissolved, some in a shorter, some in a longer time, and to be by the Conversion or Revolution of the Sun in it selfe, which in a Lunar Moneth, or thereabouts, finisheth its Period, caried about in a Circle, an accident great of it selfe, and greater for its Consequences.

The occasion inducing the Author to write this Treatise.

As to the other particular in the next place. * Many causes have moved me to write the present Tract, the subject whereof, is the Dispute which I held some dayes since, with some learned men of this City, about which, as your Highnesse knows, have followed many Discourses: The principall of which Causes hath been the Intimation of your Highnesse, having commended to me Writing, as a singular means to make true known from false, reall from apparent Reasons, farr better than by Disputing vocally, where the one or the other, or very often both the Disputants, through too great

greate heate, or exalting of the voyce, either are not understood, or else being transported by ostentation of not yeilding to one another, farr from the first Proposition, with the novelty, of the various Proposals, confound both themselves and their Auditors.

Moreover, it seemed to me convenient to informe your Highnesse of all the sequell, concerning the Controversie of which I treat, as it hath been advertised often already by others: and because the Doctrine which I follow, in the discussion of the point in hand, is different from that of *Aristotle*; and interferes with his Principles, I have considered that against the Authority of that most famous Man, which amongst many makes all suspected that comes not from the Schooles of the Peripateticks, its farr better to give ones Reasons by the Pen than by word of mouth, and therfore I resolved to write the present discourse: in which yet I hope to demonstrate that it was not out of capritiousnesse, or for that I had not read or understood *Aristotle*, that I sometimes swerve from his opinion, but because severall Reasons perswade me to it, and the same *Aristotle* hath tought me to fix my judgment on that which is grounded upon Reason, and not on the bare Authority of the Master; and it is most certaine according to the sentence of *Alcinoos*, that philosophating should be free. Nor is the resolution of our Question in my judgment without some benefit to the Universall, forasmuch as treating whether the figure of Solids operates, or not, in their going, or not going to the bottome in Water, in occurrences of building Bridges or other Fabricks on the Water, which happen commonly in affairs of grand import, it may be of great availe to know the truth.

Aristotle preferrs Reason to the Authority of an Author.

The benefit of this Argument.

I say therfore, that being the last Summer in company with certain Learned men, it was said in the argumentation; That Condensation was the propriety of Cold, and there was alledged for instance, the example of Ice: now I at that time said, that, in my judgment, the Ice should be rather Water rarified than condensed, and my reason was, because Condensation begets diminution of Mass, and augmentation of gravity, and Rarifaction causeth greater Lightness, and augmentarion of Masse: and Water in freezing, encreaseth in Masse, and the Ice made thereby is lighter than the Water on which it swimmeth.

Condensation the Propriety of Cold, according to the Peripateticks.

Ice rather water rarified, than condensed, and why:

What I say, is manifest, because, the medium subtracting from the whole Gravity of Sollids the weight of such another Masse of the said Medium; as Archimedes *proves in his * First Booke* De Insidentibus Humido; *when ever the Masse of the said Solid encreaseth by Distraction, the more shall the* Medium *detract from its entire Gravity; and lesse, when by Compression it shall be condensed and reduced to a lesse Masse.*

* In lib: 1. of Natation of Bodies Prop. 7.

Figure operates not in the Natation of Sollids.

It was answered me, that that proceeded not from the greater Levity, but from the Figure, large and flat, which not being able to penetrate the Resistance of the Water, is the cause that it submergeth not. I replied, that any piece of Ice, of whatsoever Figure, swims upon the Water, a manifest signe, that its being never so flat and broad, hath not any part in its floating: and added, that it was a manifest proofe hereof to see a piece of Ice of very broad Figure being thrust to the botome of the Water, suddenly return to flote atoppe, which had it been more grave, and had its swimming proceeded from its Forme, unable to penetrate the Resistance of the *Medium*, that would be altogether impossible; I concluded therefore, that the Figure was in sort a Cause of the Natation or Submersion of Bodies, but the greater or lesse Gravity in respect of the Water: and therefore all Bodyes heavier than it of what Figure soever they be, indifferently go to the bottome, and the lighter, though of any figure, float indifferently on the top: and I suppose that those which hold otherwise, were induced to that beliefe, by seeing how that diversity of Formes or Figures, greatly altereth the Velosity, and Tardity of Motion; so that Bodies of Figure broad and thin, descend far more leasurely into the Water, than those of a more compacted Figure, though both made of the same Matter: by which some might be induced to believe that the Dilatation of the Figure might reduce it to such amplenesse that it should not only retard but wholly impede and take away the Motion, which I hold to be false. Upon this Conclusion, in many dayes discourse, was spoken much, and many things, and divers Experiments produced, of which your Highnesse heard, and saw some, and in this discourse shall have all that which hath been produced against my Assertion, and what hath been suggested to my thoughts on this matter, and for confirmation of my Conclusion: which if it shall suffice to remove that (as I esteem hitherto false) Opinion, I shall thinke I have not unprofitably spent my paynes and time. and although that come not to passe, yet ought I to promise another benefit to my selfe, namely, of attaining the knowledge of the truth, by hearing my Fallacyes confuted, and true demonstrations produced by those of the contrary opinion.

And to proceed with the greatest plainness and perspicuity that I can possible, it is, I conceive, necessary, first of all to declare what is the true, intrinsecall, and totall Cause, of the ascending of some Sollid Bodyes in the Water, and therein floating; or on the contrary, of their sinking. and so much the rather in asmuch as I cannot satisfie my selfe in that which *Aristotle* hath left written on this Subject.

The cause of the Natation & sub-

I say then the Cause why some Sollid Bodyes descend to the Bottom

Bottom of Water, is the excesse of their Gravity, above the Gravity of the Water; and on the contrary, the excess of the Waters Gravity above the Gravity of those, is the Cause that others do not descend, rather that they rise from the Bottom, and ascend to the Surface. This was subtilly demonstrated by *Archimedes* in his Book Of the NATATION of BODIES: Conferred afterwards by a very grave Author, but, if I erre not invisibly, as below for defence of him, I shall endeavour to prove. mersion of Solids in the Water.

I, with a different Method, and by other meanes, will endeavour to demonstrate the same, reducing the Causes of such Effects to more intrinsecall and immediate Principles, in which also are discovered the Causes of some admirable and almost incredible Accidents, as that would be, that a very little quantity of Water, should be able, with its small weight, to raise and sustain a Solid Body, an hundred or a thousand times heavier than it.

And because demonstrative Order so requires, I shall define certain Termes, and afterwards explain some Propositions, of which, as of things true and obvious, I may make use of to my present purpose.

DEFINITION I.

I then call equally Grave in specie, *those Matters of which equall Masses weigh equally.*

As if for example, two Balls, one of Wax, and the other of some Wood of equall Masse, were also equall in Weight, we say, that such Wood, and the Wax are *in specie* equally grave.

DEFINITION II.

But equally grave in Absolute Gravity, we call two Sollids, weighing equally, though of Mass they be unequall.

As for example, a Mass of Lead, and another of Wood, that weigh each ten pounds, I call equall in Absolute Gravity, though the Mass of the Wood be much greater then that of the Lead.

And, consequently, less Grave in specie.

DEFINITION III.

I call a Matter more Grave in specie *than another, of which a Mass, equall to a Mass of the other, shall weigh more.*

And

And so I say, that Lead is more grave *in specie* than Tinn, because if you take of them two equall Masses, that of the Lead weigheth more.

DEFINITION IV.

But I call that Body more grave absolutely than this, if that weigh more than this, without any respect had to the Masses.

And thus a great piece of Wood is said to weigh more than a little lump of Lead, though the Lead be *in specie* more heavy than the Wood. And the same is to be understood of the less grave *in specie*, and the less grave absolutely.

These Termes defined, I take from the Mechanicks two Principles: the first is, that

AXIOME. I.

Weights absolutely equall, moved with equall Velocity, are of equall Force and Moment in their operations.

DEFINITION. V.

Moment, amongst Mechanicians, signifieth that Vertue, that Force, or that Efficacy, with which the Mover moves, and the Moveable resists.

Which Vertue dependes not only on the simple Gravity, but on the Velocity of the Motion, and on the diverse Inclinations of the Spaces along which the Motion is made: For a descending Weight makes a greater Impetus *in a Space much declining, than in one less declining; and in summe, what ever is the occasion of such Vertue, it ever retaines the name of* Moment; *nor in my Judgement, is this sence new in our Idiome, for, if I mistake not, I think we often say; This is a weighty businesse, but the other is of small moment: and we consider lighter matters and let pass those of Moment; a Metaphor, I suppose, taken from the Mechanicks.*

As for example, two weights equall in absolute Gravity, being put into a Ballance of equall Arms, they stand in *Equilibrium*, neither one going down, nor the other up: because the equality of the Distances of both, from the Centre on which the Ballance is supported, and about which it moves, causeth that those weights, the said Ballance moving, shall in the same Time move equall Spaces, that is, shall move with equall Velocity, so that there is no reason for which this

this Weight should descend more than that, or that more than this; and therefore they make an *Equilibrium*, and their Moments continue of semblable and equall Vertue.

The second Principle is; That

AXIOME II.

The Moment and Force of the Gravity, is encreased by the Velocity of the Motion.

So that Weights absolutely equall, but conjoyned with Velocity unequall, are of Force, Moment and Vertue unequall: and the more potent, the more swift, according to the proportion of the Velocity of the one, to the Velocity of the other. Of this we have a very pertinent example in the Balance or Stiliard of unequall Arms, at which Weights absolutely equall being suspended, they do not weigh down, and gravitate equally, but that which is at a greater distance from the Centre, about which the Beam moves, descends, raising the other, and the Motion of this which ascends is slow, and the other swift: and such is the Force and Vertue, which from the Velocity of the Mover, is conferred on the Moveable, which receives it, that it can exquisitely compensate, as much more Weight added to the other slower Moveable: so that if of the Arms of the Balance, one were ten times as long as the other, whereupon in the Beames moving about the Centre, the end of that would go ten times as far as the end of this, a Weight suspended at the greater distance, may sustain and poyse another ten times more grave absolutely than it: and that because the Stiliard moving, the lesser Weight shall move ten times faster than the bigger. It ought alwayes therefore to be understood, that Motions are according to the same Inclinations, namely, that if one of the Moveables move perpendicularly to the Horizon, then the other makes its Motion by the like Perpendicular; and if the Motion of one were to be made Horizontally; that then the other is made along the same Horizontall plain: and in summe, alwayes both in like Inclinations. This proportion between the Gravity and Velocity is found in all Mechanicall Instruments: and is considered by *Aristotle*, as a Principle in his *Mechanicall Questions*; whereupon we also may take it for a true Assumption, That

AXIOME III.

Weights absolutely unequall, do alternately counterpoyse and become of equall Moments, as oft as their Gravities, with contrary proportion, answer to the Velocity of their Motions.

That

That is to say, that by how much the one is less grave than the other; by so much is it in a constitution of moving more swiftly than that.

Having prefatically explicated these things, we may begin to enquire, what Bodyes those are which totally submerge in Water, and go to the Bottom, and which those that by constraint float on the top, so that being thrust by violence under Water, they return to swim, with one part of their Mass visible above the Surface of the Water: and this we will do by considering the respective operation of the said Solids, and of Water: Which operation followes the Submersion and sinking; and this it is, That in the Submersion that the Solid maketh, being depressed downwards by its proper Gravity, it comes to drive away the water from the place where it successively subenters, and the water repulsed riseth and ascends above its first levell, to which Ascent on the other side it, as being a grave Body of its own nature, resists: And because the descending Solid more and more immerging, greater and greater quantity of Water ascends, till the whole Sollid be submerged; its necessary to compare the Moments of the Resistance of the water to Ascension, with the Moments of the pressive Gravity of the Solid: And if the Moments of the Resistance of the water, shall equalize the Moments of the Solid, before its totall Immersion; in this case doubtless there shall be made an *Equilibrium*, nor shall the Body sink any farther. But if the Moment of the Solid, shall alwayes exceed the Moments wherewith the repulsed water successively makes Resistance, that Solid shall not only wholly submerge under water, but shall descend to the Bottom. But if, lastly, in the instant of totall Submersion, the equality shall be made between the Moments of the prement Solid, and the resisting Water; then shall rest, ensue, and the said Solid shall be able to rest indifferently, in whatsoever part of the water. By this time is manifest the necessity of comparing the Gravity of the water, and of the Solid; and this comparison might at first sight seem sufficient to conclude and determine which are the Solids that float a-top, and which those that sink to the Bottom in the water, asserting that those shall float which are lesse grave *in specie* than the water, and those submerge, which are *in specie* more grave. For it seems in appearance, that the Sollid in sinking continually, raiseth so much Water in Mass, as answers to the parts of its own Bulk submerged: whereupon it is impossible, that a Solid less grave *in specie*, than water, should wholly sink, as being unable to raise a weight greater than its own, and such would a Mass of water equall to its own Mass be. And likewise it seems necessary, that the graver Solids do go to the Bottom, as being of a Force more than sufficient for the raising a Masse of water, equall to its own, though inferiour in weight. Nevertheless the business succeeds otherwise: and though

How the submersion of Solids in the Water, is effected.

What Solids shall float on the Water.

What Solids shall sinke to the botome.

What Solids shall rest in all places of the Water.

The Gravitie of the Water and Solid must be compared in all Problems of Natation of Bodies.

though the Conclusions are true, yet are the Causes thus assigned deficient, nor is it true, that the Solid in submerging, raiseth and repulseth Masses of Water, equall to the parts of it self submerged; but the Water repulsed, is alwayes less than the parts of the Solid submerged : and so much the more by how much the Vessell in which the Water is contained is narrower : in such manner that it hinders not, but that a Solid may submerge all under Water, without raising so much Water in Mass, as would equall the tenth or twentieth part of its own Bulk : like as on the contrary, a very small quantity of Water, may raise a very great Solid Mass, though such Solid should weigh absolutely a hundred times as much, or more, than the said Water, if so be that the Matter of that same Solid be *in specie* less grave than the Water. And thus a great Beam, as suppose of a 1000 weight, may be raised and born afloat by Water, which weighs not 50 : and this happens when the Moment of the Water is compensated by the Velocity of its Motion.

The water repulsed is ever less than the parts of the Sollid submerged.

A small quantity of water, may float a very great Solid Mass.

But because such things, propounded thus in abstract, are somewhat difficult to be comprehended, it would be good to demonstrate them by particular examples; and for facility of demonstration, we will suppose the Vessels in which we are to put the Water, and place the Solids, to be inviron'd and included with sides erected perpendicular to the Plane of the Horizon, and the Solid that is to be put into such vessell to be either a streight Cylinder, or else an upright Prisme.

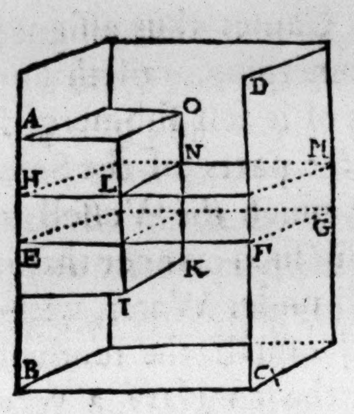
The which proposed and declared, I proceed to demonstrate the truth of what hath been hinted, forming the ensuing Theoreme.

THEOREME I.

The Mass of the Water which ascends in the submerging of a Solid, Prisme or Cylinder, or that abaseth in taking it out, is less than the Mass of the said Solid, so depressed or advanced : and hath to it the same proportion, that the Surface of the Water circumfusing the Solid, hath to the same circumfused Surface, together with the Base of the Solid.

The Proportion of the water raised to the Solid submerged.

Let the Vessell be A B C D, and in it the Water raised up to the Levell E F G, before the Solid Prisme H I K be therein immerged; but after that it is depressed under Water, let the Water be raised as high as the Levell L M, the Solid H I K shall then be all under Water, and the Mass of the elevated Water shall be L G, which is less than the



Masse of the Solid depressed, namely of HIK, being equall to the only part EIK, which is contained under the first Levell EFG. Which is manifest, because if the Solid HIK be taken out, the Water IG shall return into the place occupied by the Mass EIK, where it was continuate before the submersion of the Prisme. And the Mass LG being equall to the Mass EK: adde thereto the Mass EN, and it shall be the whole Mass EM, composed of the parts of the Prisme EN, and of the Water NF, equall to the whole Solid HIK: And, therefore, the Mass LG shall have the same proportion to EM, as to the Mass HIK: But the Mass LG hath the same proportion to the Mass EM, as the Surface LM hath to the Surface MH: Therefore it is manifest, that the Mass of Water repulsed LG, is in proportion to the Mass of the Solid submerged HIK; as the Surface LM, namely, that of the Water ambient about the Sollid, to the whole Surfaoe HM, compounded of the said ambient water, and the Base of the Prisme HN. But if we suppose the first Levell of the Water the according to the Surface HM, and the Prisme allready submerged HIK; and after to be taken out and raised to EAO, and the Water to be faln from the first Levell HLM as low as EFG; It is manifest, that the Prisme EAO being the same with HIK, its superiour part HO, shall be equall to the inferiour EIK: and remove the common part EN, and, consequently, the Mass of the Water LG is equall to the Mass HO; and, therefore, less than the Solid, which is without the Water, namely, the whole Prisme EAO, to which likewise, the said Mass of Water abated LG, hath the same proportion, that the Surface of the Waters circumfused LM hath to the same circumfused Surface, together with the Base of the Prisme AO: which hath the same demonstration with the former case above.

And from hence is inferred, that the Mass of the Water, that riseth in the immersion of the Solid, or that ebbeth in elevating it, is not equall to all the Mass of the Solid, which is submerged or elevated, but to that part only, which in the immersion is under the first Levell of the Water, and in the elevation remaines above the first Levell: Which is that which was to be demonstrated. We will now pursue the things that remain.

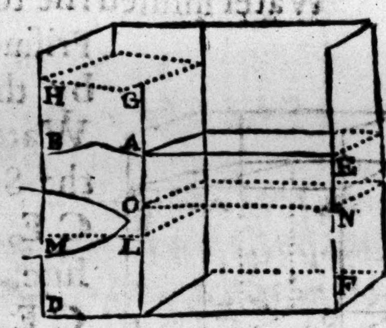
And first we will demonstrate that,

THEO-

THEOREME II.

When in one of the above said Vessels, of what ever breadth, whether wide or narrow, there is placed such a Prisme or Cylinder, inviron'd with Water, if we elevate that Solid perpendicularly, the Water circumfused shall abate, and the Abatement of the Water, shall have the same proportion to the Elevation of the Prisme, as one of the Bases of the Prisme, hath to the Surface of the Water Circumfused.

The proportion of the water abated, to the Solid raised.



IMagine in the Vessell, as is aforesaid, the Prisme A C D B to be placed, and in the rest of the Space the Water to be diffused as far as the Levell E A: and raising the Solid, let it be transferred to G M, and let the Water be abased from E A to N O: I say, that the descent of the Water, measured by the Line A O, hath the same proportion to the rise of the Prisme, measured by the Line G A, as the Base of the Solid G H hath to the Surface of the Water N O. The which is manifest: because the Mass of the Solid G A B H, raised above the first Levell E A B, is equall to the Mass of Water that is abased E N O A. Therefore, E N O A and G A B H are two equall Prismes; for of equall Prismes, the Bases answer contrarily to their heights: Therefore, as the Altitude A O is to the Altitude A G, so is the Superficies or Base G H to the Surface of the Water N O. If therefore, for example, a Pillar were erected in a waste Pond full of Water, or else in a Well, capable of little more then the Mass of the said Pillar, in elevating the said Pillar, and taking it out of the Water, according as it riseth, the Water that invirons it will gradually abate, and the abasement of the Water at the instant of lifting out the Pillar, shall have the same proportion, that the thickness of the Pillar hath to the excess of the breadth of the said Pond or Well, above the thickness of the said Pillar: so that if the breadth of the Well were an eighth part larger than the thickness of the Pillar, and the breadth of the Pond twenty five times as great as the said thickness, in the Pillars ascending one foot, the water in the Well shall descend seven foot, and that in the Pond only $\frac{1}{24}$ of a foot.

This Demonstrated, it will not be difficult to shew the true cause, how it comes to pass, that,

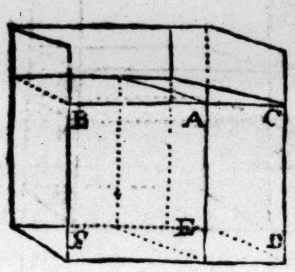
Why a Solid less grave *in specie* than water, stayeth not under water, in very small depths.

 THEO-

THEOREME III.

A Prisme or regular Cylinder, of a substance specifically less grave than Water, if it should be totally submerged in Water, stayes not underneath, but riseth, though the Water circumfused be very little, and in absolute Gravity, never so much inferiour to the Gravity of the said Prisme.

Let then the Prisme A E F B, be put into the Vessell C D F B, the same being less grave *in specie* than the Water: and let the Water infused rise to the height of the Prisme: I say, that the Prisme left at liberty, it shall rise, being born up by the Water circumfused C D E A. For the Water C E being specifically more grave than the Solid A F, the absolute weight of the water C E, shall have greater proportion to the absolute weight of the Prisme A F, than the Mass C E hath to the Mass A F (in regard the Mass hath the same proportion to the Mass, that the weight absolute hath to the weight absolute, in case the Masses are of the same Gravity *in specie*.) But the Mass C E is to the Mass A F, as the Surface of the water A C, is to the Superficies, or Base of the Prisme A B; which is the same proportion as the ascent of the Prisme when it riseth, hath to the descent of the water circumfused C E.



Therefore, the absolute Gravity of the water C E, hath greater proportion to the absolute Gravity of the Prisme A F; than the Ascent of the Prisme A F, hath to the descent of the said water C E. The Moment, therefore, compounded of the absolute Gravity of the water C E, and of the Velocity of its descent, whilst it forceably repulseth and raiseth the Solid A F, is greater than the Moment compounded of the absolute Gravity of the Prisme A F, and of the Tardity of its ascent, with which Moment it contrasts and resists the repulse and violence done it by the Moment of the water: Therefore, the Prisme shall be raised.

The Proportion according to which the Submersion & Natation of Solids is made.

It followes, now, that we proceed forward to demonstrate more particularly, how much such Solids shall be inferiour in Gravity to the water elevated; namely, what part of them shall rest submerged, and what shall be visible above the Surface of the water: but first it is necessary to demonstrate the subsequent Lemma.

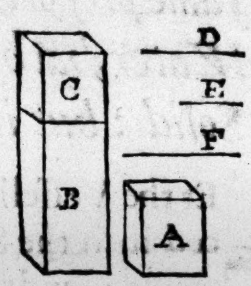
LEMM-

LEMMA I.

The absolute Gravities of Solids, have a proportion compounded of the proportions of their specificall Gravities, and of their Masses.

The absolute Gravity of Solids, are in a proportion compounded of their Specifick Gravities, and of their Masses.

LEt A and B be two Solids. I say, that the Absolute Gravity of A, hath to the Absolute Gravity of B, a proportion compounded of the proportions of the specificall Gravity of A, to the Specificall Gravity of B, and of the Mass A to the Mass B. Let the Line D have the same proportion to E, that the specifick Gravity of A, hath to the specifick Gravity of B; and let E be to F, as the Mass A to the Mass B: It is manifest, that the proportion of D to F, is compounded of the proportions D and E; and E and F. It is requisite, therefore, to demonstrate, that as D is to F, so the absolute Gravity of A, is to the absolute Gravity of B. Take the Solid C, equall in Mass to the Solid A, and of the same Gravity *in specie* with the Solid B. Because, therefore, A and C are equall in Mass, the absolute Gravity of A, shall have to the absolute Gravity of C, the same proportion, as the specificall Gravity of A, hath to the specificall Gravity of C, or of B, which is the same *in specie*; that is, as D is to E. And, because, C and B are of the same Gravity *in specie*, it shall be, that as the absolute weight of C, is to the absolute weight of B, so the Mass C, or the Mass A, is to the Mass B; that is, as the Line E to the Line F. As therefore, the absolute Gravity of A, is to the absolute Gravity of C, so is the Line D to the Line E: and, as the absolute Gravity of C, is to the absolute Gravity of B, so is the Line E to the Line F: Therefore, by Equality of proportion, the absolute Gravity of A, is to the absolute Gravity of B, as the Line D to the Line F: which was to be demonstrated. I proceed now to demonstrate, how that,



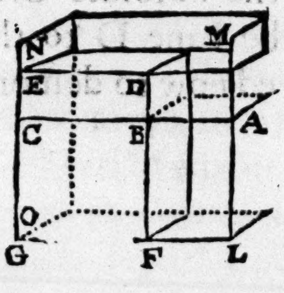
THEO-

THEOREME IV.

The proportion of water requisite to make a Solid swim.

If a Solid, Cylinder, or Prisme, lesse grave Specifically than the Water, being put into a Vessel, as above, of whatsoever greatnesse, and the Water, be afterwards infused, the Solid shall rest in the bottom, unraised, till the Water arrive to that part of the Altitude, of the said Prisme, to which its whole Altitude hath the same proportion, that the Specificall Gravity of the Water, hath to the Specificall Gravity of the said Solid: but infusing more Water, the Solid shall ascend.

LEt the Vessell be M L G N of any bigness, and let there be placed in it the Solid Prisme D F G E, less grave *in specie* than the water; and look what proportion the Specificall Gravity of the water, hath to that of the Prisme, such let the Altitude D F, have to the Altitude F B. I say, that infusing water to the Altitude F B, the Solid D G shall not float, but shall stand in *Equilibrium*, so, that that every little quantity of water, that is infused, shall raise it. Let the water, therefore, be infused to the Levell A B C, and; because the Specifick Gravity of the Solid D G, is to the Specifick Gravity of the water, as the altitude B F is to the altitude F D; that is, as the Mass B G to the Mass G D; as the proportion of the Mass B G is to the Mass G D, as the proportion of the Mass G D is to the Mass A F, they compose the Proportion of the Mass B G to the Mass A F. Therefore, the Mass B G is to the Mass A F, in a proportion compounded of the proportions of the Specifick Gravity of the Solid G D, to the Specifick Gravity of the water, and of the Mass G D to the Mass A F: But the same proportions of the Specifick Gravity of G D, to the Specifick Gravity of the water, and of the Mass G D to the Mass A F, do also by the precedent *Lemma*, compound the proportion of the absolute Gravity of the Solid D G, to the absolute Gravity of the Mass of the water A F: Therefore, as the Mass B G is to the Mass A F, so is the Absolute Gravity of the Solid D G, to the Absolute Gravity of the Mass of the water A F. But as the Mass B G is to the Mass A F; so is the Base of the Prisme D E, to the Surface of the water A B; and so is the descent of the water A B, to the Elevation of the Prisme D G; Therefore, the descent of the water



water

water is to the elevation of the Prisme, as the absolute Gravity of the Prisme, is to the absolute Gravity of the water: Therefore, the Moment resulting from the absolute Gravity of the water A F, and the Velocity of the Motion of declination, with which Moment it forceth the Prisme D G, to rise and ascend, is equall to the Moment that results from the absolute Gravity of the Prisme D G, and from the Velocity of the Motion, wherewith being raised, it would ascend: with which Moment it resists its being raised: because, therefore, such Moments are equall, there shall be an *Equilibrium* between the water and the Solid. And, it is manifest, that putting a little more water unto the other A F, it will increase the Gravity and Moment, whereupon the Prisme D G, shall be overcome, and elevated till that the only part B F remaines submerged. Which is that that was to be demonstrated.

COROLLARY I.

By what hath been demonstrated, it is manifest, that Solids less grave in specie *than the water, submerge only so far, that as much water in Mass, as is the part of the Solid submerged, doth weigh absolutely as much as the whole Solid.*

How far Solids less grave *in specie* than water, do submerge.

For, it being supposed, that the Specificall Gravity of the water, is to the Specificall Gravity of the Prisme D G, as the Altitude D F, is to the Altitude F B; that is, as the Solid D G is to the Solid B G; we might easily demonstrate, that as much water in Mass as is equall to the Solid B G, doth weigh absolutely as much as the whole Solid D G; For, by the *Lemma* foregoing, the Absolute Gravity of a Mass of water, equall to the Mass B G, hath to the Absolute Gravity of the Prisme D G, a proportion compounded of the proportions, of the Mass B G to the Mass G D, and of the Specifick Gravity of the water, to the Specifick Gravity of the Prisme: But the Gravity *in specie* of the water, to the Gravity *in specie* of the Prisme, is supposed to be as the Mass G D to the Mass G B. Therefore, the Absolute Gravity of a Mass of water, equall to the Mass B G, is to the Absolute Gravity of the Solid D G, in a proportion compounded of the proportions, of the Mass B G to the Mass G D, and of the Mass D G to the Mass G B; which is a proportion of equalitie. The Absolute Gravity, therefore, of a Mass of Water equall to the part of the Mass of the Prisme B G, is equall to the Absolute Gravity of the whole Solid D G.

COROL-

COROLLARY II.

A Rule to equilibrate Solids in the water.

It followes, moreover, that a Solid less grave than the water, being put into a Vessell of any imaginable greatness, and water being circumfused about it to such a height, that as much water in Mass, as is the part of the Solid submerged, doe weigh absolutely as much as the whole Solid; it shall by that water be justly sustained, be the circumfused Water in quantity greater or lesser.

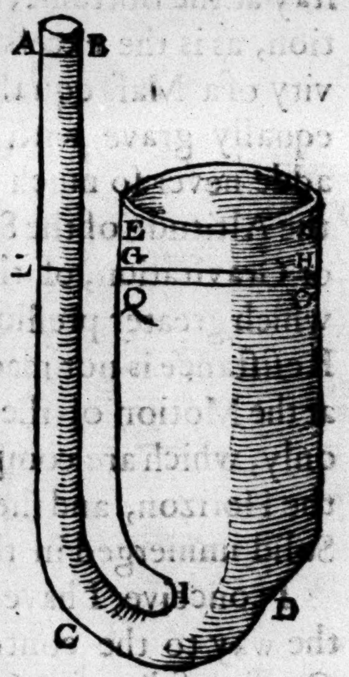
For, if the Cylinder or Prisme M, less grave than the water, *v. gra.* in Subsequiteriall proportion, shall be put into the capacious Vessell A B C D, and the water raised about it, to three quarters of its height, namely, to its Levell A D: it shall be sustained and exactly poysed in *Equilibrium.* The same will happen; if the Vessell E N S F were very small, so, that between the Vessell and the Solid M, there were but a very narrow space, and only capable of so much water, as the hundredth part of the Mass M, by which it should be likewise raised and erected, as before it had been elevated to three fourths of the height of the Solid: which to many at the first sight, may seem a notable Paradox, and beget a conceit, that the Demonstration of these effects, were sophisticall and fallacious: but, for those who so repute it, the Experiment is a means that may fully satisfie them. But he that shall but comprehend of what Importance Velocity of Motion is, and how it exactly compensates the defect and want of Gravity, will cease to wonder, in considering that at the elevation of the Solid M, the great Mass of water A B C D abateth very little, but the little Mass of water E N S F decreaseth very much, and in an instant, as the Solid M before did rise, howbeit for a very short space: Whereupon the Moment, compounded of the small Absolute Gravity of the water E N S F, and of its great Velocity in ebbing, equalizeth the Force and and Moment, that results from the composition of the immense Gravity of the water A B C D, with its great slownesse of ebbing; since that in the Elevation of the Sollid M, the abasement of the lesser water E S, is performed just so much more swiftly than the great Mass of water A C, as this is more in Mass than that which we thus demonstrate.

The proportion according to which water riseth and falls in different Vessels at the Immersion and Elevation of Solids.

In the rising of the Solid M, its elevation hath the same proportion to the circumfused water E N S F, that the Surface of the said water, hath to the Superficies or Base of the said Solid M; which Base hath the same proportion to the Surface of the water A D, that the abasement

ment or ebbing of the water A C, hath to the rise or elevation of the said Solid M. Therefore, by Perturbation of proportion, in the ascent of the said Solid M, the abasement of the water A B C D, to the abasement of the water E N S F, hath the same proportion, that the Surface of the water E F, hath to the Surface of the water A D; that is, that the whole Mass of the water E N S F, hath to the whole Mass A B C D, being equally high: It is manifest, therefore, that in the expulsion and elevation of the Solid M, the water E N S F shall exceed in Velocity of Motion the water A B C D, asmuch as it on the other side is exceeded by that in quantity: whereupon their Moments in such operations, are mutually equall.

And, for ampler confirmation, and clearer explication of this, let us consider the present Figure, (which if I be not deceived, may serve to detect the errors of some Practick Mechanitians, who upon a false foundation some times attempt impossible enterprizes,) in which, unto the large Vessell E I D F, the narrow Funnell or Pipe I C A B is continued, and suppose water infused into them, unto the Levell L G H, which water shall rest in this position, not without admiration in some, who cannot conceive how it can be, that the heavie charge of the great Mass of water G D, pressing downwards, should not elevate and repulse the little quantity of the other, contained in the Funnell or Pipe C L, by which the descent of it is resisted and hindered: But such wonder shall cease, if we begin to suppose the water G D to be abased only to Q D, and shall afterwards consider, what the water C L hath done, which to give place to the other, which is descended from the Levell G H, to the Levell Q O, shall of necessity have ascended in the same time, from the Levell L unto A B. And the ascent L B, shall be so much greater than the descent G Q, by how much the breadth of the Vessell G D, is greater than that of the Funnell I C; which, in summe, is as much as the water G D, is more than the water L C: but in regard that the Moment of the Velocity of the Motion, in one Moveable, compensates that of the Gravity of another, what wonder is it, if the swift ascent of the lesser Water C L, shall resist the slow descent of the greater G D?



The same, therefore, happens in this operation, as in the Stilliard, in which a weight of two pounds counterpoyseth an other of 200, as often as that shall move in the same time, a space 100 times greater than this: which falleth out when one Arme of the Beam is an

hundred times as long as the other. Let the erroneous opinion of those therefore cease, who hold that a Ship is better, and easier born up in a great abundance of water, then in a lesser quantity, (*this was believed by* Aristotle *in his Problems*, *Sect.* 23, *Probl.* 2.) it being on the contrary true, that its possible, that a Ship may as well float in ten Tun of water, as in an Ocean.

A ship flotes as well in ten Tun of water as in an Ocean.

But following our matter, I say, that by what hath been hitherto demonstrated, we may understand how, that

A Solid specifically graver than the water, cannot be born up by any quantity of it.

COROLLARY III.

One of the above named Solids, when more grave in specie *than the water, can never be sustained, by any whatever quantity of it.*

For having seen how that the Moment wherewith such a Solid, as grave *in specie* as the water, contrasts with the Moment of any Mass of water whatsoever, is able to retain it, even to its totall Submersion, without its ever ascending; it remaineth, manifest, that the water is far less able to raise it up, when it exceeds the same *in specie*: so, that though you infuse water till its totall Submersion, it shall still stay at the Bottome, and with such Gravity, and Resistance to Elevation, as is the excess of its Absolute Gravity, above the Absolute Gravity of a Mass equall to it, made of water, or of a Matter *in specie* equally grave with the water: and, though you should moreover adde never so much water above the Levell of that which equalizeth the Altitude of the Solid, it shall not, for all that, encrease the Pression, or Gravitation, of the parts circumfused about the said Solid, by which greater pression, it might come to be repulsed; because, the Resistance is not made, but only by those parts of the water, which at the Motion of the said Solid do also move, and these are those only, which are comprehended by the two Superficies equidistant to the Horizon, and their parallels, that comprehend the Altitude of the Solid immerged in the water.

I conceive, I have by this time sufficiently declared and opened the way to the contemplation of the true, intrinsecall and proper Causes of diverse Motions, and of the Rest of many Solid Bodies in diverse *Mediums*, and particularly in the water, shewing how all in effect, depend on the mutuall excesses of the Gravity of the Moveables and of the *Mediums*: and, that which did highly import, removing the Objection, which peradventure would have begotten much doubting, and scruple in some, about the verity of my Conclusion, namely, how that notwithstanding, that the excess of the Gravity of the water, above the Gravity of the Solid, demitted into it, be the cause of its floating and rising from the Bottom to the Surface, yet a quantity of water, that weighs not ten pounds, can raise a Solid

Solid that weighs above 100 pounds: in that we have demonstrated, That it sufficeth, that such difference be found between the Specificall Gravities of the *Mediums* and Moveables, let the particular and absolute Gravities be what they will: insomuch, that a Solid, provided that it be Specifically less grave than the water, although its absolute weight were 1000 pounds, yet may it be born up and elevated by ten pounds of water, and less: and on the contrary, another Solid, so that it be Specifically more grave than the water, though in absolute Gravity it were not above a pound, yet all the water in the Sea, cannot raise it from the Bottom, or float it. This sufficeth me, for my present occasion, to have, by the above declared Examples, discovered and demonstrated, without extending such matters farther, and, as I might have done, into a long Treatise: yea, but that there was a necessity of resolving the above proposed doubt, I should have contented my self with that only, which is demonstrated by *Archimedes*, in his first Book *De Insidentibus humido*: where in generall termes he infers and confirms the same Conclusions, namely, that Solids (*a*) less grave than water, swim or float upon it, the (*b*) more grave go to the Bottom, and the (*c*) equally grave rest indifferently in all places, yea, though they should be wholly under water.

Of Natation
(a) *Lib.* 1. *Prop.* 4.
(b) *Id. Lib.* 1. *Prop.* 3.
(c) *Id. Lib.* 1. *Prop.* 3.

The Authors defence of *Archimedes* his Doctrine, against the oppositions of *Buonamico*.

But, because that this Doctrine of *Archimedes*, perused, transcribed and examined by *Signor Francesco Buonamico*, in his *fifth Book of Motion, Chap.* 29, and afterwards by him confuted, might by the Authority of so renowned, and famous a Philosopher, be rendered dubious, and suspected of falsity; I have judged it necessary to defend it, if I am able so to do, and to clear *Archimedes*, from those censures, with which he appeareth to be charged. *Buonamico* rejecteth the Doctrine of *Archimedes*, first, as not consentaneous with the Opinion of *Aristotle*, adding, that it was a strange thing to him, that the Water should exceed the Earth in Gravity, seeing on the contrary, that the Gravity of water, increaseth, by means of the participation of Earth. And he subjoyns presently after, that he was not satisfied with the Reasons of *Archimedes*, as not being able with that Doctrine, to assign the cause whence it comes, that a Boat and a Vessell, which otherwise, floats above the water, doth sink to the Bottom, if once it be filled with water; that by reason of the equality of Gravity, between the water within it, and the other water without, it should stay a top; but yet, neverthelesse, we see it to go to the Bottom.

His first Objection against the Doctrine of *Archimedes*.

His Second Objection.

His third Objection.

He farther addes, that *Aristotle* had clearly confuted the Ancients, who said, that light Bodies moved upwards, driven by the impulse of the more grave Ambient: which if it were so, it should seem of necessity to follow, that all naturall Bodies are by nature heavy,

His fourth Objection.
The Antients denyed Absolute Levity.

and none light: For that the same would befall the Fire and Air, if put in the Bottom of the water. And, howbeit, *Aristotle* grants a Pulsion in the Elements, by which the Earth is reduced into a Sphericall Figure, yet neverthelesse, in his judgement, it is not such that it can remove grave Bodies from their naturall places, but rather, that it send them toward the Centre, to which (as he somewhat obscurely continues to say,) the water principally moves, if it in the interim meet not with something that resists it, and, by its Gravity, thrusts it out of its place: in which case, if it cannot directly, yet at least as well as it can, it tends to the Centre: but it happens, that light Bodies by such Impulsion, do all ascend upward: but this properly they have by nature, as also, that other of swimming. He concludes, lastly, that he concurs with *Archimedes* in his Conclusions; but not in the Causes, which he would referre to the facile and difficult Separation of the *Medium*, and to the predominance of the Elements, so that when the Moveable superates the power of the *Medium*; as for example, Lead doth the Continuity of water, it shall move thorow it, else not.

The causes of Natation & Submersion, according to the Peripateticks.

This is all that I have been able to collect, as produced against *Archimedes* by *Signor Buonamico*: who hath not well observed the Principles and Suppositions of *Archimedes*; which yet must be false, if the Doctrine be false, which depends upon them; but is contented to alledge therein some Inconveniences, and some Repugnances to the Doctrine and Opinion of *Aristotle*. In answer to which Objections, I say, first, That the being of *Archimedes* Doctrine, simply different from the Doctrine of *Aristotle*, ought not to move any to suspect it, there being no cause, why the Authority of this should be preferred to the Authority of the other: but, because, where the decrees of Nature are indifferently exposed to the intellectuall eyes of each, the Authority of the one and the other, loseth all anthenticalness of Perswasion, the absolute power residing in Reason; therefore I pass to that which he alledgeth in the second place, as an absurd consequent of the Doctrine of *Archimedes*, namely, That water should be more grave than Earth. But I really find not, that ever *Archimedes* said such a thing, or that it can be rationally deduced from his Conclusions: and if that were manifest unto me, I verily believe, I should renounce his Doctrine, as most erroneous. Perhaps this Deduction of *Buonamico*, is founded upon that which he citeth of the Vessel, which swims as long as its voyd of water, but once full it sinks to the Bottom, and understanding it of a Vessel of Earth, he infers against *Archimedes* thus: Thou sayst that the Solids which swim, are less grave than water: this Vessell swimmeth: therefore, this Vessell is lesse grave than water. If this be the Illation. I easily answer, granting that this Vessell is lesse grave than water, and denying the other consequence, namely,

The Authors answer to the first Objection.

The Authors answer to the second Objection.

namely, that Earth is less Grave than Water. The Vessel that swims occupieth in the water, not only a place equall to the Mass of the Earth, of which it is formed; but equall to the Earth and to the Air together, contained in its concavity. And, if such a Mass compounded of Earth and Air, shall be less grave than such another quantity of water, it shall swim, and shall accord with the Doctrine of *Archimedes*; but if, again, removing the Air, the Vessell shall be filled with water, so that the Solid put in the water, be nothing but Earth, nor occupieth other place, than that which is only possest by Earth, it shall then go to the Bottom, by reason that the Earth is heavier than the water: and this corresponds well with the meaning of *Archimedes*. See the same effect illustrated, with such another Experiment, In pressing a Viall Glass to the Bottom of the water, when it is full of Air, it will meet with great resistance, because it is not the Glass alone, that is pressed under water, but together with the Glass a great Mass of Air, and such, that if you should take as much water, as the Mass of the Glass, and of the Air contained in it, you would have a weight much greater than that of the Viall, and of its Air: and, therefore, it will not submerge without great violence: but if we demit only the Glass into the water, which shall be when you shall fill the Glass with water, then shall the Glass descend to the Bottom; as superiour in Gravity to the water.

Returning, therefore, to our first purpose; I say, that Earth is more grave than water, and that therefore, a Solid of Earth goeth to the bottom of it; but one may possibly make a composition of Earth and Air, which shall be less grave than a like Mass of Water; and this shall swim: and yet both this and the other experiment shall very well accord with the Doctrine of *Archimedes*. But because that in my judgment it hath nothing of difficulty in it, I will not positively affirme that *Signor Buonamico*, would by such a discourse object unto *Archimedes* the absurdity of inferring by his doctrine, that Earth was less grave than Water, though I know not how to conceive what other accident he could have induced thence.

Perhaps such a Probleme (in my judgement false) was read by *Signor Buonamico* in some other Author, by whom peradventure it was attributed as a singular propertie, of some particular Water, and so comes now to be used with a double errour in confutation of *Archimedes*, since he saith no such thing, nor by him that did say it was it meant of the common Element of Water.

The third difficulty in the doctrine of *Archimedes* was, that he could not render a reason whence it arose, that a piece of Wood, and a Vessell of Wood, which otherwise floats, goeth to the bottom, if filled with Water. *Signor Buonamico* hath supposed that a Vessell of Wood, and of Wood that by nature swims, as before is said, goes

The Authors answer to the third Objection.

goes to the bottom, if it be filled with water; of which he in the following Chapter, which is the 30 of the fifth Book copiously discourseth: but I (speaking alwayes without diminution of his singular Learning) dare in defence of *Archimedes* deny this experiment, being certain that a piece of Wood which by its nature sinks not in Water, shall not sinke though it be turned and converted into the forme of any Vessell whatsoever, and then filled with Water: and he that would readily see the Experiment in some other tractable Matter, and that is easily reduced into several Figures, may take pure Wax, and making it first into a Ball or other solid Figure, let him adde to it so much Lead as shall just carry it to the bottome, so that being a graine less it could not be able to sinke it, and making it afterwards into the forme of a Dish, and filling it with Water, he shall finde that without the said Lead it shall not sinke, and that with the Lead it shall descend with much slowness: & in short he shall satisfie himself, that the Water included makes no alteration. I say not all this while, but that its possible of Wood to make Barkes, which being filled with water, sinke; but that proceeds not through its Gravity, encreased by the Water, but rather from the Nailes and other Iron Workes, so that it no longer hath a Body less grave than Water, but one mixt of Iron and Wood, more grave than a like Masse of Water. Therefore let *Signor Buonamico* desist from desiring a reason of an effect, that is not in nature: yea if the sinking of the Woodden Vessell when its full of Water, may call in question the Doctrine of *Archimedes*, which he would not have you to follow, is on the contrary consonant and agreeable to the Doctrine of the Peripateticks, since it aptly assignes a reason why such a Vessell must, when its full of Water, descend to the bottom; converting the Argument the other way, we may with safety say that the Doctrine of *Archimedes* is true, since it aptly agreeth with true experiments, and question the other, whose Deductions are fastened upon erroneouss Conclusions. As for the other point hinted in this same Instance, where it seemes that *Benonamico* understands the same not only of a piece of wood, shaped in the forme of a Vessell, but also of massie Wood, which filled, *scilicet*, as I believe, he would say, soaked and steeped in Water, goes finally to the bottom that happens in some porose Woods, which, while their Porosity is replenished with Air, or other Matter less grave than Water, are Masses specificially less grave than the said Water, like as is that Viall of Glass whilest it is full of Air: but when, such light Matter departing, there succeedeth Water into the same Porosities and Cavities, there results a compound of Water and Glass more grave than a like Mass of Water: but the excess of its Gravity consists in the Matter of the Glass, and not in the Water, which cannot be graver than it self: so that which remaines of the Wood, the Air of its Cavi-

ties departing, if it shall be more grave *in specie* than Water, fil but its Porosities with Water, and you shal have a Compost of Water and of Wood more grave than Water, but not by vertue of the Water received into and imbibed by the Porosities, but of that Matter of the Wood which remains when the Air is departed: and being such it shall, according to the Doctrine of *Archimedes*, goe to the bottom, like as before, according to the same Doctrine it did swim.

As to that finally which presents it self in the fourth place, namely, that the *Ancients* have been heretofore confuted by *Aristotle*, who denying Positive and Absolute Levity, and truely esteeming all Bodies to be grave, said, that that which moved upward was driven by the circumambient Air, and therefore that also the Doctrine of *Archimedes*, as an adherent to such an Opinion was convicted and confuted: I answer first, that *Signor Buonamico* in my judgement hath imposed upon *Archimedes*, and deduced from his words more than ever he intended by them, or may from his Propositions be collected, in regard that *Archimedes* neither denies, nor admitteth Positive Levity, nor doth he so much as mention it: so that much less ought *Buonamico* to inferre, that he hath denyed that it might be the Cause and Principle of the Ascension of Fire, and other Light Bodies: having but only demonstrated, that Solid Bodies more grave than Water descend in it, according to the excess of their Gravity above the Gravity of that, he demonstrates likewise, how the less grave ascend in the same Water, accordng to its excess of Graty, above the Gravity of them. So that the most that can be gathered from the Demonstration of *Archimedes* is, that like as the excess of the Gravity of the Moveable above the Gravity of the Water, is the Cause that it descends therein, so the excess of the Gravity of the water above that of the Moveable, is a sufficient Cause why it descends not, but rather betakes it self to swim: not enquiring whether of moving upwards there is, or is not any other Cause contrary to Gravity: nor doth *Archimedes* discourse less properly than if one should say: If the South Winde shall assault the Barke with greater *Impetus* than is the violence with which the Streame of the River carries it towards the South, the motion of it shall be towards the North: but if the *Impetus* of the Water shall overcome that of the Winde, its motion shall be towards the South. The discourse is excellent and would be unworthily contradicted by such as should oppose it, saying: Thou mis-alledgest as Cause of the motion of the Bark towards the South, the *Impetus* of the Stream of the Water above that of the South Winde; mis-alledgest I say, for it is the Force of the North Winde opposite to the South, that is able to drive the Bark towards the South. Such an Objection would be superfluous, because he which alledgeth for Cause of the Motion the stream of the Water, denies not but

The Authors answer to the fourth Objection.

Of Natation, Lib. 1. Prop. 7.

Of Natation, Lib. 1. Prop. 4.

but that the Winde opposite to the South may do the same, but only affirmeth that the force of the Water prevailing over the South Wind, the Bark shall move towards the South: and saith no more than is true. And just thus when *Archimedes* saith, that the Gravity of the Water prevailing over that by which the moveable descends to the Bottom, such moveable shall be raised from the Bottom to the Surface alledgeth a very true Cause of such an Accident, nor doth he affirm or deny that there is, or is not, a vertue contrary to Gravity, called by some Levity, that hath also a power of moving some Matters upwards. Let therefore the Weapons of *Signor Buonamico* be directed against *Plato*, and other *Ancients*, who totally denying *Levity*, and taking all Bodies to be grave, say that the Motion upwards is made, not from an intrinsecal Principle of the Moveable, but only by the Impulse of the *Medium*; and let *Archimedes* and his Doctrine escape him, since he hath given him no Cause of quarelling with him. But if this Apologie, produced in defence of *Archimedes*, should seem to some insufficient to free him from the Objections and Arguments, produced by *Aristotle* against *Plato*, and the other *Ancients*, as if they did also fight against *Archimedes*, alledging the Impulse of the Water as the Cause of the swimming of some Bodies less grave than it, I would not question, but that I should be able to maintaine the Doctrine of *Plato* and those others to be most true, who absolutely deny Levity, and affirm no other Intrinsecal Principle of Motion to be in Elementary Bodies save only that towards the Centre of the Earth, nor no other Cause of moving upwards, speaking of that which hath the resemblance of natural Motion, but only the repulse of the *Medium*, fluid, and exceeding the Gravity of the Moveable: and as to the Reasons of *Aristotle* on the contrary, I believe that I could be able fully to answer them, and I would assay to do it, if it were absolutely necessary to the present Matter, or were it not too long a Digression for this short Treatise. I will only say, that if there were in some of our Ellementary Bodies an Intrinsecall Principle and Naturall Inclination to shun the Centre of the Earth, and to move towards the Concave of the Moon, such Bodies, without doubt, would more swiftly ascend through those *Mediums* that least oppose the Velocity of the Moveable, and these are the more tenuous and subtle; as is, for example, the Air in comparison of the Water, we daily proving that we can with farre more expeditious Velocity move a Hand or a Board to and again in one than in the other: nevertheless, we never could finde any Body, that did not ascend much more swiftly in the water than in the Air. Yea of Bodies which we see continually to ascend in the Water, there is none that having arrived to the confines of the Air, do not wholly lose their Motion; even the Air it self, which rising with great Celerity through the Water, being once come to its Region it loseth all

Plato denyeth Positive Levity.

The Authors defence of the doctrine of *Plato* and the *Ancients*, who absolutely deny Levity:

According to *Plato* there is no Principle of the Motion of descent in Naturall Bodies, save that to the Centre.

No cause of the motion of Ascent, save the Impulse of the *Medium*, exceeding the Moveable in Gravitie.

Bodies ascend much swifter in the Water, than in the Air.

All Bodies ascending through Water, lose their Motion, comming to the confines of the Air.

Im-

And, howbeit, Experience shewes, that the Bodies, successively less grave, do most expeditiously ascend in water, it cannot be doubted, but that the Ignean Exhalations do ascend more swiftly through the water, than doth the Air: which Air is seen by Experience to ascend more swiftly through the Water, than the Fiery Exhalations through the Air: Therefore, we must of necessity conclude, that the said Exhalations do much more expeditiously ascend through the Water, than through the Air; and that, consequently, they are moved by the Impulse of the Ambient *Medium*, and not by an intrinsick Principle that is in them, of avoiding the Centre of the Earth; to which other grave Bodies tend.

The lighter Bodies ascend more swiftly through Water. Fiery Exhalations ascend thorow the Water more swiftly than doth the Air; & the Air ascends more swiftly thorow the Water, than Fire throw the Air.

To that which for a finall conclusion, *Signor Buonamico* produceth of going about to reduce the descending or not descending, to the easie and uneasie Division of the *Medium*, and to the predominancy of the Elements: I answer, as to the first part, that that cannot in any manner be admitted as a Cause, being that in none of the Fluid *Mediums*, as the Air, the Water, and other Liquids, there is any Resistance against Division, but all by every the least Force, are divided and penetrated, as I will anon demonstrate: so, that of such Resistance of Division there can be no Act, since it self is not in being. As to the other part, I say, that the predominancy of the Elements in Moveables, is to be considered, as far as to the excesse or defect of Gravity, in relation to the *Medium*: for in that Action, the Elements operate not, but only, so far as they are grave or light: therefore, to say that the Wood of the Firre sinks not, because Air predominateth in it, is no more than to say, because it is less grave than the Water. Yea, even the immediate Cause, is its being less grave than the Water: and it being under the predominancy of the Air, is the Cause of its less Gravity: Therefore, he that alledgeth the predominancy of the Element for a Cause, brings the Cause of the Cause, and not the neerest and immediate Cause. Now, who knows not that the true Cause is the immediate, and not the mediate? Moreover, he that alledgeth Gravity, brings a Cause most perspicuous to Sence: The cause we may very easily assertain our selves; whether Ebony, for example, and Firre, be more or less grave than water: but whether Earth or Air predominates in them, who shall make that manifest? Certainly, no Experiment can better do it than to observe whether they swim or sink. So, that he who knows, not whether such a Solid swims, unless when he knows that Air predominates in it, knows not whether it swim, unless he sees it swim, for then he knows that it swims, when he knows that it is Air that predominates, but knows not that Air hath the predominance, unless he sees it swim: therefore, he knows not if it swims, till such time as he hath seen it swim.

The Authors confutation of the Peripateticks Causes of Natation & Submersion.

Water & other fluids void of Resistance against Division.

The predominancy of Elements in Moveables to be considered only in relation to their excess or defect of Gravity in reference to the *Medium*.

The immediate Cause of Natation is that the Moveable is less grave than the Water.

The Peripateticks alledge for the reason of Natation the Cause of the Cause.

Gravity a Cause most perspicuous to sence.

Let us not then despise those Hints, though very dark, which Reason, after some contemplation, offereth to our Intelligence, and lets be content to be taught by *Archimedes*, that then any Body shall submerge in water, when it shall be specifically more grave than it, and that if it shall be less grave, it shall of necessity swim, and that it will rest indifferently in any place under water, if its Gravity be perfectly like to that of the water.

Lib. 1. of Natation Prop. 7. Id. Lib. 1. Prop. 4.

These things explained and proved, I come to consider that which offers it self, touching what the Diversity of figure given unto the said Moveable hath to do with these Motions and Rests; and proceed to affirme, that,

Id. Lib. 1. Prop. 3.

THEOREME V.

The diversity of Figures given to this or that Solid, cannot any way be a Cause of its absolute Sinking or Swimming.

Diversity of Figure no Cause of its absolute Natation or Submersion.

SO that if a Solid being formed, for example, into a Sphericall Figure, doth sink or swim in the water, I say, that being formed into any other Fi ure, the same figure in the same water, shall sink or swim: nor can such its Motion by the Expansion or by other mutation of Figure, be impeded or taken away.

The Expansion of Figure, retards the Velocity of the ascent or descent of the Moveable in the water; but doth not deprive it of all Motion.

The Expansion of the Figure may indeed retard its Velocity, as well of ascent as descent, and more and more according as the said Figure is reduced to a greater breadth and thinness: but that it may be reduced to such a form as that that same matter be wholly hindred from moving in the same water, that I hold to be impossible. In this I have met with great contradictors, who producing some Experiments, and in perticular a thin Board of Ebony, and a Ball of the same Wood, and shewing how the Ball in Water descended to the bottom, and the Board being put lightly upon the Water submerged not, but rested; have held, and with the Authority of *Aristotle*, confirmed themselves in their Opinions, that the Cause of that Rest was the breadth of the Figure, u able by its small weight to pierce and penetrate the Resistance of the Waters Crassitude, which Resistance is readily overcome by the other Sphericall Figure.

This is the Principal point in the present Question, in which I perswade my self to be on the right side.

Therefore, beginning to investigate with the examination of exquisite Experiments that really the Figure doth not a jot alter the descent or Ascent of the same Solids, and having already demonstrated that the greater or less Gravity of the Solid in relation to the Gravity of the *Medium* is the cause of Descent or Ascent: when ever we would

would make proof of that, which about this Effect the diversity of Figure worketh, its necessary to make the Experiment with Matter wherein variety of Gravities hath no place. For making use of Matters which may be different in their Specifical Gravities, and meeting with varieties of effects of Ascending and Descending, we shall alwayes be left unsatisfied whether that diversity derive it self really from the sole Figure, or else from the divers Gravity also. We may remedy this by takeing one only Matter, that is tractable and easily reduceable into every sort of Figure. Moreover, it wil be an excellent expedient to take a kinde of Matter, exactly alike in Gravity unto the Water: for that Matter, as far as pertaines to the Gravity, is indifferent either to Ascend or Descend; so that we may presently observe any the least difference that derives it self from the diversity of Figure.

An Experiment in Wax, that proveth Figure to have no Operation in Natation & Submersion.

Now to do this, Wax is most apt, which, besides its incapacity of receiveing any sensible alteration from its imbibing of Water, is ductile or pliant, and the same piece is easily reduceable into all Figures: and being *in specie* a very inconsiderable matter inferiour in Gravity to the Water, by mixing therewith a little of the fileings of Lead it is reduced to a Gravity exactly equall to that of the Water.

This Matter prepared, and, for example, a Ball being made thereof as bigge as an Orange or biger, and that made so grave as to sink to the bottom, but so lightly, that takeing thence one only Grain of Lead, it returnes to the top, and being added, it submergeth to the bottom, let the same Wax afterwards be made into a very broad and thin Flake or Cake; and then, returning to make the same Experiment, you shall see that it being put to the bottom, it shall, with the Grain of Lead rest below, and that Grain deducted, it shall ascend to the very Surface, and added again it shall dive to the bottom. And this same effect shall happen alwaies in all sort of Figures, as wel regular as irregular: nor shall you ever finde any that will swim without the removall of the Grain of Lead, or sinke to the bottom unless it be added: and, in short, about the going or not going to the Bottom, you shall discover no diversity, although, indeed, you shall about the quick and slow descent: for the more expatiated and distended Figures move more slowly aswel in the diveing to the bottom as in the rising to the top; and the other more contracted and compact Figures, more speedily. Now I know not what may be expected from the diversity of Figures, if the most contrary to one another operate not so much as doth a very small Grain of Lead, added or removed.

An objection against the Experiment in Wax.

Me thinkes I hear some of the Adversaries to raise a doubt upon my produced Experiment. And first, that they offer to my consideration, that the Figure, as a Figure simply, and disjunct from the Matter workes not any effect, but requires to be conjoyned with the Matter;

and, furthermore, not with every Matter, but with those only, wherewith it may be able ro execute the desired operation. Like as we see it verified by Experience, that the Acute and sharp Angle is more apt to cut, than the Obtuse; yet alwaies provided, that both the one and the other, be joyned with a Matter apt to cut, as for example, with Steel. Therefore, a Knife with a fine and sharp edge, cuts Bread or Wood with much ease, which it will not do, if the edge be blunt and thick: but he that will instead of Steel, take Wax, and mould it into a Knife, undoubtedly shall never know the effects of sharp and blunt edges: because neither of them will cut, the Wax being unable by reason of its flexibility, to overcome the hardness of the Wood and Bread. And, therefore, applying the like discourse to our purpose, they say, that the difference of Figure will shew different effects, touching Natation and Submersion, but not conjoyned with any kind of Matter, but only with those Matters which, by their Gravity, are apt to resist the Velocity of the water, whence he that would elect for the Matter, Cork or other light wood, unable, through its Levity, to superate the Crassitude of the water, and of that Matter should forme Solids of divers Figures, would in vain seek to find out what operation Figure hath in Natation or Submersion; because all would swim, and that not through any property of this or that Figure, but through the debility of the Matter, wanting so much Gravity, as is requisite to superate and overcome the Density and Crassitude of the water.

An Experiment in Ebany, brought to disprove the Experiment in Wax.

Its needfull, therefore, if wee would see the effect wrought by the Diversity of Figure, first to make choice of a Matter of its nature apt to penetrate the Crassitude of the water. And, for this effect, they have made choice of such a Matter, as fit, that being readily reduced into Sphericall Figure, goes to the Bottom; and it is Ebony, of which they afterwards making a small Board or Splinter, as thin as a Lath, have illustrated how that this, put upon the Surface of the water, rests there without descending to the Bottom: and making, on the otherside, of the same wood a Ball, no less than a hazell Nut, they shew, that this swims not, but descendes. From which Experiment, they think they may frankly conclude, that the Breadth of the Figure in the flat Lath or Board, is the cause of its not descending to the Bottom, forasmuch as a Ball of the same Matter, not different from the Board in any thing but in Figure, submergeth in the same water to the Bottom. The discourse and the Experiment hath really so much of probability and likelyhood of truth in it, that it would be no wonder, if many perswaded by a certain cursory observation, should yield credit to it; nevertheless, I think I am able to discover, how that it is not free from falacy.

Beginning, therefore, to examine one by one, all the particulars that have

have been produced, I say, that Figures, as simple Figures, not only operate not in naturall things, but neither are they ever seperated from the Corporeall substance : nor have I ever alledged them stript of sensible Matter, like as also I freely admit, that in our endeavouring to examine the Diversity of Accidents, dependant upon the variety of Figures, it is necessary to apply them to Matters, which obstruct not the various operations of those various Figures : and I admit and grant, that I should do very ill, if I would experiment the influence of Acutenesse of edge with a Knife of Wax, applying it to cut an Oak, because there is no Acuteness in Wax able to cut that very hard wood. But yet such an Experiment of this Knife, would not be besides the purpose, to cut curded Milk, or other very yielding Matter : yea, in such like Matters, the Wax is more commodious than Steel ; for finding the diversity depending upon Angles, more or less Acute, for that Milk is indifferently cut with a Raisor, and with a Knife, that hath a blunt edge. It needs, therefore, that regard be had, not only to the hardness, solidity or Gravity of Bodies, which under divers figures, are to divide and penetrate some Matters, but it forceth also, that regard be had, on the other side, to the Resistance of the Matters, to be divided and penetrated. But since I have in making the Experiment concerning our Contest, chosen a Matter which penetrates the Resistance of the water; and in all figures descendes to the Bottome, the Adversaries can charge me with no defect; yea, I have propounded so much a more excellent Method than they, in as much as I have removed all other Causes, of descending or not descending to the Bottom, and retained the only sole and pure variety of Figures, demonstrating that the same Figures all descende with the only alteration of a Grain in weight : which Grain being removed, they return to float and swim; it is not true, therefore, (resuming the Example by them introduced) that I have gon about to experiment the efficacy of Acuteness, in cutting with Matters unable to cut, but with Matters proportioned to our occasion; since they are subjected to no other variety, then that alone which depends on the Figure more or less acute.

Figure is unseperable from Corporeall Substance.

But let us proceed a little farther, and observe, how that indeed the Consideration, which, they say, ought to be had about the Election of the Matter, to the end, that it may be proportionate for the making of our experiment, is needlesly introduced, declaring by the example of Cutting, that like as Acuteness is insufficient to cut, unless when it is in a Matter hard and apt to superate the Resistance of the wood or other Matter, which we intend to cut; so the aptitude of descending or not descending in water, ought and can only be known in those Matters, that are able to overcome the Renitence, and superate the Crassitude of the water. Unto which, I say, that to make distinction and election, more of this than of that Matter, on which to

The answer to the Objection against the Experiment of the Wax.

impress

impreſs the Figures for cutting or penetrating this or that Body, as the ſolidity or obdurateneſs of the ſaid Bodies ſhall be greater or leſs, is very neceſſary : but withall I ſubjoyn, that ſuch diſtinction, election and caution would be ſuperfluous and unprofitable, if the Body to be cut or penetrated, ſhould have no Reſiſtance, or ſhould not at all withſtand the Cutting or Penitration : and if the Knife were to be uſed in cutting a Miſt or Smoak, one of Paper would be equally ſerviceable with one of *Damaſcus* Steel : and ſo by reaſon the water hath not any Reſiſtance againſt the Penitration of any Solid Body, all choice of Matter is ſuperfluous and needleſs, and the Election which I ſaid above to have been well made of a Matter reciprocall in Gravity to water, was not becauſe it was neceſſary, for the overcoming of the craſſitude of the water, but its Gravity, with which only it reſiſts the ſinking of Solid Bodies : and for what concerneth the Reſiſtance of the craſſitude, if we narrowly conſider it, we ſhall find that all Solid Bodies, as well thoſe that ſink, as thoſe that ſwim, are indifferently accomodated and apt to bring us to the knowledge of the truth in queſtion. Nor will I be frighted out of the belief of theſe Concluſions, by the Experiments which may be produced againſt me, of many ſeverall Woods, Corks, Galls, and, moreover, of ſubtle ſlates and plates of all ſorts of Stone and Mettall, apt by means of their Naturall Gravity, to move towards the Centre of the Earth, the which, nevertheleſs, being impotent, either through the Figure (as the Adverſaries thinke) or through Levity, to break and penetrate the Continuity of the parts of the water, and to diſtract its union, do continue to ſwimm without ſubmerging in the leaſt : nor on the other ſide, ſhall the Authority of *Ariſtotle* move me, who in more than one place, affirmeth the contrary to this, which Experience ſhews me.

No Solid of ſuch Levity, nor of ſuch Figure, but that it doth penetrate the Craſſitude of the Water.

Bodies of all Figures, laid upon the water, do penetrate its Craſſitude, and in what proportion.

I return, therefore, to aſſert, that there is not any Solid of ſuch Levity, nor of ſuch Figure, that being put upon the water, doth not divide and penetrate its Craſſitude : yea if any with a more perſpicatious eye, ſhall return to obſerve more exactly the thin Boards of Wood, he ſhall ſee them to be with part of their thickneſs under water, and not only with their inferiour Superficies, to kiſſe the Superiour of the water, as they of neceſſity muſt have believed, who have ſaid, that ſuch Boards ſubmerge not, as not being able to divide the Tenacity of the parts of the water : and, moreover, he ſhall ſee, that ſubtle ſhivers of Ebony, Stone or Metall, when they float, have not only broak the Continuity of the water, but are with all their thickneſs, under the Surface of it ; and more and more, according as the Matters are more grave : ſo that a thin Plate of Lead, ſhall be lower than the Surface of the circumfuſed water, by at leaſt twelve times the thickneſs of the Plate, and Gold ſhall dive

below

below the Levell of the water, almost twenty times the thickness of the Plate, as I shall anon declare.

But let us proceed to evince, that the water yields and suffers it self to be penetrated by every the lightest Body; and therewithall demonstrate, how, even by Matters that submerge not, we may come to know that Figure operates nothing about the going or not going to the Bottom, seeing that the water suffers it self to be penetrated equally by every Figure.

Make a Cone, or a Piramis of Cypress, of Firre, or of other Wood of like Gravity, or of pure Wax, and let its height be somewhat great, namely a handfull, or more, and put it into the water with the Base downwards: first, you shall see that it will penetrate the water, nor shall it be at all impeded by the largeness of the Base, nor yet shall it sink all under water, but the part towards the point shall lye above it: by which shall be manifest, first, that that Solid forbeares not to sink out of an inabillity to divide the Continuity of the water, having already divided it with its broad part, that in the opinion of the Adversaries is the less apt to make the division. The Piramid being thus fixed, note what part of it shall be submerged, and revert it afterwards with the point downwards, and you shall see that it shall not dive into the water more than before, but if you observe how far it shall sink, every person expert in Geometry, may measure, that those parts that remain out of the water, both in the one and in the other Experiment are equall to an hair: whence he may manifestly conclude, that the acute Figure which seemed most apt to part and penetrate the water, doth not part or penetrate it more than the large and spacious.

The Experiment of a Cone, demitted with its Base, and after with its Point downwards.

And he that would have a more easie Experiment, let him take two Cylinders of the same Matter, one long and small, and the other short, but very broad, and let him put them in the water, not distended, but erect and endways: he shall see, if he diligently measure the parts of the one and of the other, that in each of them the part submerged, retains exactly the same proportion to that out of the water, and that no greater part is submerged of that long and small one, than of the other more spacious and broad: howbeit, this rests upon a very large, and that upon a very little Superficies of water: therefore the diversity of Figure, occasioneth neither facility, nor difficulty, in parting and penetrating the Continuity of the water; and, consequently, cannot be the Cause of the Natation or Submersion. He may likewise discover the non-operating of variety of Figures, in arising from the Bottom of the water, towards the Surface, by taking Wax, and tempering it with a competent quantity of the filings of Lead, so that it may become a considerable matter graver than the water: then let him make

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it into a Ball, and thrust it unto the Bottom of the water; and fasten to it as much Cork, or other light matter, as just serveth to raise it, and draw it towards the Surface: for afterwards changing the same Wax into a thin Cake, or into any other Figure, that same Cork shall raise it in the same manner to a hair.

This silenceth not my Antagonists, but they say, that all the discourse hitherto made by me little importeth to them, and that it serves their turn, that they have demonstrated in one only particular, and in what matter, and under what Figure pleaseth them, namely, in a Board and in a Ball of Ebony, that this put in the water, descends to the Bottom, and that stays atop to swim: and the Matter being the same, and the two Bodies differing in nothing but in Figure, they affirm, that they have with all perspicuity demonstrated and sensibly manifested what they undertook; and lastly, that they have obtained their intent. Neverthelesse, I believe, and thinke, I can demonstrate, that that same Experiment proveth nothing against my Conclusion.

In Experiments of Natation, the Solid is to be put into, not upon the water.

And first, it is false, that the Ball descends, and the Board not: for the Board shall also descend, if you do to both the Figures, as the words of our Question requireth; that is, if you put them both into the water.

The Question of Natation stated.

The words were these. That the Antagonists having an opinion, that the Figure would alter the Solid Bodies, in relation to the descending or not descending, ascending or not ascending in the same Medium, *as* v. gr. *in the same water, in such sort, that, for Example, a Solid that being of a Sphericall Figure, shall descend to the Bottom, being reduced into some other Figure, shall not descend: I holding the contrary, do affirm, that a Corporeall Solid Body, which reduced into a Sphericall Figure, or any other, shall go to the Bottom, shall do the like under whatsoever other Figure, &c.*

Place defined according to *Aristotle*.

But to be in the water, implies to be placed in the water, and by *Aristotles* own Definition of place, to be placed, importeth to be invironed by the Superficies of the Ambient Body, therefore, then shall the two Figures be in the water, when the Superficies of the water, shall imbrace and inviron them: but when the Adversaries shew the Board of Ebony not descending to the Bottom, they put it not into the water, but upon the water, where being by a certain impediment (as by and by we will shew) retained, it is invironed, part by water, and part by air, which thing is contrary to our agreement, that was, that the Bodies should be in the water, and not part in water, and part in air.

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The which is again made manifest, by the questions being put as well about the things which go to the Bottom, as those which arise from the Bottom to swimme, and who sees not that things placed in the Bottom, must have water about them.

It is now to be noted, that the Board of Ebany and the Ball, put *into* the water, both sink, but the Ball more swiftly, and the Board more slowly; and slower and slower, according as it shall be more broad and thin, and of this Tardity the breadth of the Figure is the true Cause: But these broad Boards that slowly descend, are the same, that being put lightly upon the water, do swimm: Therefore, if that were true which the Adversaries affirm, the same numerical Figure, would in the same numericall water, cause one while Rest, and another while Tardity of Motion, which is impossible: for every perticular Figure which descends to the Bottom, hath of necessity its own determinate Tardity and slowness, proper and naturall unto it, according to which it moveth, so that every other Tardity, greater or lesser is improper to its nature: if, therefore, a Board, as suppose of a foot square, descendeth naturally with six degrees of Tardity, it is impossible, that it should descend with ten or twenty, unless some new impediment do arrest it. Much less can it, by reason of the same Figure rest, and wholly cease to move; but it is necessary, that when ever it resteth, there do some greater impediment intervene than the breadth of the Figure. Therefore, it must be somewhat else, and not the Figure, that stayeth the Board of Ebany above water, of which Eigure the only Effect is the retardment of the Motion, according to which it descendeth more slowly than the Ball. Let it be confessed, therefore, rationally discoursing, that the true and sole Cause of the Ebanys going to the Bottom, is the excess of its Gravity above the Gravity of the water: and the Cause of the greater or less Tardity, the breadth of this Figure, or the contractedness of that: but of its Rest, it can by no means be allowed, that the quallity of the Figure, is the Cause thereof: aswell, because, making the Tardity greater, according as the Figure more dilateth, there cannot be so immense a Dilatation, to which there may not be found a correspondent immence Tardity without redusing it to Nullity of Motion; as, because the Figures produced by the Antagonists for effecters of Rest, are the self same that do also go to the Bottom.

The confutation of the Experiment in the Ebany.

Every perticular Figure hath its own peculiar Tardity.

I will not omit another reason, founded also upon Experience, and if I deceive not my self, manifestly concluding, how that the Introduction of the breadth or amplitude of Figure, and the Resistance of the water against penetration, have nothing to do in the Effect of descending, or ascending, or resting in the water. *Take a piece of wood or other Matter, of which a Ball ascends from the Bottom of the water to

* The Figure & Resistance of the Medium against Division, have nothing to do with the Effect of Natation or Submersion, by an Experiment in Wallnut tree.

to the Surface, more ſlowly than a Ball of Ebony of the ſame bigneſſe, ſo that it is manifeſt, that the Ball of Ebony more readily divideth the water in deſcending, than the other in aſcending; as for Example, let the Wood be Walnut-tree. Then take a Board of Walnut-tree, like and equall to that of Ebony of the Antagoniſts, which ſwims; and if it be true, that this floats above water, by reaſon of the Figure, unable through its breadth, to pierce the Craſſitude of the ſame, the other of Wallnut-tree, without all queſtion, being thruſt unto the Bottom, will ſtay there, as leſs apt, through the ſame impediment of Figure, to divide the ſaid Reſiſtance of the water. But if we ſhall find, and by experience ſee, that not only the thin Board, but every other Figure of the ſame Wallnut-tree will return to float, as undoubtedly we ſhall, then I muſt deſier my oppoſers to forbear to attribute the floating of the Ebony, unto the Figure of the Board, in regard that the Reſiſtance of the water is the ſame, as well to the aſcent, as to the deſcent, and the force of the Wallnut-trees aſcenſion, is leſſe than the Ebonys force in going to the Bottom.

An Experiment in Gold, to prove the non-operating of Figure in Natation and Submerſion.

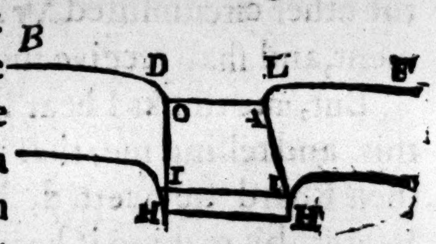
Nay, I will ſay more, that if we ſhall conſider Gold in compariſon of water, we ſhall find, that it exceeds it in Gravity almoſt twenty times, ſo that the Force and Impetus, wherewith a Ball of Gold goes to the Bottom, is very great. On the contrary, there want not matters, as Virgins Wax, and ſome Woods, which are not above a fiftieth part leſs grave than water, whereupon their Aſcenſion therein is very ſlow, and a thouſand times weaker than the *Impetus* of the Golds deſcent: yet notwithſtanding, a plate of Gold ſwims without deſcending to the Bottom, and, on the contrary, we cannot make a Cake of Wax, or thin Board of Wood, which put in the Bottom of the Water, ſhall reſt there without aſcending. Now if the Figure can obſtruct the Penetration, and impede the deſcent of Gold, that hath ſo great an *Impetus*, how can it chooſe but ſuffice to reſiſt the ſame Penetration of the other matter in aſcending, when as it hath ſcarce a thouſandth part of the *Impetus* that the Gold hath in deſcending? Its therefore, neceſſary, that that which ſuſpends the thin Plate of Gold, or Board of Ebony, upon the water, be ſome thing that is wanting to the other Cakes and Boards of Matters leſs grave than the water; ſince that being put to the Bottom, and left at liberty, they riſe up to the Surface, without any obſtruction: But they want not for flatneſs and breadth of Figure: Therefore, the ſpaciouſneſſe of the Figure, is not that which makes the Gold and Ebony to ſwim.

And, becauſe, that the exceſs of their Gravity above the Gravity of the water, is queſtionleſs the Cauſe of the ſinking of the flat piece of Ebony, and the thin Plate of Gold, when they go to the Bottom, therefore, of neceſſity, when they float, the Cauſe of their ſtaying above water, proceeds from Levity, which in that caſe, by ſome Accident,

peradventure

peradventure not hitherto observed, cometh to meet with the said Board, rendering it no longer as it was before, whilst it did sink more ponderous than the water, but less.

Now, let us return to take the thin Plate of Gold, or of Silver, or the thin Board of Ebony, and let us lay it lightly upon the water, so that it stay there without sinking, and diligently observe its effect. And first, see how false the assertion of *Aristotle*, and our oponents is, to wit, that it stayeth above water, through its unability to pierce and penetrate the Resistance of the waters Crassitude: for it will manifestly appear, not only that the said Plates have penetrated the water, but also that they are a considerable matter lower than the Surface of the same, the which continueth eminent, and maketh as it were a Rampert on all sides, round about the said Plates, the profundity of which they stay swimming: and, according as the said Plates shall be more grave than the water, two, four, ten or twenty times, it is necessary, that their Superficies do stay below the universall Surface of the water, so much more, than the thickness of those Plates, as we shal more distinctly shew anon. In the mean space, for the more easie understanding of what I say, observe with me a little the present Scheme: in which let us suppose the Surface of the water to be distended, according to the Lines F L D B, upon which if one shall put a board of matter specifically more grave than water, but so lightly that it submerge not, it shall not rest any thing above, but shall enter with its whole thickness into the water: and, moreover, shall sink also, as we see by the Board A I, O I, whose breadth is wholly sunk into the water, the little Ramperts of water L A and D O incompassing it, whose Superficies is notably higher than the Superficies of the Board. See now whether it be true, that the said Board goes not to the Bottom, as being of Figure unapt to penetrate the Crassitude of the water.



But, if it hath already penetrated, and overcome the Continuity of the water, & is of its own nature more grave than the said water, why doth it not proceed in its sinking, but stop and suspend its self within that little dimple or cavitie, which with its ponderosity it hath made in the water? I answer; because that in submerging it self, so far as till its Superficies come to the Levell with that of the water, it loseth a part of its Gravity, and loseth the rest of it as it submergeth & descends beneath the Surface of the water, which maketh Ramperts and Banks round about it, and it sustaines this loss by means of its drawing after it, and carrying along with it, the Air that is above it, and by Contact adherent to it, which Air succeeds to fill the Cavity that is invironed by the Ramperts of water: so that that which in this case descends and is placed in the water, is not only the Board of Ebony or Plate of Iron,

Why solids having penitrated the Water, do not proceed to a totall Submersion.

but a composition of Ebony and Air, from which resulteth a Solid no longer superiour in Gravity to the water, as was the simple Ebony, or the simple Gold. And, if we exactly consider, what, and how great the Solid is, that in this Experiment enters into the water, and contrasts with the Gravity of the same, it will be found to be all that which we find to be beneath the Surface of the water, the which is an aggregate and Compound of a Board of Ebony, and of almost the like quantity of Air, or a Mass compounded of a Plate of Lead, and ten or twelve times as much Air. But, Genrlemen, you that are my Antagonists in our Question, we require the Identity of Matter, and the alteration only of the Figure; therefore, you mnst remove that Air, which being conjoyned with the Board, makes it become another Body less grave than the Water, and put only the Ebony into the Water, and you shall certainly see the Board descend to the Bottom; and, if that do not happen, you have got the day. And to seperate the Air from the Ebony, there needs no more but only to bath the Superficies of the said Board with the same Water: for the Water being thus interposed between the Board and the Air, the other circumfused Water shall run together without any impediment, and shall receive into it the sole and bare Ebony, as it was to do.

How to seperate the Air from Solids in demitting them into the water.

But, me thinks I hear some of the Adversaries cunningly opposing this, and telling me, that they will not yield, by any means, that their Board be wetted, because the weight added thereto by the Water, by making it heavier than it was before, draws it to the Bottom, and that the addition of new weight is contrary to our agreement, which was, that the Matter be the same.

To this, I answer, first; that treating of the operation of Figure in Bodies put into the Water, none can suppose them to be put into the Water without being wet; nor do I desire more to be done to the Board, then I will give you leave to do to the Ball. Moreover, it is untrue, that the Board sinks by vertue of the new Weight added to it by the Water, in the single and slight bathing of it: for I will put ten or twenty drops of Water upon the same Board, whilst it is sustained upon the water; which drops, because not conjoyned with the other Water circumfused, shall not so encrease the weight of it, as to make it sink: but if the Board being taken out, and all the water wiped off that was added thereto, I should bath all its Superficies with one only very small drop, and put it again upon the water, without doubt it shall sink, the other Water running to cover it, not being retained by the superiour Air; which Air by the interposition of the thin vail of water, that takes away its Contiguity unto the Ebony, shall without Renitence be seperated, nor doth it in the least oppose the succession of the other Water: but rather, to speak better, it shall descend freely; because it shall be all invironed and covered

with

with water, as soon as its superiour Superficies, before vailed with water, doth arrive to the Levell of the universall Surface of the said water. To say, in the next place, that water can encrease the weight of things that are demitted into it, is most false; for water hath no Gravity in water, since it descends not: yea, if we would well consider what any immense Mass of water doth put upon a grave Body; that is placed in it, we shall find experimentally, that it, on the contrary, will rather in a great part deminish the weight of it; and that we may be able to lift an huge Stone from the Bottom of the water, which the water being removed, we are not able to stir. Nor let them tell me by way of reply, that although the superposed water augment not the Gravity of things that are in it, yet it increaseth the ponderosity of those that swim, and are part in the water and part in the Air, as is seen, for Example, in a Brass Ketle, which whilst it is empty of water, and replenished only with Air shall swim, but pouring of Water therein, it shall become so grave, that it shall sink to the Bottom, and that by reason of the new weight added thereto. To this I will return answer, as above, that the Gravity of the Water, contained in the Vessel is not that which sinks it to the Bottom, but the proper Gravity of the Brass, superiour to the Specificall Gravity of the Water: for if the Vessel were less grave than water, the Ocean would not suffice to submerge it. And, give me leave to repeat it again, as the fundamentall and principall point in this Case, that the Air contained in this Vessel before the infusion of the Water, was that which kept it a-float, since that there was made of it, and of the Brass, a Composition less grave than an equall quantity of Water: and the place that the Vessel occupyeth in the Water whilst it floats, is not equall to the Brass alone, but to the Brass and to the Air together, which filleth that part of the Vessel that is below the Levell of the water: Moreover, when the Water is infused, the Air is removed, and there is a composition made of Brass and of water, more grave *in specie* than the simple water, but not by vertue of the water infused, as having greater Specifick Gravity than the other water, but through the proper Gravity of the Brass, and through the alienation of the Air. Now, as he that should say that Brass, that by its nature goes to the Bottom, being formed into the Figure of a Ketle, acquireth from that Figure a vertue of lying in the Water without sinking, would say that which is false; because that Brass fashioned into any whatever Figure, goeth always to the Bottom, provided, that that which is put into the water be simple Brass; and it is not the Figure of the Vessel that makes the Brass to float, but it is because that that is not purely Brass which is put into the water, but an aggregate of Brass and of Air: so is it neither more nor less false, that a thin Plate of Brass

Water hath no Gravity in Water.

Water deminisheth the Gravity of Solids immerged therein.

The Experiment of a Brass Ketle swiming when empty, & sinking when full, alledged to prove that water gravitates in water, answered.

An Ocean sufficeth not to sink a Vessel specifically less grave than water.

Air, the Cause of the Natation of empty Vessels of Matters graver *in specie* than the water.

Neither Figure, nor the breadth of Figure, is the Cause of Natation.

or of Ebony, swims by vertue of its dilated & broad Figure: for the truth is, that it bares up without submerging, because that that which is put in the water, is not pure Brass or simple Ebony, but an aggregate of Brass and Air, or of Ebony and Air. And, this is not contrary unto my Conclusion, the which, (having many a time seen Vessels of Mettall, and thin pieces of diverse grave Matters float, by vertue of the Air conjoyned with them) did affirm, That Figure was not the Cause of the Natation or Submersion of such Solids as were placed in the water. Nay more, I cannot omit, but must tell my Antagonists, that this new conceit of denying that the Superficies of the Board should be bathed, may beget in a third person an opinion of a poverty of Arguments of defence on their part, since that such bathing was never insisted upon by them in the beginning of our Dispute, and was not questioned in the least, being that the Originall of the discourse arose upon the swiming of Flakes of Ice, wherein it would be simplicity to require that their Superficies might be dry: besides, that whether these pieces of Ice be wet or dry they alwayes swim, and as the Adversaries say, by reason of the Figure.

Some peradventure, by way of defence, may say, that wetting the Board of Ebony, and that in the superiour Superficies, it would, though of it self unable to pierce and penetrate the water, be born downwards, if not by the weight of the additionall water, at least by that desire and propension that the superiour parts of the water have to re-unite and rejoyn themselves: by the Motion of which parts, the said Board cometh in a certain manner, to be depressed downwards.

The Bathed Solid descends not out of any affectation of union in the upper parts of the water.

This weak Refuge will be removed, if we do but consider, that the repugnancy of the inferiour parts of the water, is as great against Dis-union, as the Inclination of its superiour parts is to union: nor can the uper unite themselves without depressing the board, nor can it descend without disuniting the parts of the nether Water: so that it doth follow, by necessary consequence, that for those respects, it shall not descend. Moreover, the same that may be said of the upper parts of the water, may with equall reason be said of the nether; namely, that desiring to unite, they shall force the said Board upwards.

A Magnetisme in the *Air*, by which it bears up those Solids in the water, that are contiguous with it.

Happily, some of these Gentlemen that dissent from me, will wonder, that I affirm, that the contiguous superiour Air is able to sustain that Plate of *Brass* or of Silver, that stayeth above water; as if I would in a certain sence allow the Air, a kind of Magnetick vertue of sustaining the grave *Bodies*, with which it is contiguous. To satisfie all I may, to all doubts, I have been considering how by some other sensible Experiment I might demonstrate, how truly that little contiguous and superiour Air sustaines those Solids, which being by

nature

nature apt to descend to the Bottom, being placed lightly on the water submerge not, unless they be first thorowly bathed; and have found, that one of these Bodies having descended to the Bottom, by conveighing to it (without touching it in the least) a little Air, which conjoyneth with the top of the same; it becometh sufficient, not only, as before to sustain it, but also to raise it, and to carry it back to the top, where it stays and abideth in the same manner, till such time, as the assistance of the conjoyned Air is taken away. And to this effect, I have taken a Ball of Wax, and made it with a little Lead, so grave, that it leasurely descends to the Bottom, making with all its Superficies very smooth and pollite: and this being put gently into the water, almost wholly submergeth, there remaining vissible only a little of the very top, the which so long as it is conjoyned with the Air, shall retain the Ball a-top, but the Contiguity of the Air taken away by wetting it, it shall descend to the Bottom and there remain. Now to make it by vertue of the Air, that before sustained it to return again to the top, and stay there, thrust into the water a Glass reversed with the mouth downwards, the which shall carry with it the Air it contains, and move this towards the Ball, abasing it till such time that you see, by the transparency of the Glass, that the contained Air do arrive to the summity of the *Ball*: then gently withdraw the Glass upwards, and you shall see the *Ball* to rise, and afterwards stay on the top of the water, if you carefully part the Glass and the water without overmuch commoving and disturbing it. There is, therefore, a certain affinity between the Air and other *Bodies*, which holds them united, so, that they seperate not without a kind of violence. The same likewise is seen in the water; for if we shall wholly submerge some *Body* in it, so that it be thorowly bathed, in the drawing of it afterwards gently out again, we shall see the water follow it, and rise notably above its Surface, before it seperates from it. Solid *Bodies*, also, if they be equall and alike in Superficies, so, that they make an exact Contact without the interposition of the least Air, that may part them in the seperation and yield untill that the ambient *Medium* succeeds to replenish the place, do hold very firmly conjoyned, and are not to be seperated without great force but, because, the Air, Water, and other Liquids, very expeditiously shape themselves to contact with any Solid *Bodies*, so that their Superficies do exquisitely adopt themselves to that of the Solids, without any thing remaining between them, therefore, the effect of this Conjunction and Adherence is more manifestly and frequently observed in them, than in hard and inflexible *Bodies*, whose Superficies do very rarely conjoyn with exactness of Contact. This is therefore that Magnetick vertue, which with firm Connection conjoyneth all Bodies, that do touch withou tthe interposition of flexible fluids; and, who knows, but that that a Contact, when it is very exact, may be a sufficient Cause of the Union and Continuity of the parts of a naturall *Body*?

The Effect of the Airs Contiguity in the Natation of Solids.

The force of Contact.

An affectation of Conjunction betwixt Solids and the Air contiguous to them.

The like affectation of Conjunction betwixt Solids & the water.

Also the like affectation and Conjunction betwixt Solids themselves.

Contact may be the Cause of the Continuity of Naturall Bodies.

Now,

Now, pursuing my purpose, I say; that it needs not, that we have recourse to the Tenacity, that the parts of the water have amongst themselves, by which they resist and oppose Division, Distraction, and Seperation, because there is no such Coherence and Resistance of Division for if there were, it would be no less in the internall parts than in those nearer the superiour or externall Surface, so that the same Board, finding alwayes the same Resistance and Renitence, would no less stop in the middle of the water than about the Surface, which is false. Moreover, what Resistance can we place in the Continuity of the water, if we see that it is impossible to find any Body of whatsoever Matter, Figure or Magnitude, which being put into the water, shall be obstructed and impeded by the Tenacity of the parts of the water to one another, so, but that it is moved upwards or downwards, according as the Cause of their Motion transports it? And, what greater proof of it can we desier, than that which we daily see in Muddy waters, which being put into Vessels to be drunk, and being, after some hours setling, still, as we say, thick in the end, after four or six dayes they are wholly setled, and become pure and clear? Nor can their Resistance of Penetration stay those impalpable and insensible Atomes of Sand, which by reason of their exceeding small force, spend six dayes in descending the space of half a yard.

The settlement of Muddy Water, proveth that that Element hath no aversion to Division.

Nor let them say, that the seeing of such small Bodies, consume six dayes in descending so little a way, is a sufficient Argument of the Waters Resistance of Division; because that is no resisting of Division, but a retarding of Motion; and it would be simplicity to say, that a thing opposeth Division, and that in the same instant, it permits it self to be divided: nor doth the Retardation of Motion at all favour the Adversaries cause, for that they are to instance in a thing that wholly prohibiteth Motion, and procureth Rest; it is necessary, therefore, to find out Bodies that stay in the water, if one would shew its repugnancy to Division, and not such as move in it, howbeit but slowly.

Water cannot oppose division, and at the same time permit it self to be divided.

What then is this Crassitude of the water, with which it resisteth Division? What, I beseech you, should it be, if we (as we have said above) with all diligence attempting the reduction of a Matter into so like a Gravity with the water, that forming it into a dilated Plate it rests suspended as we have said, between the two waters, it be impossible to effect it, though we bring them to such an Equiponderance, that as much Lead as the fourth part of a Grain of Musterd-seed, added to the same expanded Plate, that in Air [*i. e. out of the water*] shall weigh four or six pounds, sinketh it to the Bottom, and being substracted, it ascends to the Surface of the water? I cannot see, (if what I say be true, as it is most certain) what minute vertue and force we can possibly find or imagine, to which the Resistance of the water against Division and Penetration

tion is not inferiour; whereupon, we must of necessity conclude that it is nothing: because, if it were of any sensible power, some large Plate might be found or compounded of a Matter alike in Gravity to the water, which not only would stay between the two waters; but, moreover, should not be able to descend or ascend without notable force. We may likewise collect the same from an other Experiment, shewing that the Water gives way also in the same manner to transversall Division; for if in a setled and standing water we should place any great Mass that goeth not to the bottom, drawing it with a single Womans Hair, we might carry it from place to place without any opposition, and this whatever Figure it hath, though that it possess a great space of water, as for instance, a great Beam would do moved side-ways. Perhaps some might oppose me and say, that if the Resistance of water against Division, as I affirm, were nothing; Ships should not need such a force of Oars and Sayles for the moving of them from place to place in a tranquile Sea, or standing Lake. To him that should make such an objection, I would reply, that the water contrasteth not against, nor simply resisteth Division, but a sudden Division, and with so much greater Renitence, by how much greater the Velocity is: and the Cause of this Resistance depends not on Crassitude, or any other thing that absolutely opposeth Division, but because that the parts of the water divided, in giving way to that Solid that is moved in it, are themselves also necessitated locally to move, some to the one side, and some to the other, and some downwards: and this must no less be done by the waves before the Ship, or other Body swimming through the water, than by the posteriour and subsequent; because, the Ship proceeding forwards, to make it self a way to receive its Bulk, it is requisite, that with the Prow it repulse the adjacent parts of the water, as well on one hand as on the other, and that it move them as much transversly, as is the half of the breadth of the Hull: and the like removall must those waves make, that succeeding the Poump do run from the remoter parts of the Ship towards those of the middle, successively to replenish the places, which the Ship in advancing forwards, goeth, leaving vacant. Now, because, all Motions are made in Time, and the longer in greater time: and it being moreover true, that those Bodies that in a certain time are moved by a certain power such a certain space, shall not be moved the same space, and in a shorter Time, unless by a greater Power: therefore, the broader Ships move slower than the narrower, being put on by an equall Force: and the same Vessel requires so much greater force of Wind, or Oars, the faster it is to move.

An hair will draw a great Mass thorow the Water; which proveth, that it hath no Resistance against transversall Division.

How Ships are moved in the water.

Bodies moved a certain space in a certain Time, by a certain power, cannot be moved the same space, and in a shorter time, but by a greater power.

But yet for all this, any great Mass swimming in a standing Lake, may be moved by any petit force; only it is true, that a lesser force more slowly moves it : but if the waters Resistance of Division, were in any manner sensible, it would follow, that the said Mass, should, notwithstanding the percussion of some sensible force, continue immoveable, which is not so. Yea, I will say farther, that should we retire our selves into the more internall contemplation of the Nature of water and other Fluids, perhaps we should discover the Constitution of their parts to be such, that they not only do not oppose Division, but that they have not any thing in them to be divided : so that the Resistance that is observed in moving through the water, is like to that which we meet with in passing through a great Throng of People, wherein we find impediment, and not by any difficulty in the Division, for that none of those persons are divided whereof the Croud is composed, but only in moving of those persons sideways which were before divided and disjoyned : and thus we find Resistance in thrusting a Stick into an heap of Sand, not because any part of the Sand is to be cut in pieces, but only to be moved and raised. Two manners of Penetration, therefore, offer themselves to us, one in Bodies, whose parts were continuall, and here Division seemeth necessary; the other in the aggregates of parts not continuall, but contiguous only, and here there is no necessity of dividing but of moving only. Now, I am not well resolved, whether water and other Fluids may be esteemed to be of parts continuall or contiguous only; yet I find my self indeed inclined to think that they are rather contiguous (if there be in Nature no other manner of aggregating, than by the union, or by the touching of the extreams :) and I am induced thereto by the great difference that I see between the Conjunction of the parts of an hard or Solid Body, and the Conjunction of the same parts when the same Body shall be made Liquid and Fluid : for if, for example, I take a Mass of Silver or other Solid and hard Mettall, I shall in dividing it into two parts, find not only the resistance that is found in the moving of it only, but an other incomparably greater, dependent on that vertue, whatever it be, which holds the parts united : and so if we would divide again those two parts into other two, and successively into others and others, we should still find a like Resistance, but ever less by how much smaller the parts to be divided shall be; but if, lastly, employing most subtile and acute Instruments, such as are the most tenuous parts of the Fire, we shall resolve it (perhaps) into its last and least Particles, there shall not be left in them any longer either Resistance of Division, or so much as a capacity of being farther divided, especially by Instruments more grosse than the acuities of Fire : and what Knife or Rasor put into well melted Silver can we finde, that will divide a thing which surpasseth the separating power of Fire? Certainly none : because either the whole shall be reduced to the most minute and ultimate Divisions, or if there remain parts capable still of other Subdivisions,

The parts of Liquids, so farre from resisting Division, that they contain not any thing that may be divided.

The Resistance a Solid findeth in moving through the water, like to that we meet with in passing through a throng of people:

Or in thrusting a Stick into an heap of Sand.

Two kinds of Penetration, one in Bodies continuall, the other in Bodies only contiguous.

Water consists not of continuall, but only of contiguous parts.

See what satisfaction he hath given, as to this point, in Lib. de Motu. Dial. 2.

Great difference betwixt the Conjunction of the parts of a Body when Solid, and when fluid.

divisions, they cannot receive them, but only from acuter Divisors than Fire; but a Stick or Rod of Iron, moved in the melted Metall, is not such a one. Of a like Constitution and Consistence, I account the parts of Water, and other Liquids to be, namely, incapable of Division by reason of their Tenuity; or if not absolutely indivisible, yet at least not to be divided by a Board, or other Solid Body, palpable unto the hand, the Sector being alwayes required to be more sharp than the Solid to be cut. Solid Bodies, therefore, do only move, and not divide the Water, when put into it; whose parts being before divided to the extreamest minuity, and therefore capable of being moved, either many of them at once, or few, or very few, they soon give place to every small Corpuscle, that descends in the same: for that, it being little and light, descending in the Air, and arriving to the Surface of the Water, it meets with Particles of Water more small, and of less Resistance against Motion and Extrusion, than is its own prement and extrusive force, whereupon it submergeth, and moveth such a portion of them, as is proportionate to its Power. There is not, therefore, any Resistance in Water against Division, nay, there is not in it any divisible parts. I adde, moreover, that in case yet there should be any small Resistance found (which is absolutely false) haply in attempting with an Hair to move a very great natant Machine, or in essaying by the addition of one small Grain of Lead to sink, or by removall of it to raise a very broad Plate of Matter, equall in Gravity with Water, (which likewise will not happen, in case we proceed with dexterity) we may observe that that Resistance is a very different thing from that which the Adversaries produce for the Cause of the Natation of the Plate of Lead or Board of Ebony, for that one may make a Board of Ebony, which being put upon the Water swimmeth, and cannot be submerged, no not by the addition of an hundred Grains of Lead put upon the same, and afterwards being bathed, not only sinks, though the said Lead be taken away, but though moreover a quantity of Cork, or of some other light Body fastened to it, sufficeth not to hinder it from sinking unto the bottome: so that you see, that although it were granted that there is a certain small Resistance of Division found in the substance of the Water, yet this hath nothing to do with that Cause which supports the Board above the Water, with a Resistance an hundred times greater than that which men can find in the parts of the Water: nor let them tell me, that only the Surface of the Water hath such Resistance, and not the internall parts, or that such Resistance is found greatest in the beginning of the Submersion, as it also seems that in the beginning, Motion meets with greater opposition, than in the continuance of it; because, first, I will permit, that the Water be stirred, and that the superiour parts be mingled with the middle, and inferiour parts, or that those above be wholly removed, and those in the middle only made use off, and yet you shall see the effect for all

Water consists of parts that admit of no farther division.

Solids dimitted into the water, do onely move, and not divide it.

If there were any Resistance of Division in water, it must needs be small, in that it is overcome by an Hair, a Grain of Lead, or a slight bathing of the Solid.

The uper parts of the Water, do no more resist Division, than the middle or lowest parts.

Waters Resistance of division, not greater in the begining of the Submersion.

all that, to be still the same: Moreover, that Hair which draws a Beam through the Water, is likewise to divide the upperparts, and is also to begin the Motion, and yet it begins it, and yet it divides it: and finally, let the Board of Ebony be put in the midway, betwixt the bottome and the top of the Water, and let it there for a while be suspended and setled, and afterwards let it be left at liberty, and it will instantly begin its Motion, and will continue it unto the bottome. Nay, more, the Board so soon as it is dimitted upon the Water, hath not only begun to move and divide it, but is for a good space dimerged into it.

Let us receive it, therefore, for a true and undoubted Conclusion, That the Water hath not any Renitence against simple Division, and that it is not possible to find any Solid Body, be it of what Figure it will, which being put into the Water, its Motion upwards or downwards, according as it exceedeth, or shall be exceeded by the Water in Gravity (although such excesse and difference be insensible) shall be prohibited, and taken away, by the Crassitude of the said Water. When, therefore, we see the Board of Ebony, or of other Matter, more grave than the Water, to stay in the Confines of the Water and Air, without submerging, we must have recourse to some other Originall, for the investing the Cause of that Effect, than to the breadth of the Figure, unable to overcome the Renitence with which the Water opposeth Division, since there is no Resistance; and from that which is not in being, we can expect no Action. It remains most true, therefore, as we have said before, that this so succeeds, for that that which in such manner put upon the water, not the same Body with that which is put *into* the Water: because this which is put *into* the Water, is the pure Board of Ebony, which for that it is more grave than the Water, sinketh, and that which is put *upon* the Water, is a Composition of Ebony, and of so much Air, that both together are specifically less grave than the Water, and therefore they do not descend.

I will farther confirm this which I say. Gentlemen, my Antagonists, we are agreed, that the excess or defect of the Gravity of the Solid, unto the Gravity of the Water, is the true and proper Cause of Natation or Submersion.

Great Caution to be had in experimenting the operation of Figure in Natation.

Now, if you will shew that besides the former Cause, there is another which is so powerfull, that it can hinder and remove the Submersion of those very Solids, that by their Gravity sink, and if you will say, that this is the breadth or ampleness of Figure, you are oblieged, when ever you would shew such an Experiment, first to make the circumstances certain, that that Solid which you put into the Water, be not less grave *in specie* than it, for if you should not do so, any one might with reason say, that not the Figure, but the Levity was the cause of that Natation. But I say, that when you shall dimit

mit a Board of Ebony into the Water, you do not put therein a Solid more grave *in specie* than the Water, but one lighter, for besides the Ebony, there is in the Water a Mass of Air, united with the Ebony, and such, and so light, that of both there results a Composition less grave than the Water: See, therefore, that you remove the Air, and put the Ebony alone into the Water, for so you shall immerge a Solid more grave then the Water, and if this shall not go to the Bottom, you have well Philosophized, and I ill.

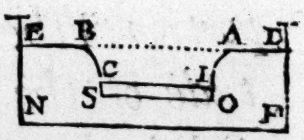
Now, since we have found the true Cause of the Natation of those Bodies, which otherwise, as being graver than the Water, would descend to the bottom, I think, that for the perfect and distinct knowledge of this business, it would be good to proceed in a way of discovering demonstratively those particular Accidents that do attend these effects, and,

PROBL. I.

To finde what proportion severall Figures of different Matters ought to have, unto the Gravity of the Water, that so they may be able by vertue of the Contiguous Air to stay afloat.

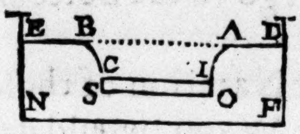
To finde the proportion Figures ought to have to the waters Gravity, that by help of the contiguous Air, they may swim.

LEt, therefore, for better illustration, D F N E be a Vessell, wherein the water is contained, and suppose a Plate or Board, whose thickness is comprehended between the Lines I C and O S, and let it be of Matter exceeding the water in Gravity, so that being put upon the water, it dimergeth and abaseth below the Levell of the said water, leaving the little Banks A I and B C, which are at the greatest height they can be, so that if the Plate I S should but descend any little space farther, the little Banks or Ramparts would no longer consist, but expulsing the Air A I C B, they would diffuse themselves over the Superficies I C, and would submerge the Plate. The height AIBC is therefore the greatest profundity that the little Banks of water admit of. Now I say, that from this, and from the proportion in Gravity, that the Matter of the Plate hath to the water, we may easily finde of what thickness, at most, we may make the said Plates, to the end, they may be able to bear up above water: for if the Matter of the Plate or Board I S were, for Example, as heavy again as the water, a Board of that Matter shall be, at the most of a thickness equall to the greatest height of the Banks, that is, as thick as A I is high: which we will thus demonstrate. Let the Solid I S be double in Gravity to the water, and let it be a regular



Prisme

Prisme, or Cylinder, to wit, that hath its two flat Superficies, superiour and inferiour, alike and equall, and at Right Angles with the other laterall Superficies, and let its thickness I O be equall to the greatest Altitude of the Banks of water: I say, that if it be put upon the water, it will not submerge: for the Altitude A I being equall to the Altitude I O, the Mass of the Air A B C I shall be equall to the Mass of the Solid C I O S: and the whole Mass A O S B double to the Mass I S; And since the Mass of the Air A C, neither encreaseth nor diminisheth the Gravity of the Mass I S, and the Solid I S was supposed double in Gravity to the water; Therefore as much water as the Mass submerged A O S B, compounded of the Air A I C B, and of the Solid I O S C, weighs just as much as the same submerged Mass A O S B: but when such a Mass of water, as is the submerged part of the Solid, weighs as much as the said Solid, it descends not farther, but resteth, as by (*a*) *Archimedes*, and above by us, hath been demonstrated: Therefore, I S shall descend no farther, but shall rest. And if the Solid I S shall be Sesquialter in Gravity to the water, it shall float, as long as its thickness be not above twice as much as the greatest Altitude of the Ramparts of water, that is, of A I. For I S being Sesquialter in Gravity to the water, and the Altitude O I, being double to I A, the Solid submerged A O S B, shall be also Sesquialter in Mass to the Solid I S. And because the Air A C, neither increaseth nor diminisheth the ponderosity of the Solid I S: Therefore, as much water in quantity as the submerged Mass AOSB, weighs as much as the said Mass submerged: And, therefore, that Mass shall rest. And briefly in generall.



Of Natation Lib. 1. Prop. 3.

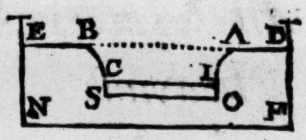
THEOREME. VI.

The proportion of the greatest thickness of Solids, beyond which encreased they sink.

When ever the excess of the Gravity of the Solid above the Gravity of the Water, shall have the same proportion to the Gravity of the Water, that the Altitude of the Rampart, hath to the thickness of the Solid, that Solid shall not sink, but being never so little thicker it shall.

LEt the Solid I S be superior in Gravity to the water, and of such thickness, that the Altitude of the Rampart A I, be in proportion to the thickness of the Solid I O, as the excess of the Gravity of the said Solid I S, above the Gravity of a Mass of water equall to the Mass I S, is to the Gravity of the Mass of water equall to the Mass

Mass I S. I say, that the Solid I S shall not sinke, but being never so little thicker it shall go to the bottom : For being that as A I is to I O, so is the Excess of the Gravity of the Solid I S, above the Gravity of a Mass of water equall to the Mass I S, to the Gravity of the said Mass of water : Therefore, compounding, as A O is to O I, so shall the Gravity of the Solid I S, be to the Gravity of a Mass of water equall to the Mass I S: And, converting, as I O is to O A, so shall the Gravity of a Mass of water equall to the Mass I S, be to the Gravity of the Solid I S : But as I O is to O A, so is a Mass of water I S, to a Mass of water equall to the Mass A B S O : and so is the Gravity of a Mass of water I S, to the Gravity of a Mass of water A S: Therefore as the Gravity of a Mass of water, equall to the Mass I S, is to the Gravity of the Solid I S, so is the same Gravity of a Mass of water I S, to the Gravity of a Mass of Water A S: Therefore the Gravity of the Solid I S, is equall to the Gravity of a Mass of water equall to the Mass A S: But the Gravity of the Solid I S, is the same with the Gravity of the Solid A S, compounded of the Solid I S, and of the Air A B C I. Therefore the whole compounded Solid A O S B, weighs as much as the water that would be comprised in the place of the said Compound A O S B : And, therefore, it shall make an *Equilibrium* and rest, and that same Solid I O S C shall sinke no farther. But if its thickness I O should be increased, it would be necessary also to encrease the Altitude of the Rampart A I, to maintain the due proportion : But by what hath been supposed, the Altitude of the Rampart A I, is the greatest that the Nature of the Water and Air do admit, without the waters repulsing the Air adherent to the Superficies of the Solid I C, and possessing the space A I C B : Therefore, a Solid of greater thickness than I O, and of the same Matter with the Solid I S, shall not rest without submerging, but shall descend to the bottome : which was to be demonstrated.



In consequence of this that hath been demonstrated, sundry and various Conclusions may be gathered, by which the truth of my principall Proposition comes to be more and more confirmed, and the imperfection of all former Argumentations touching the present Question cometh to be discovered.

And first we gather from the things demonstrated, that,

COROL-

THEOREME VII.

The heaviest Bodies may swimme.

All Matters, how heavy soever, even to Gold it self, the heaviest of all Bodies, known by us, may float upon the Water.

BEcause its Gravity being considered to be almost twenty times greater than that of the water, and, moreover, the greatest Altitude that the Rampart of water can be extended to, without breaking the Contiguity of the Air, adherent to the Surface of the Solid, that is put upon the water being predetermined, if we should make a Plate of Gold so thin, that it exceeds not the nineteenth part of the Altitude of the said Rampart, this put lightly upon the water shall rest, without going to the bottom: and if Ebony shall chance to be in sesquiseptimall proportion more grave than the water, the greatest thicknes that can be allowed to a Board of Ebony, so that it may be able to stay above water without sinking, would be seaven times more than the height of the Rampart Tinn, *v. gr.* eight times more grave than water, shall swimm as oft as the thicknes of its Plate, exceeds not the 7th part of the Altitude of the Rampart.

He elsewhere cites this as a Proposition, therefore I make it of that number.

And here I will not omit to note, as a second Corrollary dependent upon the things demonstrated, that,

THEOREME VIII.

Natation and Submersion, collected from the thicknes, excluding the length and breadth of Plates.

The Expansion of Figure not only is not the Cause of the Natation of those grave Bodies, which otherwise do submerge, but also the determining what be those Boards of Ebony, or Plates of Iron or Gold that will swimme, depends not on it, rather that same determination is to be collected from the only thicknes of those Figures of Ebony or Gold, wholly excluding the consideration of length and breadth, as having no wayes any share in this Effect.

IT hath already been manifested, that the only cause of the Natation of the said Plates, is the reduction of them to be less grave than the water, by means of the connexion of that Air, which descendeth together with them, and possesseth place in the water; which place so occupyed, if before the circumfused water diffuseth it self to fill it, it be capable of as much water, as shall weigh equall with the Plate, the Plate shall remain suspended, and sinke no farther.

Now

Now let us see on which of these three dimensions of the Solid depends the terminating, what and how much the Mass of that ought to be, that so the assistance of the Air contiguous unto it, may suffice to render it specifically less grave than the water, whereupon it may rest without Submersion. It shall undoubtedly be found, that the length and breadth have not any thing to do in the said determination, but only the height, or if you will the thickness: for, if we take a Plate or Board, as for Example, of Ebony, whose Altitude hath unto the greatest possible Altitude of the Rampart, the proportion above declared, for which cause it swims indeed, but yet not if we never so little increase its thickness; I say, that retaining its thickness, and encreasing its Superficies to twice, four times, or ten times its bigness, or diminishing it, by dividing it into four, or six, or twenty, or a hundred parts, it shall still in the same manner continue to float: but encreasing its thickness only a Hairs breadth, it will alwaies submerge, although we should multiply the Superficies a hundred and a hundred times. Now forasmuch as that this is a Cause, which being added, we adde also the Effect, and being removed, it is removed; and by augmenting or lessening the length or breadth in any manner, the effect of going, or not going to the bottom, is not added or removed: I conclude, that the greatness and smalness of the Superficies hath no influence upon the Natation or Submersion. And that the proportion of the Altitude of the Ramparts of Water, to the Altitude of the Solid, being constituted in the manner aforesaid, the greatness or smalness of the Superficies, makes not any variation, is manifest from that which hath been above demonstrated, and from this, that, *The Prisms and Cylinders which have the same Base, are in proportion to one another as their heights*: Whence Cylinders or Prismes, namely, the Board, be they great or little, so that they be all of equall thickness, have the same proportion to their Conterminall Air, which hath for Base the said Superficies of the Board, and for height the Ramparts of water; so that alwayes of that Air, and of the Board, Solids are compounded, that in Gravity equall a Mass of water equall to the Mass of the Solids, compounded of Air, and of the Board: whereupon all the said Solids do in the same manner continue afloat. We will conclude in the third place, that,

Prismes and Cylinders having the same Base, are to one another as their heights.

THEOREME. IX.

All Figures of all Matters, float by hep of the Rampart replenished with Air, and some but only touch the water.

All sorts of Figures of whatsoever Matter, albeit more grave than the Water, do by Benefit of the said Rampart, not only float, but some Figures, though of the gravest Matter, do stay wholly above Water, wetting only the inferiour Surface that toucheth the Water.

And these shall be all Figures, which from the inferiour Base upwards, grow lesser and lesser; the which we shall exemplifie for this time in Piramides or Cones, of which Figures the passions are common. We will demonstrate therefore, that,

It is possible to form a Piramide, of any whatsoever Matter proposed, which being put with its Base upon the Water, rests not only without submerging, but without wetting it more then its Base.

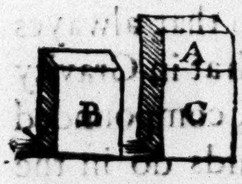
For the explication of which it is requisite, that we first demonstrate the subsequent Lemma, namely, that,

LEMMA II.

Solids whose Masses are in contrary proportion to their Specifick Gravities, are equall in absolute Gravity.

Solids whose Masses answer in proportion contrarily to their Specificall Gravities, are equall in Absolute Gravities.

Let A C and B be two Solids, and let the Mass A C be to the Mass B, as the Specificall Gravity of the Solid B, is to the Specificall Gravity of the Solid A C: I say, the Solids A C and B are equall in absolute weight, that is, equally grave. For if the Mass A C be equall to the Mass B, then, by the Assumption, the Specificall Gravity of B, shall be equall to the Specificall Gravity of A C, and being equall in Mass, and of the same Specificall Gravity they shall absolutely weigh one as much as another.



But if their Masses shall be unequall, let the Mass A C be greater, and in it take the part C, equall to the Mass B. And, because the Masses B and C are equall; the Absolute weight of B, shall have the same proportion to the Absolute weight of C, that the Specificall Gravity of B, hath to the Specificall Gravity of C; or of C A, which is the same *in specie*: But look what proportion the Specificall Gravity of B, hath to the Specificall Gravity of C A, the like proportion, by the Assumption, hath the Mass C A, to the Mass B; that is, to the Mass C:

Therefore,

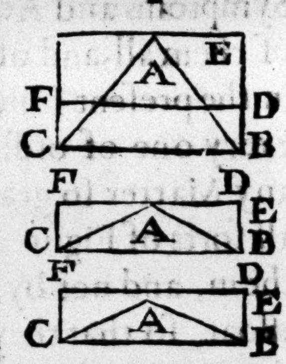
Therefore, the absolute weight of B, to the absolute weight of C, is as the Mass A C to the Mass C: But as the Mass AC, is to the Mass C, so is the absolute weight of A C, to the absolute weight of C: Therefore the absolute weight of B, hath the same proportion to the absolute weight of C, that the absolute weight of A C, hath to the absolute weight of C: Therefore, the two Solids A C and B are equall in absolute Gravity: which was to be demonstrated. Having demonstrated this, I say,

THEOREME X.

That it is possible of any assigned Matter, to form a Piramide or Cone upon any Base, which being put upon the Water shall not submerge, nor wet any more than its Base.

There may be Cones and Piramides of any *Matter*, which demitted into the water, rest only their Bases.

LEt the greatest possible Altitude of the Rampart be the Line D B, and the Diameter of the Base of the Cone to be made of any Matter assigned B C, at right angles to D B: And as the Specificall Gravity of the Matter of the Piramide or Cone to be made, is to the Specificall Gravity of the water, so let the Altitude of the Rampart D B, be to the third part of the Piramide or Cone A B C, described upon the Base, whose Diameter is B C: I say, that the said Cone A B C, and any other Cone, lower then the same, shall rest upon the Surface of the water B C without sinking. Draw D F parallel to B C, and suppose the Prisme or Cylinder E C, which shall be tripple to the Cone A B C.



And, because the Cylinder D C hath the same proportion to the Cylinder C E, that the Altitude D B, hath to the Altitude B E: But the Cylinder C E, is to the Cone A B C, as the Altitude E B is to the third part of the Altitude of the Cone: Therefore, by Equality of proportion, the Cylinder D C is to the Cone A B C, as D B is to the third part of the Altitude B E: But as D B is to the third part of B E, so is the Specificall Gravity of the Cone A B C, to the Specificall Gravity of the water: Therefore, as the Mass of the Solid D C, is to the Mass of the Cone A B C, so is the Specificall Gravity of the said Cone, to the Specificall Gravity of the water: Therefore, by the precedent Lemma, the Cone A B C weighs in absolute Gravity as much as a Mass of Water equall to the Mass D C: But the water which by the imposition of the Cone A B C, is driven out of its place, is as much as would precisely lie in the place D C, and is equall in weight to the Cone that displaceth it: Therefore, there shall be an *Equilibrium*, and the Cone shall rest without farther submerging. And its manifest,

 COROL.

COROLARY I.

Amongst Cones of the same Base, those of least Altitude shall sink the least.

That making upon the same Basis, a Cone of a less Altitude, it shall be also less grave, and shall so much the more rest without Submersion.

COROLARY II.

There may be Cones and Piramides of any Matter, which demitted with the Point downwards do float a top.

It is manifest, also, that one may make Cones and Piramids of any Matter whatsoever, more grave than the water, which being put into the water, with the Apix or Point downwards, rest without Submersion.

BEcause if we reassume what hath been above demonstrated, of Prisms and Cylinders, and that on Bases equall to those of the said Cylinders, we make Cones of the same Matter, and three times as high as the Cylinders, they shall rest afloat, for that in Mass and Gravity they shall be equall to those Cylinders, and by having their Bases equall to those of the Cylinders, they shall leave equall Masses of Air included within the Ramparts. This, which for Example sake hath been demonstrated, in Prisms, Cylinders, Cones and Piramids, might be proved in all other Solid Figures, but it would require a whole Volume (such is the multitude and variety of their Symptoms and Accidents) to comprehend the particuler demonstration of them all, and of their severall Segments: but I will to avoid prolixity in the present Discourse, content my self, that by what I have declared every one of ordinary Capacity may comprehend, that there is not any Matter so grave, no not Gold it self, of which one may not form all sorts of Figures, which by vertue of the superiour Air adherent to them, and not by the Waters Resistance of Penetration, do remain afloat, so that they sink not. Nay, farther, I will shew, for removing that Error, that,

THEOREME XI.

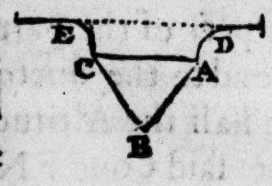
A Piramide or Cone, demitted with the Point downwards shal swim, with its Base downward shall sink.

A Piramide or Cone put into the Water, with the Point downward shall swimme, and the same put with the Base downwards shall sinke, and it shall be impossible to make it float.

NOw the quite contrary would happen, if the difficulty of Penetrating the water, were that which had hindred the descent, for that the said Cone is far apter to pierce and penetrate with its sharp Point, than with its broad and spacious Base.

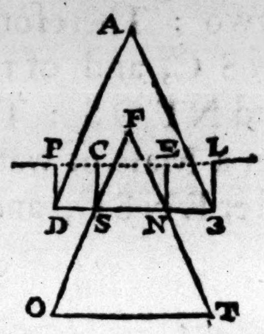
And, to demonstrate this, let the Cone be *A B C*, twice as grave as the water, and let its height be tripple to the height of the Rampart *D A E C*: I say, first, that being put lightly into the water with the Point

Point downwards, it ſhall not deſcend to the bottom: for the Aeriall Cylinder contained betwixt the Ramparts *DACE*, is equall in Maſs to the Cone *ABC*; ſo that the whole Maſs of the Solid compounded of the Air *DACE*, and of the Cone *ABC*, ſhall be double to the Cone *ACB*: And,



becauſe the Cone *ABC* is ſuppoſed to be of Matter double in Gravity to the water, therefore as much water as the whole Maſſe *DABCE*, placed beneath the Levell of the water, weighs as much as the Cone *ABC*: and, therefore, there ſhall be an *Equilibrium*, and the Cone *ABC* ſhall deſcend no lower. Now, I ſay farther, that the ſame Cone placed with the Baſe downwards, ſhall ſink to the bottom, without any poſſibility of returning again, by any means to ſwimme.

Let, therefore, the Cone be *ABD*, double in Gravity to the water, and let its height be tripple the height of the Rampart of water LB: It is already manifeſt, that it ſhall not ſtay wholly out of the water, becauſe the Cylinder being comprehended betwixt the Ramparts *LBDP*, equall to the Cone *ABD*, and the Matter of the Cone, beig double in Gravity to the water, it is evident that the weight of the ſaid Cone ſhall be double to the weight of the Maſs of water equall to the Cylinder *LBDP*: Therefore it ſhall not reſt in this ſtate, but ſhall deſcend.



COROLARY I.

I ſay farther; that much leſſe ſhall the ſaid Cone ſtay afloat, if one immerge a part thereof.

Much leſs ſhall the ſaid Cone ſwim, if one immerge a part thereof.

WHich you may ſee, comparing with the water as well the part that ſhall immerge as the other above water. Let us therefore of the Cone ABD, ſubmerge th part NTOS, and advance the Point NSF above water. The Altitude of the Cone FNS, ſhall either be more than half the whole Altitude of the Cone FTO, or it ſhall not be more: if it ſhall be more than half, the Cone FNS ſhall be more than half of the Cylinder ENSC: for the Altitude of the Cone FNS, ſhall be more than Seſquialter of the Altitude of the Cylinder ENSC: And, becauſe the Matter of the Cone is ſuppoſed to be double in Specificall Gravity to the water, the water which would be contained within the Rampart ENSC, would be leſs grave abſolutely than the Cone FNS; ſo that the whole Cone FNS cannot be ſuſtained by the Rampart: But the part immerged NTOS, by being double in Specificall Gravity to the water, ſhall

tend

tend to the bottom: Therefore, the whole Cone F T O, as well in respect of the part submerged, as the part above water shall descend to the bottom. But if the Altitude of the Point F N S, shall be half the Altitude of the whole Cone F T O, the same Altitude of the said Cone F N S shall be Sesquialter to the Altitude E N: and, therefore, E N S C shall be double to the Cone F N S; and as much water in Mass as the Cylinder E N S C, would weigh as much as the part of the Cone F N S. But, because the other immerged part N T O S, is double in Gravity to the water, a Mass of water equall to that compounded of the Cylinder E N S C, and of the Solid N T O S, shall weigh less than the Cone F T O, by as much as the weight of a Mass of water equall to the Solid N T O S: Therefore, the Cone shall also descend. Again, because the Solid N T O S, is septuple to the Cone F N S, to which the Cylinder E S is double, the proportion of the Solid N T O S, shall be to the Cylinder E N S C, as seaven to two: Therefore, the whole Solid compounded of the Cylinder E N S C, and of the Solid N T O S, is much less than double the Solid N T O S: Therefore, the single Solid N T O S, is much graver than a Mass of water equall to the Mass, compounded of the Cylinder E N S C, and of N T O S.

COROLARY II.

Part of the Cones towards the Cuspis removed, it shall still sink.

From whence it followeth, that though one should remove and take away the part of the Cone F N S, the sole remainder N T O S would go to the bottom.

COROLARY III.

The more the Cone is immerged, the more impossible is its floating.

And if we should more depress the Cone F T O, it would be so much the more impossible that it should sustain it self afloat, the part submerged N T O S still encreasing, and the Mass of Air contained in the Rampart diminishing, which ever grows less, the more the Cone submergeth.

That Cone, therefore, that with its Base upwards, and its *Cuspis* downwards doth swimme, being dimitted with its Base downward must of necessity sinke. They have argued farre from the truth, therefore, who have ascribed the cause of Natation to waters resistance of Division, as to a passive principle, and to the breadth of the Figure, with which the division is to be made, as the Efficient.

I come in the fourth place, to collect and conclude the reason of that which I have proposed to the Adversaries, namely,

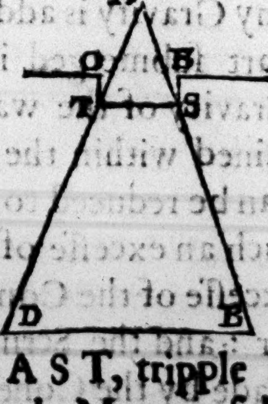
THEO-

THEOREME XII.

That it is possible to form Solid Bodies, of what Figure and greatness soever, that of their own Nature goe to the Bottome; But by the help of the Air contained in the Rampart, rest without submerging.

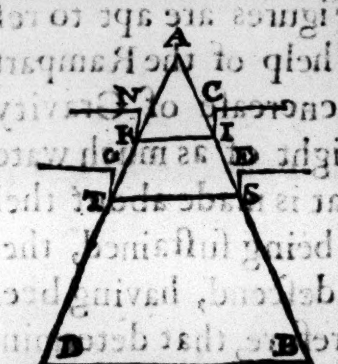
Solids of any Figure & greatnesse, that naturally sink, may by help of the Air in the Rampart swimme.

THe truth of this Proposition is sufficiently manifest in all those Solid Figures, that determine in their uppermost part in a plane Superficies : for making such Figures of some Matter specifically as grave as the water, putting them into the water, so that the whole Mass be covered, it is manifest, that they shall rest in all places, provided, that such a Matter equall in weight to the water, may be exactly adjusted : and they shall by consequence, rest or lie even with the Levell of the water, without making any Rampart. If, therefore, in respect of the Matter, such Figures are apt to rest without submerging, though deprived of the help of the Rampart, it is manifest, that they may admit so much encrease of Gravity, (without encreasing their Masses) as is the weight of as much water as would be contained within the Rampart, that is made about their upper plane Surface : by the help of which being sustained, they shall rest afloat, but being bathed, they shall descend, having been made graver than the water. In Figures, therefore, that determine above in a plane, we may cleerly comprehend, that the Rampart added or removed, may prohibit or permit the descent : but in those Figures that go lessening upwards towards the top, some Persons may, and that not without much seeming Reason, doubt whether the same may be done, and especially by those which terminate in a very acute Point, such as are your Cones and small Piramids. Touching these, therefore, as more dubious than the rest, I will endeavour to demonstrate, that they also lie under the same Accident of going, or not going to the Bottom, be they of any whatever bignes. Let therefore the Cone be ABD, made of a matter specifically as grave as the water ; it is manifest that being put all under water, it shall rest in all places (alwayes provided, that it shall weigh exactly as much as the water, which is almost impossible to effect) and that any small weight being added to it, it shall sink to the bottom ; but if it shall descend downwards gently, I say, that it shall make the Rampart ESTO, and that there shall stay out of the water the point AST, tripple in height to the Rampart ES : which is manifest, for the Matter of the



Cone

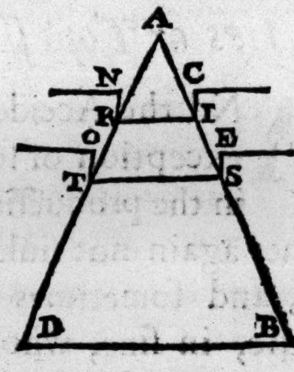
Cone weighing equally with the water, the part submerged *S B D T*, becomes indifferent to move downwards or upwards; and the Cone *A S T*, being equall in Mass to the water that would be contained in the concave of the Rampart *E S T O*, shall be also equall unto it in Gravity: and, therefore, there shall be a perfect *Equilibrium*, and, consequently, a Rest. Now here ariseth a doubt, whether the Cone *A B D* may be made heavier, in such sort, that when it is put wholly under water, it goes to the bottom, but yet not in such sort, as to take from the Rampart the vertue of sustaining it that it sink not, and, the reason of the doubt is *this*: that although at such time as the Cone *A B D* is specifically as grave as the water, the Rampart *E S T O* sustaines it, not only when the point *A S T* is tripple in height to the Altitude of the Rampart *E S*, but also when a lesser part is above water; [for although in the Descent of the Cone the Point *A S T* by little and little diminisheth, and so likewise the Rampart *E S T O*, yet the Point diminisheth in greater proportion than the Rampart, in that it diminisheth according to all the three Dimensions, but the Rampart according to two only, the Altitude still remaining the same; or, if you will, because the Cone *S T* goes diminishing, according to the proportion of the cubes of the Lines that do successively become the Diameters of the Bases of emergent Cones, and the Ramparts diminish according to the proportion of the Squares of the same Lines; whereupon the proportions of the Points are alwayes Sesquialter of the proportions of the Cylinders, contained within the Rampart; so that if, for Example, the height of the emergent Point were double, or equall to the height of the Rampart, in these cases, the Cylinder contained within the Rampart, would be much greater than the said Point, because it would be either sesquialter or tripple, by reason of which it would perhaps serve over and above to sustain the whole Cone, since the part submerged would no longer weigh any thing;] yet, neverthelesse, when any Gravity is added to the whole Mass of the Cone, so that also the part submerged is not without some excesse of Gravity above the Gravity of the water, it is not manifest, whether the Cylinder contained within the Rampart, in the descent that the Cone shall make, can be reduced to such a proportion unto the emergent Point, and to such an excesse of Mass above the Mass of it, as to compensate the excesse of the Cones Specificall Gravity above the Gravity of the water: and the Scruple ariseth, because that howbeit in the descent made by the Cone, the emergent Point *A S T* diminisheth, whereby there is also a diminution of the excess of the Cones Gravity above



the

the Gravity of the water, yet the case stands so, that the Rampart doth also contract it self, and the Cylinder contained in it doth deminish. Neverthelesse it shall be demonstrated, how that the Cone *ABD* being of any supposed bignesse, and made at the first of a Matter exactly equall in Gravity to the Water, if there may be affixed to it some Weight, by means of which it may descend to the bottom, when submerged under water, it may also by vertue of the Rampart stay above without sinking.

Let, therefore, the Cone *ABD* be of any supposed greatnesse, and alike in specificall Gravity to the water. It is manifest, that being put lightly into the water, it shall rest without descending; and it shall advance above water, the Point *AST*, tripple in height to the height of the Rampart *ES*: Now, suppose the Cone *ABD* more depressed, so that it advance above water, only the Point *AIR*, higher by half than the Point *AST*, with the Rampart about it *CIRN*. And, because, the Cone *ABD* is to the Cone *AIR*, as the cube of the Line *ST* is to the cube of the Line *IR*, but the Cylinder *ESTO*, is to the Cylinder *CIRN*, as the Square of *ST* to the Square of *IR*, the Cone *AST* shall be Octuple to the Cone *AIR*, and the Cylinder *ESTO*, quadruple to the Cylinder *CIRN*: But the Cone *AST*, is equall to the Cylinder *ESTO*: Therefore, the Cylinder *CIRN*, shall be double to the Cone *AIR*: and the water which might be contained in the Rampart *CIRN*, would be double in Mass and in Weight to the Cone *AIR*, and, therefore, would be able to sustain the double of the Weight of the Cone *AIR*: Therefore, if to the whole Cone *ABD*, there be added as much Weight as the Gravity of the Cone *AIR*, that is to say, the eighth part of the weight of the Cone *AST*, it also shall be sustained by the Rampart *CIRN*, but without that it shall go to the bottome: the Cone *ABD*, being, by the addition of the eighth part of the weight of the Cone *AST*, made specifically more grave than the water. But if the Altitude of the Cone *AIR*, were two thirds of the Altitude of the Cone *AST*, the Cone *AST* would be to the Cone *AIR*, as twenty seven to eight; and the Cylinder *ESTO*, to the Cylinder *CIRN*, as nine to four, that is, as twenty seven to twelve; and, therefore, the Cylinder *CIRN*, to the Cone *AIR*, as twelve to eight; and the excess of the Cylinder *CIRN*, above the Cone *AIR*, to the Cone *AST*, as four to twenty seven: therefore if to the Cone *ABD* be added so much weight as is the four twenty sevenths of the weight of the Cone *AST*, which is a little more then its seventh part, it also shall continue to swimme, and



the height of the emergent Point shall be double to the height of the Rampart. This that hath been demonstrated in Cones, exactly holds in Piramides, although the one or the other should be very sharp in their Point or Cuspis: From whence we conclude, that the same Accident shall so much the more easily happen in all other Figures, by how much the less sharp the Tops shall be, in which they determine, being assisted by more spacious Ramparts.

Natation easiest effected in Figures broad toward the top.

THEOREME XIII.

All Figures, therefore, of whatever greatnesse, may go, and not go, to the Bottom, according as their Summities or Tops shall be bathed or not bathed.

All Figures sink or swim, upon bathing or not bathing of their tops.

AND this Accident being common to all sorts of Figures, without exception of so much as one. Figure hath, therefore, no part in the production of this Effect, of sometimes sinking, and sometimes again not sinking, but only the being sometimes conjoyned to, and sometimes seperated from, the supereminent Air: which cause, in fine, who so shall rightly, and, as we say, with both his Eyes, consider this business, will find that it is reduced to, yea, that it really is the same with, the true, Naturall and primary cause of Natation or Submersion; to wit, the excess or deficiency of the Gravity of the water, in relation to the Gravity of that Solid Magnitude, that is demitted into the water. For like as a Plate of Lead, as thick as the back of a Knife, which being put into the water by it self alone goes to the bottom, if upon it you fasten a piece of Cork four fingers thick, doth continue afloat, for that now the Solid that is demitted in the water, is not, as before, more grave than the water, but less, so the Board of Ebony, of its own nature more grave than water; and, therefore, descending to the bottom, when it is demitted by it self alone into the water, if it shall be put upon the water, conjoyned with an Expanded vail of Air, that together with the Ebony doth descend, and that it be such, as that it doth make with it a compound less grave than so much water in Mass, as equalleth the Mass already submerged and depressed beneath the Levell of the waters Surface, it shall not descend any farther, but shall rest, for no other than the universall and most common cause, which is that Solid Magnitudes, less grave *in specie* than the water, go not to the bottom.

So that if one should take a Plate of Lead, as for Example, a finger thick, and an handfull broad every way, and should attempt to make it swimme, with putting it lightly on the water, he would lose his Labour, because that if it should be depressed an Hairs breadth be-

yond the possible Altitude of the Ramparts of water, it would dive and sink; but if whilst it is going downwards, one should make certain Banks or Ramparts about it, that should hinder the defusion of the water upon the said Plate, the which Banks should rise so high, as that they might be able to contain as much water, as should weigh equally with the said Plate, it would, without all Question, descend no lower, but would rest, as being sustained by vertue of the Air contained within the aforesaid Ramparts: and, in short, there would be a Vessell by this means formed with the bottom of Lead. But if the thinness of the Lead shall be such, that a very small height of Rampart would suffice to contain so much Air, as might keep it afloat, it shall also rest without the Artificiall Banks or Ramparts, but yet not without the Air, because the Air by it self makes Banks sufficient for a small height, to resist the Superfusion of the water: so that that which in this case swimmes, is as it were a Vessell filled with Air, by vertue of which it continueth afloat.

I will, in the last place, with an other Experiment, attempt to remove all difficulties, if so be there should yet be any doubt left in any one, touching the opperation of this *Continuity of the Air, with the thin Plate which swims, and afterwards put an end to this part of my discourse.

*Or rather Contiguity,

I suppose my self to be questioning with some of my Oponents.

An Experiment of the operation of Figures, in encreasing or lessening of the Airs Resistance of Division.

Whether Figure have any influence upon the encrease or diminution of the Resistance in any Weight against its being raised in the Air, and I suppose, that I am to maintain the Affirmative, asserting that a Mass of Lead, reduced to the Figure of a Ball, shall be raised with less force, then if the same had been made into a thinne and broad Plate, because that it in this spacious Figure, hath a great quantity of Air to penetrate, and in that other, more compacted and contracted very little: and to demonstrate the truth of such my Opinion, I will hang in a small thred first the Ball or Bullet, and put that into the water, tying the thred that upholds it to one end of the Ballance that I hold in the Air, and to the other end I by degrees adde so much Weight, till that at last it brings up the Ball of Lead out of the water: to do which, suppose a Gravity of thirty Ounces sufficeth; I afterwards reduce the said Lead into a flat and thinne Plate, the which I likewise put into the water, suspended by three threds, which hold it parallel to the Surface of the water, and putting in the same manner, Weights to the other end, till such time as the Plate comes to be raised and drawn out of the water: I finde that thirty six ounces will not suffice to seperate it from the water, and raise it thorow the Air: and arguing from this Experiment, I affirm, that I have fully demonstrated the truth of my Proposition. Here my Oponents desires me to look down, shewing me a thing

which I had not before observed, to wit, that in the Ascent of the Plate out of the water, it draws after it another Plate (*if I may so call it*) of water, which before it divides and parts from the inferiour Surface of the Plate of Lead, is raised above the Levell of the other water, more than the thickness of the back of a Knife : Then he goeth to repeat the Experiment with the Ball, and makes me see, that it is but a very small quantity of water, which cleaves to its compacted and contracted Figure : and then he subjoynes, that its no wonder, if in seperating the thinne and broad Plate from the water, we meet with much greater Resistance, than in seperating the Ball, since together with the Plate, we are to raise a great quantity of water, which occurreth not in the Ball : He telleth me moreover, how that our Question is, whether the Resistance of Elevation be greater in a dilated Plate of Lead, than in a Ball, and not whether more resisteth a Plate of Lead with a great quantity of water, or a Ball with a very little water : He sheweth me in the close, that the putting the Plate and the Ball first into the water, to make proofe thereby of their Resistance in the Air, is besides our case, which treats of Elivating in the Air, and of things placed in the Air, and not of the Resistance that is made in the Confines of the Air and water, and by things which are part in Air and part in water : and lastly, they make me feel with my hand, that when the thinne Plate is in the Air, and free from the weight of the water, it is raised with the very same Force that raiseth the Ball. Seeing, and understanding these things, I know not what to do, unless to grant my self convinced, and to thank such a Friend, for having made me to see that which I never till then observed : and, being advertised by this same Accident, to tell my Adversaries, that our Question is, whether a Board and a Ball of Ebony, equally go to the bottom in water, and not a Ball of Ebony and a Board of Ebony, joyned with another flat Body of Air : and, farthermore, that we speak of sinking, and not sinking to the bottom, in water, and not of that which happeneth in the Confines of the water and Air to Bodies that be part in the Air, and part in the water ; nor much less do we treat of the greater or lesser Force requisite in seperating this or that Body from the Air; not omitting to tell them, in the last place, that the Air doth resist, and gravitate downwards in the water, just so much as the water (if I may so speak) gravitates and resists upwards in the Air, and that the same Force is required to sinke a Bladder under water, that is full of Air, as to raise it in the Air, being full of water, removing the consideration of the weight of that Filme or Skinne, and considering the water and the Air only. And it is likewise true, that the same Force is required to sink a Cup or such like Vessell under water, whilst it is full of Air, as to raise it above the Superficies of the water, keeping it

it with the mouth downwards; whilst it is full of water, which is constrained in the same manner to follow the Cup which contains it, and to rise above the other water into the Region of the Air, as the Air is forced to follow the same Vessell under the Surface of the water, till that in this case the water, surmounting the brimme of the Cup, breaks in, driving thence the Air, and in that case, the said brimme coming out of the water, and arriving to the Confines of the Air, the water falls down, and the Air sub-enters to fill the cavity of the Cup: upon which ensues, that he no less transgresses the Articles of the *Convention*, who produceth a Plate conjoyned with much Air, to see if it descend to the bottom in water, then he that makes proof of the Resistance against Elevation in Air with a Plate of Lead, joyned with a like quantity of water.

I have said all that I could at present think of, to maintain the Assertion I have undertook. It remains, that I examine that which *Aristotle* hath writ of this matter towards the end of his Book *De Cælo*; wherein I shall note two things: the one that it being true as hath been demonstrated, that Figure hath nothing to do about the moving or not moving it self upwards or downwards, it seemes that *Aristotle* at his first falling upon this Speculation, was of the same opinion, as in my opinion may be collected from the examination of his words. 'Tis true, indeed, that in essaying afterwards to render a reason of such effect, as not having in my conceit hit upon the right, (which in the second place I will examine) it seems that he is brought to admit the largenesse of Figure, to be interessed in this operation. As to the first particuler, hear the precise words of *Aristotle*.

Aristotles opinion touching the Operation of Figure examined.

Aristot. de Cælo, Lib. 4. Cap. 6.

Figures are not the Causes of moving simply upwards or downwards, but of moving more slowly or swiftly, and by what means this comes to pass, it is not difficult to see.

Aristotle makes not Figure the cause of Motion absolutely, but of swift or slow motion,

Lib. 4. Cap. 6. Text. 42.

Here first I note, that the terms being four, which fall under the present consideration, namely, Motion, Rest, Slowly and Swiftly: And *Aristotle* naming Figures as Causes of Tardity and Velocity, excluding them from being the Cause of absolute and simple Motion, it seems necessary, that he exclude them on the other side, from being the Cause of Rest, so that his meaning is this. Figures are not the Causes of moving or not moving absolutely, but of moving quickly or slowly: and, here, if any should say the mind of *Aristotle* is to exclude Figures from being Causes of Motion, but yet not from being Causes of Rest, so that the sence would be to remove from Figures, there being the Causes of moving simply, but yet not there being Causes of Rest, I would demand, whether we ought with *Aristotle* to understand, that all Figures universally, are, in some manner, the causes of Rest in those Bodies, which otherwise would move, or else some particular Figures only, as for Example, broad

and

and thinne Figures : If all indifferently, then every Body shall rest : because every Body hath some Figure, which is false : but if some particular Figures only may be in some manner a Cause of Rest, as, for Example, the broad, then the others would be in some manner the Causes of Motion : for if from seeing some Bodies of a contracted Figure move, which after dilated into Plates rest, may be inferred, that the Amplitude of Figure hath a part in the Cause of that Rest; so from seeing such like Figures rest, which afterwards contracted move, it may with the same reason be affirmed, that the united and contracted Figure, hath a part in causing Motion, as the remover of that which impeded it : The which again is directly opposite to what *Aristotle* saith, namely, that Figures are not the Causes of Motion. Besides, if *Aristotle* had admitted and not excluded Figures from being Causes of not moving in some Bodies, which moulded into another Figure would move, he would have impertinently propounded in a dubitative manner, in the words immediately following, whence it is, that the large and thinne Plates of Lead or Iron, rest upon the water, since the Cause was apparent, namely, the Amplitude of Figure. Let us conclude, therefore, that the meaning of *Aristotle* in this place is to affirm, that Figures are not the Causes of absolutely moving or not moving, but only of moving swiftly or slowly : which we ought the rather to believe, in regard it is indeed a most true conceipt and opinion. Now the mind of *Aristotle* being such, and appearing by consequence, rather contrary at the first sight, then favourable to the assertion of the Oponents, it is necessary, that their Interpretation be not exactly the same with that, but such, as being in part understood by some of them, and in part by others, was set down : and it may easily be indeed so, being an Interpretation consonent to the sence of the more famous Interpretors, which is, that the Adverbe *Simply* or *Absolutely*, put in the Text, ought not to be joyned to the Verbe to *Move*, but with the Noun *Causes* : so that the purport of *Aristotles* words, is to affirm, That Figures are not the Causes absolutely of moving or not moving, but yet are Causes *Secundum quid*, *viz.* in some sort ; by which means, they are called Auxiliary and Concomitant Causes : and this Proposition is received and asserted as true by *Signor Buonamico Lib.* 5. *Cap.* 28. where he thus writes. *There are other Causes concomitant, by which some things float, and others sink, among which the Figures of Bodies hath the first place*, &c.

Concerning this Proposition, I meet with many doubts and difficulties, for which me thinks the words of *Aristotle* are not capable of such a construction and sence, and the difficulties are these.

First in the order and dispsure of the words of *Aristotle*, the particle *Simpliciter*, or if you will *absolutè*, is conjoyned with the Verb

to move, and seperated from the Noun *Causes*, the which is a great presumption in my favour, seeing that the writing and the Text saith, Figures are not the Cause of moving simply upwards or downwards, but of quicker or slower Motion : and, saith not; Figures are not simply the Causes of moving upwards or downwards, and when the words of a Text receive, transposed, a sence different from that which they sound, taken in the order wherein the Author disposeth them, 'it is not convenient to inverte them. And who will affirm that *Aristotle* desiring to write a Proposition, would dispose the words in such sort, that they should import a different, nay, a contrary sence? contrary, I say, because understood as they are written; they say, that Figures are not the Causes of Motion, but inverted, they say, that Figures are the Causes of Motion, &c.

Moreover, if the intent of *Aristotle* had been to say, that Figures are not simply the Causes of moving upwards or downwards, but only Causes *Secundum quid*, he would not have adjoyned those words, *but they are Causes of the more swift or slow Motion*; yea, the subjoining this would have been not only superfluous but false, for that the whole tenour of the Proposition would import thus much. Figures are not the absolute Causes of moving upwards or downwards, but are the absolute Cause of the swift or slow Motion; which is not true : because the primary Causes of greater or lesser Velocity, are by *Aristotle* in the 4th of his *Physicks*, *Text.* 71. attributed to the greater or lesser Gravity of Moveables, compared among themselves, and to the greater or lesser Resistance of the *Medium's*, depending on their greater or less Crassitude : and these are inserted by *Aristotle* as the primary Causes; and these two only are in that place nominated : and Figure comes afterwards to be considered, *Text.* 74. rather as an Instrumentall Cause of the force of the Gravity, the which divides either with the Figure, or with the *Impetus*; and, indeed, Figure by it self without the force of Gravity or Levity, would opperate nothing.

I adde, that if *Aristotle* had an opinion that Figure had been in some sort the Cause of moving or not moving, the inquisition which he makes immediately in a doubtfull manner, whence it comes, that a Plate of Lead flotes, would have been impertinent; for if but just before he had said, that Figure was in a certain sort the Cause of moving or not moving, he needed not to call in Question, by what Cause the Plate of Lead swims, and then ascribing the Cause to its Figure; and framing a discourse in this manner. Figure is a Cause *Secundum quid* of not sinking : but, now, if it be doubted, for what Cause a thin Plate of Lead goes not to the bottom; it shall be answered, that that proceeds from its Figure : a discourse which

which would be indecent in a Child, much more in *Aristotle*; For where is the occasion of doubting? And who sees not, that if *Aristotle* had held, that Figure was in some sort a Cause of Natation, he would without the least Hesitation have writ; That Figure is in a certain sort the Cause of Natation, and therefore the Plate of Lead in respect of its large and expatiated Figure swims; but if we take the proposition of *Aristotle* as I say, and as it is written, and as indeed it is true, the ensuing words come in very oppositely, as well in the introduction of swift and slow, as in the question, which very pertinently offers it self, and would say thus much.

Figures are not the Cause of moving or not moving simply upwards or downwards, but of moving more quickly or slowly: But if it be so, the Cause is doubtfull, whence it proceeds, that a Plate of Lead or of Iron broad and thin doth swim, &c. And the occasion of the doubt is obvious, because it seems at the first glance, that the Figure is the Cause of this Natation, since the same Lead, or a less quantity, but in another Figure, goes to the bottom, and we have already affirmed, that the Figure hath no share in this effect.

Lastly, if the intent of *Aristotle* in this place had been to say, that Figures, although not absolutely, are at least in some measure the Cause of moving or not moving: I would have it considered, that he names no less the Motion upwards, than the other downwards: and because in exemplifying it afterwards, he produceth no other Experiments than of a Plate of Lead, and Board of Ebony, Matters that of their own Nature go to the bottom, but by vertue (as our Adversaries say) of their Figure, rest afloat; it is fit that they should produce some other Experiment of those Matters, which by their Nature swims, but retained by their Figure rest at the bottom. But since this is impossible to be done, we conclude, that *Aristotle* in this place, hath not attributed any action to the Figure of simply moving or not moving.

But though he hath exquisitely Philosophiz'd, in investigating the solution of the doubts he proposeth, yet will I not undertake to maintain, rather various difficulties, that present themselves unto me, give me occasion of suspecting that he hath not entirely displaid unto us, the true Cause of the present Conclusion: which difficulties I willpropound one by one, ready to change opinion, when ever I am shewed, that the Truth is different from what I say; to the confession whereof I am much more inclinable than to contradiction.

Aristotle erred in affirming a Needle dimitted long wayes to sink.

Aristotle having propounded the Question, whence it proceeds, that broad Plates of Iron or Lead, float or swim; he addeth (as it were strengthening the occasion of doubting) forasmuch as other things, less, and less grave, be they round or long, as for instance a

Needle

Needle go to the bottom. Now I here doubt, or rather am certain, that a Needle put lightly upon the water, rests afloat, no less than the thin Plates of Iron or Lead. I cannot believe, albeit it hath been told me, that some to defend *Aristotle* should say, that he intends a Needle demitted not longwayes but endwayes, and with the Point downwards; nevertheless, not to leave them so much as this, though very weak refuge, and which in my judgement *Aristotle* himself would refuse, I say it ought to be understood, that the Needle must be demitted, according to the Dimension named by *Aristotle*, which is the length: because, if any other Dimension than that which is named, might or ought to be taken, I would say, that even the Plates of Iron and Lead, sink to the bottom, if they be put into the water edgewayes and not flatwayes. But because *Aristotle* saith, broad Figures go not to the bottom, it is to be understood, being demitted broadwayes: and, therefore, when he saith, long Figures as a Needle, albeit light, rest not afloat, it ought to be understood of them when demitted longwayes.

Moreover, to say that Aristotle *is to be understood of the Needle demitted with the Point downwards, is to father upon him a great impertinency; for in this place he saith, that little Particles of Lead or Iron, if they be round or long as a Needle, do sink to the bottome; so that by his Opinion, a Particle or small Grain of Iron cannot swim: and if he thus believed, what a great folly would it be to subjoyn, that neither would a Needle demitted endwayes swim? And what other is such a Needle, but many such like Graines accumulated one upon another? It was too unworthy of such a man to say, that one single Grain of Iron could not swim, and that neither can it swim, though you put a hundred more upon it.*

Lastly, either *Aristotle* believed, that a Needle demitted longwayes upon the water, would swim, or he believed that it would not swim: If he believed it would not swim, he might well speak as indeed he did; but if he believed and knew that it would float, why, together with the dubious Problem of the Natation of broad Figures, though of ponderous Matter, hath he not also introduced the Question; whence it proceeds, that even long and slender Figures, howbeit of Iron or Lead do swim? And the rather, for that the occasion of doubting seems greater in long and narrow Figures, than in broad and thin, as from *Aristotles* not having doubted of it, is manifested.

No lesser an inconvenience would they fasten upon *Aristotle*, who in his defence should say, that he means a Needle pretty thick, and not a small one; for take it for granted to be intended of a small one;

and it shall suffice to reply, that he believed that it would swim; and I will again charge him with having avoided a more wonderfull and intricate Probleme, and introduced the more facile and less wonderfull.

We say freely therefore, that *Aristotle* did hold, that only the broad Figure did swim, but the long and slender, such as a Needle, not. The which neverthelesse is false, as it is also false in round Bodies: because, as from what hath been predemonstrated, may be gathered, little Balls of Lead and Iron, do in like manner swim.

Aristotle affirmeth some Bodies volatile for their Minuity, Text. 42.

He proposeth likewise another Conclusion, which likewise seems different from the truth, and it is, That some things, by reason of their littleness fly in the Air, as the small dust of the Earth, and the thin leaves of beaten Gold: but in my Opinion, Experience shews us, that that happens not only in the Air, but also in the water, in which do descend, even those Particles or Atomes of Earth, that disturbe it, whose minuity is such, that they are not deservable, save only when they are many hundreds together. Therefore, the dust of the Earth, and beaten Gold, do not any way sustain themselves in the Air, but descend downwards, and only fly to and again in the same, when strong Windes raise them, or other agitations of the Air commove them: and this also happens in the commotion of the water, which raiseth its Sand from the bottom, and makes it muddy. But *Aristotle* cannot mean this impediment of the commotion, of which he makes no mention, nor names other than the lightness of such Minutiæ or Atomes, and the Resistance of the Crassitudes of the Water and Air, by which we see, that he speakes of a calme, and not disturbed and agitated Air: but in that case, neither Gold nor Earth, be they never so small, are sustained, but speedily descend.

Democritus placed the Cause of Natation in certain fiery Atomes.

Aristot. De Cœlo lib. 4. cap. 6. text. 43.

He passeth next to confute *Democritus*, which, by his Testimony would have it, that some Fiery Atomes, which continually ascend through the water, do spring upwards, and sustain those grave Bodies, which are very broad, and that the narrow descend to the bottom, for that but a small quantity of those Atomes, encounter and resist them.

Democritus confuted by *Aristotle*, text 43.

I say, *Aristotle* confutes this position, saying, that that should much more occurre in the Air, as the same *Democritus* instances against himself, but after he had moved the objection, he slightly resolves it, with saying, that those Corpuscles which ascend in the Air, make not their *Impetus* conjunctly. Here I will not say, that the reason alledged by *Democritus* is true, but I will only say, it seems in my judgement, that it is not wholly confuted by *Aristotle*, whilst he saith, that were it true, that the calid ascending Atomes, should sustain Bodies grave, but very broad, it would much more be done in the Air, than in Water, for that haply in the Opinion of *Aristotle*,

Aristotles confutation of *Democritus* refuted by the Author.

the

the said calid Atomes ascend with much greater Force and Velocity through the Air, than through the water. And if this be so, as I verily believe it is, the Objection of *Aristotle* in my judgement seems to give occasion of suspecting, that he may possibly be deceived in more than one particular: First, because those calid Atomes, (whether they be Fiery Corpuscles, or whether they be Exhalations, or in short, whatever other matter they be, that ascends upwards through the Air) cannot be believed to mount faster through Air, than through water: but rather on the contrary, they peradventure move more impetuously through the water, than through the Air, as hath been in part demonstrated above. And here I cannot finde the reason, why *Aristotle* seeing, that the descending Motion of the same Moveable, is more swift in Air, than in water, hath not advertised us, that from the contrary Motion, the contrary should necessarily follow; to wit, that it is more swift in the water, than in the Air: for since that the Moveable which descendeth, moves swifter through the Air, than through the water, if we should suppose its Gravity gradually to diminish, it would first become such, that descending swiftly through the Air, it would descend but slowly through the water: and then again, it might be such, that descending in the Air, it should ascend in the water: and being made yet less grave, it shall ascend swiftly through the water, and yet descend likewise through the Air: and in short, before it can begin to ascend, though but slowly through the Air, it shall ascend swiftly through the water: how then is it true, that ascending Moveables move swifter through the Air, than through the water?

That which hath made *Aristotle* believe, the Motion of Ascent to be swifter in Air, than in water, was first, the having referred the Causes of slow and quick, as well in the Motion of Ascent, as of Descent, only to the diversity of the Figures of the Moveable, and to the more or less Resistance of the greater or lesser Crassitude, or Rarity of the *Medium*; not regarding the comparison of the Excesses of the Gravities of the Moveables, and of the *Mediums*: the which notwithstanding, is the most principal point in this affair: for if the augmentation and diminution of the Tardity or Velocity, should have only respect to the Density or Rarity of the *Medium*, every Body that descends in Air, would descend in water: because whatever difference is found between the Crassitude of the water, and that of the Air, may well be found between the Velocity of the same Moveable in the Air, and some other Velocity: and this should be its proper Velocity in the water, which is absolutely false. The other occasion is, that he did believe, that like as there is a positive and intrinsecall Quality, whereby Elementary Bodies have a propension of moving towards the Centre of the Earth, so there is another like-

Lib. 4. Cap. 5.

wise intrinsecall, whereby some of those Bodies have an *Impetus* of flying the Centre, and moving upwards: by Vertue of which intrinsecall Principle, called by him Levity, the Moveables which have that same Motion more easily penetrate the more subtle *Medium*, than the more dense: but such a Proposition appears likewise uncertain, as I have above hinted in part, and as with Reasons and Experiments, I could demonstrate, did not the present Argument importune me, or could I dispatch it in few words.

The Objection therefore of *Aristotle* against *Democritus*, whilst he saith, that if the Fiery ascending Atomes should sustain Bodies grave, but of a distended Figure, it would be more observable in the Air than in the water, because such Corpuscles move swifter in that, than in this, is not good; yea the contrary would evene, for that they ascend more slowly through the Air: and, besides their moving slowly, they ascend, not united together, as in the water, but discontinue, and, as we say, scatter: And, therefore, as *Democritus* well replyes, resolving the instance they make not their push or *Impetus* conjunctly.

Aristotle, in the second place, deceives himself, whilst he will have the said grave Bodies to be more easily sustained by the said Fiery ascending Atomes in the Air than in the Water: not observing, that the said Bodies are much more grave in that, than in this, and that such a Body weighs ten pounds in the Air, which will not in the water weigh ½ an ounce; how can it then be more easily sustained in the Air, than in the Water?

Democritus confuted by the Authour.

Let us conclude, therefore, that *Democritus* hath in this particular better Philosophated than *Aristotle*. But yet will not I affirm, that *Democritus* hath reason'd rightly, but I rather say, that there is a manifest Experiment that overthrows his Reason, and this it is, That if it were true, that calid ascending Atomes should uphold a Body, that if they did not hinder, would go to the bottom, it would follow, that we may find a Matter very little superiour in Gravity to the water, the which being reduced into a Ball, or other contracted Figure, should go to the bottom, as encountring but few Fiery Atomes; and which being distended afterwards into a dilated and thin Plate, should come to be thrust upwards by the impulsion of a great Multitude of those Corpuscles, and at last carried to the very Surface of the water: which wee see not to happen; Experience shewing us, that a Body *v. gra.* of a Sphericall Figure, which very hardly, and with very great leasure goeth to the bottom, will rest there, and will also descend thither, being reduced into whatsoever other distended Figure. We must needs say then, either that in the water, there are no such ascending Fiery Atoms, or if that such there be, that they are not able to raise and lift up any Plate of a Matter,

that without them would go to the bottom : Of which two Positions, I esteem the second to be true, understanding it of water, constituted in its naturall Coldness. But if we take a Vessel of Glass, or Brass, or any other hard matter, full of cold water, within which is put a Solid of a flat or concave Figure, but that in Gravity exceeds the water so little, that it goes slowly to the bottom; I say, that putting some burning Coals under the said Vessel, as soon as the new Fiery Atomes shall have penetrated the substance of the Vessel, they shall without doubt, ascend through that of the water, and thrusting against the foresaid Solid, they shall drive it to the Superficies, and there detain it, as long as the incursions of the said Corpuscles shall last, which ceasing after the removall of the Fire, the Solid being abandoned by its supporters, shall return to the bottom.

But *Democritus* notes, that this Cause only takes place when we treat of raising and sustaining of Plates of Matters, but very little heavier than the water, or extreamly thin : but in Matters very grave, and of some thickness, as Plates of Lead or other Mettal, that same Effect wholly ceaseth : In Testimony of which, let's observe that such Plates, being raised by the Fiery Atomes, ascend through all the depth of the water, and stop at the Confines of the Air, still staying under water : but the Plates of the Opponents stay not, but only when they have their upper Superficies dry, nor is there any means to be used, that when they are within the water, they may not sink to the bottom. The cause, therefore, of the Supernatation of the things of which *Democritus* speaks is one, and that of the Supernatation of the things of which we speak is another. But, returning to *Aristotle*, methinks that he hath more weakly confuted *Democritus*, than *Democritus* himself hath done : For *Aristotle* having propounded the Objection which he maketh against him, and opposed him with saying, that if the calid ascendent Corpuscles were those that raised the thin Plate, much more then would such a Solid be raised and born upwards through the Air, it sheweth that the desire in *Aristotle* to detect *Democritus*, was predominate over the exquisiteness of Solid Philosophizing : which desire of his he hath discovered in other occasions, and that we may not digress too far from this place, in the Text precedent to this Chapter which we have in hand; where he attempts to confute the same *Democritus*, for that he, not contenting himself with names only, had essayed more particularly to declare what things Gravity and Levity were; that is, the Causes of descending and ascending, (and had introduced Repletion and Vacuity) ascribing this to Fire, by which it moves upwards, and that to the Earth, by which it descends; afterwards attributing to the Air more of Fire, and to the water more of Earth. But *Aristotle* desiring a positive Cause, even of ascending Motion, and not as *Plato*, or

Aristotle shews his desire of finding *Democritus* in an Error, to exceed that of discovering Truth.

Cap. 5. Text 41.

or these others, a simple negation, or privation, such as Vacuity would be in reference to Repletion, argueth against *Democritus* and saith: If it be true, as you suppose, then there shall be a great Mass of water, which shall have more of Fire, than a small Mass of Air, and a great Mass of Air, which shall have more of Earth than a little Mass of water, whereby it would ensue, that a great Mass of Air, should come more swiftly downwards, than a little quantity of water: But that is never in any case soever: Therefore *Democritus* discourseth erroneously.

Id. ibid.

But in my opinion, the Doctrine of *Democritus*, is not by this allegation overthrown, but if I erre not, the manner of *Aristotle* deduction either concludes not, or if it do conclude any thing, it may with equall force be restored against himself. *Democritus* will grant to *Aristotle*, that there may be a great Mass of Air taken, which contains more Earth, than a small quantity of water, but yet will deny, that such a Mass of Air, shall go faster downwards than a little water, and that for many reasons. First, because if the greater quantity of Earth, contained in the great Mass of Air, ought to cause a greater Velocity than a less quantity of Earth, contained in a little quantity of water, it would be necessary, first, that it were true, that a greater Mass of pure Earth, should move more swiftly than a less: But this is false, though *Aristotle* in many places affirms it to be true: because not the greater absolute, but the greater specificall Gravity, is the cause of greater Velocity: nor doth a Ball of Wood, weighing ten pounds, descend more swiftly than one weighing ten Ounces, and that is of the same Matter: but indeed a Bullet of Lead of four Ounces, descendeth more swiftly than a Ball of Wood of twenty Pounds: because the Lead is more grave *in specie* than the Wood. Therefore, its not necessary, that a great Mass of Air, by reason of the much Earth contained in it, do descend more swiftly than a little Mass of water, but on the contrary, any whatsoever Mass of water, shall move more swiftly than any other of Air, by reason the participation of the terrene parts *in specie* is greater in the water, than in the Air. Let us note, in the second place, how that in multiplying the Mass of the Air, we not only multiply that which is therein of terrene, but its Fire also: whence the Cause of ascending, no less encreaseth, by vertue of the Fire, than that of descending on the account of its multiplied Earth. It was requisite in increasing the greatness of the Air, to multiply that which it hath of terrene only, leaving its Fire in its first state, for then the terrene parts of the augmented Air, overcoming the terrene parts of the small quantity of water, it might with more probability have been pretended, that the great quantity of Air, ought to descend with a greater *Impetus*, than the little quantity of water.

The greater Specificall, not the greater absolute Gravity, is the Cause of Velocity.

Any Mass of water shal move more swiftly, than any of Air, and why.

Therefore,

Therefore, the Fallacy lyes more in the Discourse of *Aristotle*, than in that of *Democritus*, who with severall other Reasons might oppose *Aristotle*, and alledge; If it be true, that the extreame Elements be one simply grave, and the other simply light, and that the mean Elements participate of the one, and of the other Nature; but the Air more of Levity, and the water more of Gravity, then there shall be a great Mass of Air, whose Gravity shall exceed the Gravity of a little quantity of water; and therefore such a Mass of Air shall descend more swiftly than that little water: But that is never seen to occurr: Therefore its not true, that the mean Elements do participate of the one, and the other quality. This argument is fallacious, no less than the other against *Democritus*.

Lastly, *Aristotle* having said, that if the Position of *Democritus* were true, it would follow, that a great Mass of Air should move more swiftly than a small Mass of water, and afterwards subjoyned, that that is never seen in any Case: methinks others may become desirous to know of him in what place this should evene, which he deduceth against *Democritus*, and what Experiment teacheth us, that it never falls out so. To suppose to see it in the Element of water, or in that of the Air is vain, because neither doth water through water, nor Air through Air move, nor would they ever by any whatever participation others assign them, of Earth or of Fire: the Earth, in that it is not a Body fluid, and yielding to the mobility of other Bodies, is a most improper place and *Medium* for such an Experiment: *Vacuum*, according to the same *Aristotle* himself, there is none, and were there, nothing would move in it: there remains the Region of Fire, but being so far distant from us, what Experiment can assure us, or hath assertained *Aristotle* in such sort, that he should as of a thing most obvious to sence, affirm what he produceth in confutation of *Democritus*, to wit, that a great Mass of Air, is moved no swifter than a little one of water? But I will dwell no longer upon this matter, whereon I have spoke sufficiently: but leaving *Democritus*, I return to the Text of *Aristotle*, wherein he goes about to render the true reason, how it comes to pass, that the thin Plates of Iron or Lead do swim on the water; and, moreover, that Gold it self being beaten into thin Leaves, not only swims in water, but flyeth too and again in the Air. He supposeth that of Continualls, some are easily divisible, others not: and that of the easily divisible, some are more so, and some less: and these he affirms we should esteem the Causes. He addes that that is easily divisible, which is well terminated, and the more the more divisible, and that the Air is more so, than the water, and the water than the Earth. And, lastly, he supposeth that in each kind, the lesse quantity is easlyer divided and broken than the greater. *De Cœlo* l. 4. c. 6. t. 44.

Here

Here I note, that the Conclusions of *Aristotle* in generall are all true, but methinks, that he applyeth them to particulars, in which they have no place, as indeed they have in others, as for Example, Wax is more easily divisible than Lead, and Lead than Silver, inasmuch as Wax receives all the terms more easlier than Lead, and Lead than Silver. Its true, moreover, that a little quantity of Silver is easlier divided than a great Mass: and all these Propositions are true, because true it is, that in Silver, Lead and Wax, there is simply a Resistance against Division, and where there is the absolute, there is also the respective. But if as well in water as in Air, there be no Renitence against simple Division, how can we say, that the water is easlier divided than the Air? We know not how to extricate our selves from the Equivocation: whereupon I return to answer, that Resistance of absolute Division is one thing, and Resistance of Division made with such and such Velocity is another. But to produce Rest, and to abate the Motion, the Resistance of absolute Division is necessary; and the Resistance of speedy Division, causeth not Rest, but slowness of Motion. But that as well in the Air, as in water, there is no Resistance of simple Division, is manifest, for that there is not found any Solid Body which divides not the Air, and also the water: and that beaten Gold, or small dust, are not able to superate the Resistance of the Air, is contrary to that which Experience shews us, for we see Gold and Dust to go waving to and again in the Air, and at last to descend downwards, and to do the same in the water, if it be put therein, and separated from the Air. And, because, as I say, neither the water, nor the Air do resist simple Division, it cannot be said, that the water resists more than the Air. Nor let any object unto me, the Example of most light Bodies, as a Feather, or a little of the pith of Elder, or water-reed that divides the Air and not the water, and from this infer, that the Ait is easlier divisible than the water; for I say unto them, that if they do well observe, they shall see the same Body likewise divide the Continuity of the water, and submerge in part, and in such a part, as that so much water in Mass would weigh as much as the whole Solid. And if they shal yet persist in their doubt, that such a Solid sinks not through inability to divide the water, I will return them this reply, that if they put it under water, and then let it go, they shall see it divide the water, and presently ascend with no less celerity, than that with which it divided the Air in descending: so that to say that this Solid ascends in the Air, but that coming to the water, it ceaseth its Motion, and therefore the water is more difficult to be divided, concludes nothing: for I, on the contrary, will propose them a piece of Wood, or of Wax, which riseth from the bottom of the water, and easily divides its Resistance, which afterwards being arrived

Archimed. De Insident. humi lib. 2. prop. 1.

ved at the Air, stayeth there, and hardly toucheth it ; whence I may aswell say, that the water is more easier divided than the Air.

I will not on this occasion forbear to give warning of another fallacy of these persons, who attribute the reason of sinking or swimming to the greater or lesse Resistance of the Crassitude of the water against Division, making use of the example of an Egg, which in sweet water goeth to the bottom, but in salt water swims ; and alledging for the cause thereof, the faint Resistance of fresh water against Division, and the strong Resistance of salt water. But if I mistake not, from the same Experiment, we may aswell deduce the quite contrary; namely, that the fresh water is more dense, and the salt more tenuous and subtle, since an Egg from the bottom of salt water speedily ascends to the top, and divides its Resistance, which it cannot do in the fresh, in whose bottom it stays, being unable to rise upwards. Into such like perplexities, do false Principles Lead men: but he that rightly Philosophating, shall acknowledge the excesses of the Gravities of the Moveables and of the Mediums, to be the Causes of those effects, will say, that the Egg sinks to the bottom in fresh water, for that it is more grave than it, and swimeth in the salt, for that its less grave than it : and shall without any absurdity, very solidly establish his Conclusions.

Therefore the reason totally ceaseth, that *Aristotle* subjoyns in the Text saying ;The things, therefore, which have great breadth remain above, because they comprehend much, and that which is greater, is not easily divided. Such discoursing ceaseth, I say, because its not true, that there is in water or in Air any Resistance of Division ; besides that the Plate of Lead when it stays, hath already divided and penetrated the Crassitude of the water, and profounded it self ten or twelve times more than its own thickness: besides that such Resistance of Division, were it supposed to be in the water, could not rationally be affirmed to be more in its superiour parts than in the middle, and lower : but if there were any difference, the inferiour should be the more dense, so that the Plate would be no less unable to penetrate the lower, than the superiour parts of the water ; nevertheless we see that no sooner do we wet the superiour Superficies of the Board or thin Piece of Wood, but it precipitatly, and without any retension, descends to the bottom. Text 45.

I believe not after all this, that any (thinking perhaps thereby to defend *Aristotle*) will say, that it being true, that the much water resists more than the little, the said Board being put lower descendeth, because there remaineth a less Mass of water to be divided by it : because if after the having seen the same Board swim in four Inches of water, and also after that in the same to sink, he shall try the same Experiment upon a profundity of ten or twenty fathom water, he shall see the very self same effect. And here I will take occasion to

remember, for the removall of an Error that is too common; That that Ship or other whatsoever Body, that on the depth of an hundred or a thousand fathom, swims with submerging only six fathom of its own height, [*or in the Sea-dialect, that draws six fathom water*] shall swim in the same manner in water, that hath but six fathom and half an Inch of depth. Nor do I on the other side, think that it can be said, that the superiour parts of the water are the more dense, although a most grave Authour hath esteemed the superiour water in the Sea to be so, grounding his opinion upon its being more salt, than that at the bottom: but I doubt the Experiment, whether hitherto in taking the water from the bottom, the Observatour did not light upon some spring of fresh water there spouting up: but we plainly see on the contrary, the fresh Waters of Rivers to dilate themselves for some miles beyond their place of meeting with the salt water of the Sea, without descending in it, or mixing with it, unless by the intervention of some commotion or turbulency of the Windes.

A Ship that in 100 Fathome water draweth 6 Fathome, shall float in 6 Fathome and ½ an Inch of depth.

But returning to *Aristotle*, I say, that the breadth of Figure hath nothing to do in this business more or less, because the said Plate of Lead, or other Matter, cut into long Slices, swim neither more nor less; and the same shall the Slices do, being cut anew into little pieces, because its not the breadth but the thickness that operates in this business. I say farther, that in case it were really true, that the Renitence to Division were the proper Cause of swimming, the Figures more narrow and short, would much better swim than the more spacious and broad, so that augmenting the breadth of the Figure, the facility of supernatation will be deminished, and decreasing, that this will encrease.

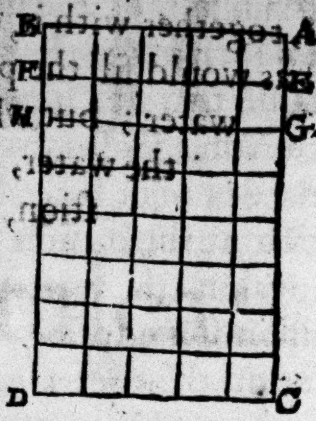
Thickness not breadth of Figure to be respected in Natation.

Were Renitence the cause of Natation, breadth of Figure would hinder the swiming of Bodies.

And for declaration of what I say, consider that when a thin Plate of Lead descends, dividing the water, the Division and discontinuation is made between the parts of the water, invironing the perimeter or Circumference of the said Plate, and according to the bigness greater or lesser of that circuit, it hath to divide a greater or lesser quantity of water, so that if the circuit, suppose of a Board, be ten Feet in sinking it flatways, it is to make the seperation and division, and to so speak, an incission upon ten Feet of water; and likewise a lesser Board that is four Feet in Perimeter, must make an incession of four Feet. This granted, he that hath any knowledge in Geometry, will comprehend, not only that a Board sawed in many long thin pieces, will much better float than when it was entire, but that all Figures, the more short and narrow they be, shall so much the better swim. Let the Board A B C D be, for Example, eight Palmes long, and five broad, its circuit shall be twenty six Palmes; and so many must the incession be, which it shall make in the water to descend therein: but if we do saw ir, as suppose into eight little

pieces

pieces, according to the Lines E F, G H, &c. making seven Segments, we must adde to the twenty six Palmes of the circuit of the whole Board, seventy others; whereupon the eight little pieces so cut and seperated, have to cut ninty six Palmes of water. And, if moreover, we cut each of the said pieces into five parts, reducing them into Squares, to the circuit of ninty six Palmes, with four cuts of eight Palmes apiece; we shall adde also sixty four Palmes, whereupon the said Squares, to descend in the water, must divide one hundred and sixty Palmes of water; but the Resistance is much greater than that of twenty six; therefore to the lesser Superficies, we shall reduce them, so much the more easily will they float: and the same will happen in all



other Figures, whose Superficies are simular amongst themselves, but different in bigness: because the [illegible] diminished or encreased, always diminish or encrease their Perimeters in subduple proportion; to wit, the Resistance that they find in penetrating the water; therefore the little pieces gradually swim, with more and more facility as their breadth is lessened.

This is manifest; for keeping still the same height of the Solid, with the same proportion as the Base encreaseth or deminisheth, doth the said Solid also encrease or diminish; whereupon the Solid more diminishing than the Circuit, the Cause of Submersion more diminisheth than the Cause of Natation: And on the contrary, the Solid more encreasing than the Circuit, the Cause of Submersion encreaseth more, that of Natation less.

And this may all be deduced out of the Doctrine of *Aristotle* against his own Doctrine.

Lastly, to that which we read in the latter part of the Text, that is to say, that we must compare the Gravity of the Moveable with the Resistance of the Medium against Division, because if the force of the Gravity exceed the Resistance of the *Medium*, the Moveable will descend, if not it will float. I need not make any other answer, but that which hath been already delivered; namely, that its not the Resistance of absolute Division, (which neither is in Water nor Air) but the Gravity of the *Medium* that must be compared with the Gravity of the Moveables; and if that of the *Medium* be greater, the Moveable shall not descend, nor so much as make a totall Submersion, but a partiall only: because in the place which it would occupy in the water, there must not remain a Body that weighs less than a like quantity of water: but if the Moveable be more grave, it shall descend to the bottom, and possess a place where it is more conformable Lib. 4. c. 6. Text 45.

 for

for it to remain, than another Body that is less grave. And this is the only true proper and absolute Cause of Natation and Submersion, so that nothing else hath part therein: and the Board of the Adversaries swimmeth, when it is conjoyned with as much Air, as, together with it, doth form a Body less grave than so much water as would fill the place that the said Compound occupyes in the water; but when they shall demit the simple Ebony into the water, according to the Tenour of our Question, it shall alwayes go to the bottom, though it were as thin as a Paper.

FINIS.

THE TROUBLESOME INVENTION OF Nicolas Tartalea:

BEING

A Generall way to recover from the bottome of the *Water*, any *SHIP* that's *Sunke*, Or any other *Ponderous Masse*, though it were a *Solid* TOWER *of Metal.*

TOGETHER WITH

An Artificiall way of DIVING, and staying a long time under *Water*, to seeke any thing *Sunke* in the greatest *DEPTHS*.

AS ALSO,

A SUPPLEMENT, Shewing a Generall and Secure Way to *Grapple*, &c. any *Submerged SHIP*.

Englished, By *THO. SALUSBURY*, Esq;

LONDON,

Printed by WILLIAM LEYBOURN, *Anno Dom.*

MDCLXIV.

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Printed by William Leybourn, Anno Dom. MDCLXIV.

To the most *Serene*, and most *Illustrious* Prince, FRANCESCO DONATO Duke of VENICE.

IT *having been told me here at* Brescia, *Most Serene and Most Illustrious Prince, that about ten years since, that a Ship full-laden did sinke near to* Malamoccho, *in about 5 Fathome of Water, and that to endeavour the recovering and getting it from thense, there had been used all those Means, and bountifull Offers and Tenders that could be imagined, aswel by the Illustrious Signory, for the Preservation of the Port, as by the chief Owners of the Ship and its Cargo: and that although there were many that had tried, and attempted the same, by sundry and divers wayes, of no small expence, and that it had been severall times well grappled and begirt, yet neverthelesse as far as I could hear, none of them were able to raise her from that small depth: And it being also told me, that of late there was another sunk again in less than four Fathome of Water, so that all its Poope and Prow, and a greate part of its Hull, was above Water, and that yet notwithstanding this also was judged by the fruitless Experiments and Expenses made about the former, to be irrecoverable, so that*

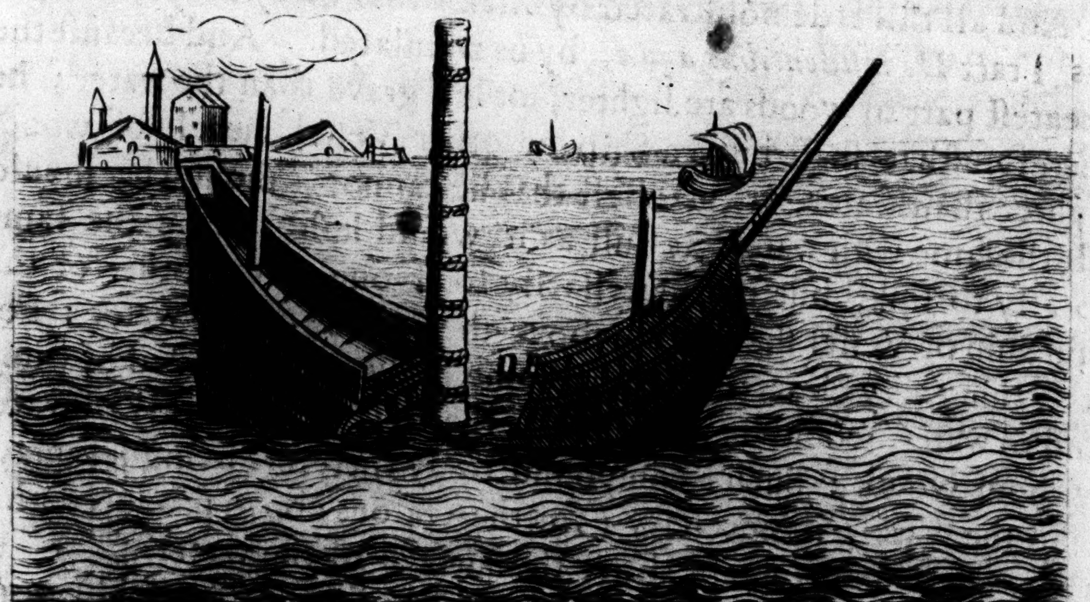
that for the clearing of the Port, it is presently resolved, that the said Ship should be broken up, & taken to pieces at low Water: and so, for ought that I hear, it hath been. Now I having considered of how great prejudice the breaking up of such a Vessel was, besides the loss of the Cargo, I deliberated about the finding of a way or Rule, that might remedy such detrimentall Occurrences: And having found out one thats generall and unquestionable, I thought fit, for the common benefit of this renowned City, to declare, and by Figures *to dilucidate the same in the present Tractate, and to offer and dedicate the same to your Highness; not as a present worthy of you (for indeed these Mechanicall Matters are exceeding disproportionate to your Highness Merits) but only with an Ambition to Enoble and Dignifie my Book with your Glorious Name; In confidence that like as the Sun doth not disdain that all sorts of Persons should make use of its light and heat, so neither will Your accustomed Humanity be offended with this my Presumption; and therefore I humbly lay my self at your Highness Feet,*

Nicolas Tartalea.

THE Industrious or Troublesome INVENTION OF Nicolas Tartalea:

BOOKE I.

The Figure of a Ship sunke according to the Relation made of that which was caused to be broken up neere Malamoccho, *as being judged irrecoverable.*



EXPLANATION I.

BEfore I come to declare the promised way to recover any laden or empty Ship when it is sunke; I think it convenient (*Most Serene and Illustrious Prince,*) first to declare the reall cause of its sinking.

Archimed. of Natation, Lib. 2. Prop. 1.

I say then; That its impossible that the water should wholly swallow or receive into it any materiall Body lighter than it self (as to species;) but it will leave or cause one part thereof to lie above the Superficies of the said water, that is uncovered by it. And as the whole Body demitted into the water, is to the part thereof, which shall be received or admitted by the water, so shall the Specificall Gravity of the water, be unto the Specificall Gravity of the said Solid Body.

Archimed. of Natation, Lib. 1. Prop. 7.

But those Solid Bodies which are more grave than the water; being demitted into the said water, suddenly make the water to give place; and not only enter wholly into the same, but they do go continually descending, till they arrive at the bottom: And they descend with so much greater Velocity, by how much they exceed the water in specificall Gravity

Archimed. of Natation, Lib. 1. Prop. III.

And those again which happen to be of the same Gravity with the water, of necessary consequence being put into it, are admitted and received totally into the same, but yet they stay in the Surface of the said water; that is, they suffer not any part to lie above the Superficies of the said water, nor much less doth the water consent to their descent to the bottom.

And all this is demonstrated by *Archimedes* of *Syracusa*, in that his Tract *De insidentibus aquæ*, by us translated. And because the greatest part of woods are lighter, or less grave than the water; he therefore that shall build a Ship or other Vessel meerly of wood, lighter than water, its manifest that he cannot (though he should fill the same with water, as full as it would hold) make the same totally to sink, but that necessarily some one part or other of the said Ship or Vessel shall stand above the Surface of the water: For its a thing very clear, that all that same Body, compounded of wood and of water, would be much lighter than if it were all only of water without wood: Such a compound Body therefore being less grave than the water, its necessary (for the reasons above produced) that a part of the same remain above the Surface of the water.

And if the said Ship or Bark shall be built (as it is usual) with Bolts, Nailes, and other Materials of Iron, and that such Ironworks be not of such quantity, as to make that Body compounded of wood and Iron, graver than the water, but that it continue still less grave than the water (as I judge all Ships and Barks to be;) The same will follow as did before, namely, that filling the said Ship with water, as full as is posible, it cannot by any means go to the bottom If then a Ship or other Vessel being wholly fill'd with water, cannot be thereby sunk to the bottom; It is a thing evident, that if such a Ship or Vessel shall be totally fill'd with a Matter lighter than the water; not only its totall sinking under that weight will

will be impossible, but also its floating in some part above the Surface of the water will be necessary : And so much the greater part shall be visible above the water, by how much the Matter of the Lading, is lighter than the water.

Therefore, if all the Cargo of a Ship (for instance) Buts of Oyl, and that no other Matters of a graver Nature than water were introduced, and that the said Ship should by some Accident be filled up with water, it is not only manifest that the Ship cannot be thereby sunk to the bottom, but that a part thereof must necessarily float above the Surface of the water : Because all that Composition of Wood, Water and Oyl, would be lighter than if it had been all simply of water. The very same would follow, if the Cargo had been soley of Wine, Wax, Camphor, Spices, or the like Matters, lighter than the water. But because the Merchandizes that fraight Ships, or other Vessels, are some (specifically) graver, and some (specifically) lighter than the water : (The graver are all sorts of Mettals, as Iron, Tinn, Lead, Brass, Copper, Silver, Gold, and infinite other Species of Commodities ; likewise the persons of Men, Stones, Ballasts, and the like :) And that also there are some sorts of Commodities that chance to differ very little in Gravity from the water : Therefore I conclude, that as oft as any Ship accidentally is fill'd with water, and so sinks by degrees to the bottom, it is necessary to grant that all the Composition, namely, of the Fraight, of the Vessel, and of the water that entered into it, is more grave, than if the composition had been all simply of water, by the reasons before alledg'd.

And therefore in such a case things graver than the water, must of necessity exceed in force those that be lighter : and by how much things graver than the water, exceed the lighter, so much the more Force will be required to recover such a Ship or other Vessel being sunk, and on the contrary, so much less Force will be required, when the Mass of the Materials more grave than the water, shall not differ much from the Mass of the less grave : provided the Recovery be undertaken in some short time after the Ship shall be sunk, For if the Ship lie many dayes under water, the delay will introduce many difficulties : One will be, that it will consolidate with and dock or work it self farther into the Mudd or Sand, which will not a little hinder its Recovery ; and again, the water will continually carry into the said Ship, Ouze, Mudd, and Sand, which Matter is much graver than the water, whereby the Ship is continually made graver as to the water, than it was at the beginning when it was first submerg'd. And moreover the corruptible Matters, which are by nature lighter than the water, will corrupt, and corrupting will change into other earthy substances much graver than the

water : insomuch that at the length, it ought to be presupposed in order to the recovery of the said Ship, as if it were solely laden with Mire, Dirt, and Sand : which doing, you will not be deceived in the operation, that is to say, preparing and working with a Force equivalent to that its Gravity. The way to know how to prepare Forces equivalent to the Gravity shall be shewn in the eight Explanation of this.

EXPLANATION II.

NOw to give beginning to the business proposed, I say, that in the Recovery of a Foundred Ship laden, or any other laden Vessel that is foundered or sunk, there interveneth more especially these three great Obstructions. The first difficulty is, how to imbreech and grapple it with such, and so many Ropes, as may suffice to bear it up; for if this either by ill chance cannot be done (whether through its being in a place two deep, or too far dockt in the Mudd or Sand) all our other labour will be frustrate and vain.

The second difficulty, when once it is grappled, is how with dexterity to seperate it from the bottom of the Sea; and this difficulty will be much greater, the Ship being in a Miry or Sandy bottom, than if it shall be in a Stony place; and it shall be also a greater difficulty to seperate it from a very deep bottom, than from a Shallow; (alwayes supposing that the two bottoms be both alike, namely, either both Stony or both Sandy;) and also far greater shall the said difficulty be in a Ship long sunk, than in one newly foundered; (as we have already said in the precedent Explanation:) But when she is once water-born, or seperated from the bottom, its an easie matter to raise her up to the Surface of the water; for then she shall not be a little aleviated in her Gravity : But the truth is, the drawing of it afterwards above the Superficies of the water, is no very easie matter, but is extream hard to be done; and this is the third difficulty; the principal cause of which two last difficulties shall be assigned by and by.

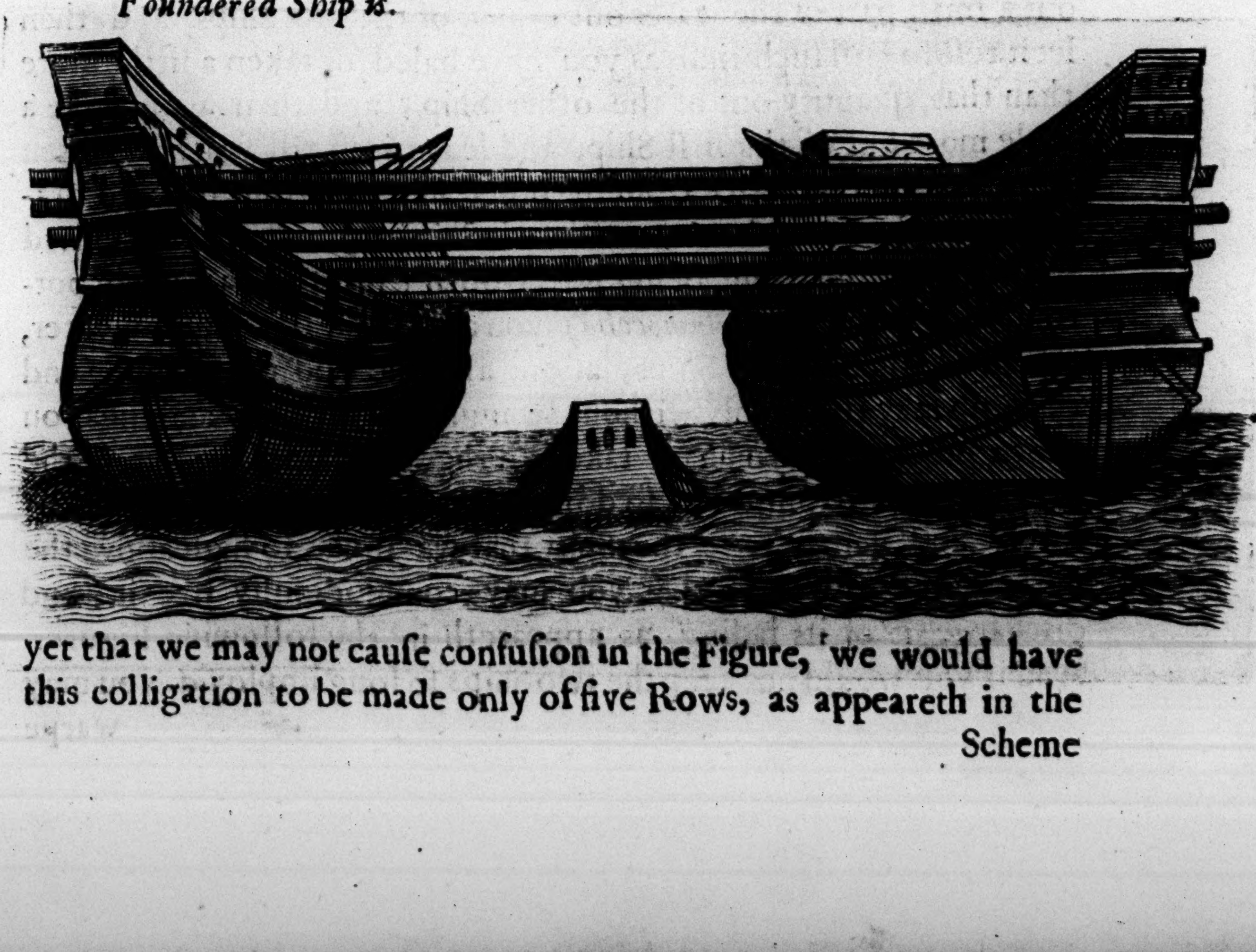
But because the means to obviate and superate the first difficulties as more * common, we shall forbear to speak of them untill the next Book. To provide, and that briefly, to the second and third impediments (which are the least known) that is, not only to seperate the Ship from the bottom, but to raise it also somewhat above the Surface of the water.

** The Author believed (as he declareth in the Epistle to the ensuing Suppliment of this his* Invention*) that the Mariners conversant in these affairs, had many wayes to imbreech a Vessel under water; and for that reason he over passeth it here, and is very cursive upon the same Point, in the second Book, , but giveth a generall Rule for it in the said Suppliment : to which the Reader is referred for fuller Satisfaction.*

And

And this is the Rule that you must observe; If the Ship be newly sunk, you must immediately, if it be possible, find two other Ships, that be each of them rather of greater bulk than the foundered Ship than less: and when you have found these two Ships, you must free them of all the inward and outward lading, and rigging, especially of those things which are by nature more grave than the water, as are the Guns, the Shot, and any kind of Ballast, which is presupposed to be in the Hold, and of other things of impediment; and when these Ships are thus cleared, you must stop all the Loop-holes, Cat-holes, Skuppers and Hauses, which you shall finde between or above Decks, graving and calking them so with Okum, and paying them with Pitch, that the water can neither get in nor out thereat. And next you must join or grapple these two Ships together with five or more Tires or Orders of thick and strong Beames tripplicated; that is, that each of the said Orders consist of three Beams, joyned lengthways; and that each of the three Beams be somewhat longer than the bredth of the Deck or Hull of each Ship; and that they be thick and strong, as being to support the Foundered Ship, as you shall see it made to appear presently: and couple the said Ships together, at such a distance from each other, that you give berth, or leave room enough betwixt for the foundered Ship to play; and you must make this couppling in such sort, that the length or side of the one Ship, look towards the length or side of the other; and albeit this conjunction or grappling may be made with many Orders or Tires of those Beams tripplicated lengthways, as was said above,

The Figurall representation of the two empty Ships, conjoyned with five Orders of Beams, and towed just over the place where the Foundered Ship is.



yet that we may not cause confusion in the Figure, we would have this colligation to be made only of five Rows, as appeareth in the Scheme

Scheme : and although the ſaid Rows of Beames cannot be all placed equidiſtant from the Surface of the water, for that the Wailes or Riſings of the two Ships are not fluſh, but cuved, it is not of any importance, ſo that they be well faſtened and ſtrengthened in thoſe places where they reſt upon the ſaid Riſings : upon which Riſings, you ſhall conjoyn the ſaid Beams, namely, the two ends of them, which two ends ſhall be the ſtrongeſt place, able to ſupport any great weight. Yet the truth is, that to fit theſe Tires of Beams, you need not have regard to make them paſs through from ſide to ſide, in that weak part of the Ships Poop and Prow, to reſt them on the Riſings or Gun-wales of the Deck of thoſe Ships, and to go croſs the Hull in thoſe places. And next you are to make upon theſe Beams, that is upon the mouths of both the Ships, a Plat-form of Planks for to ſtand upon whilſt you are about the work; leaving diverſe Scuttles or Spaces open, whereby to deſcend, aud for other uſes : And all this being done, you are to tow or hall theſe Veſſels to the place where the Ship is that did ſink, and to lay them Board and Board in ſuch faſhion, that the one may lie on one ſide of it, and the other upon the other, as in the Scheme is apparent.

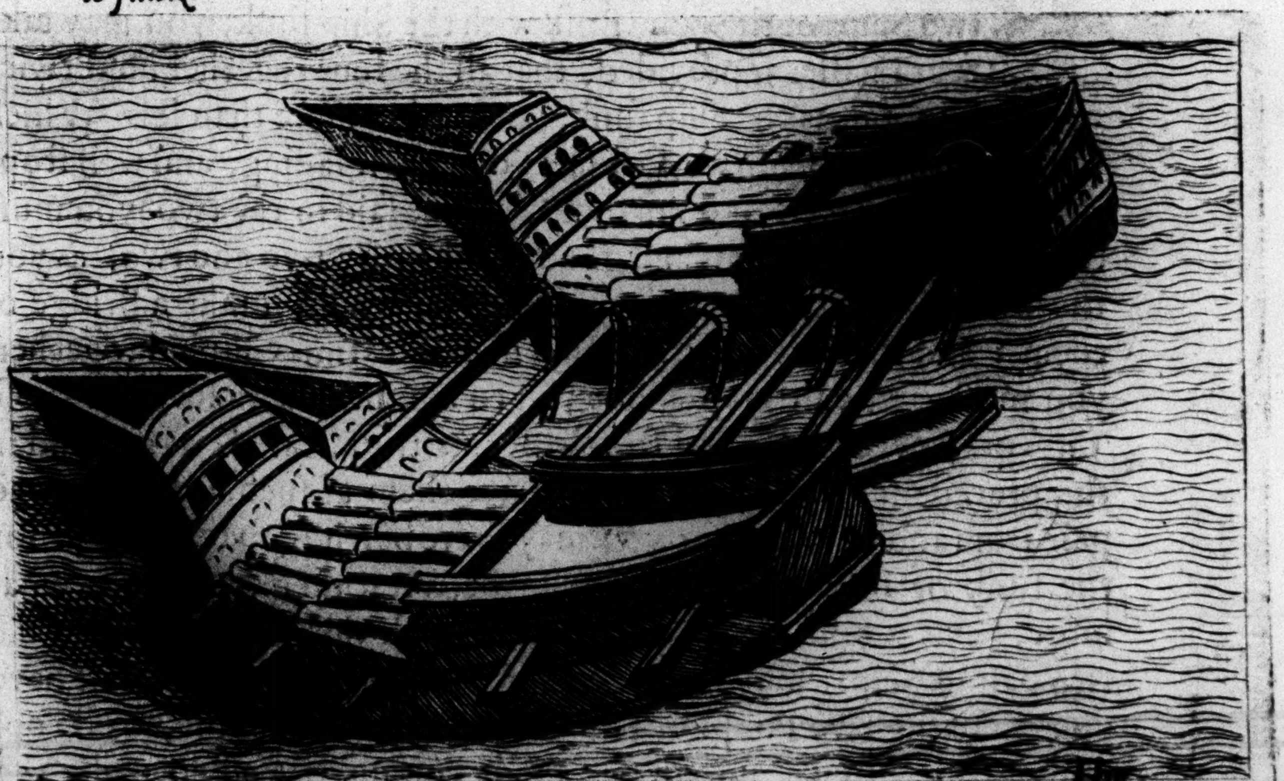
This being done, fill thoſe two Ships as full of water as they can hold or ſwim, (the way to free them with great facility and expedition, ſhall be ſhewn in the twelfth Explanation;) and being full, wait the time of low water ; that is, when the Tide returning, the Sea doth low as much as it can do ; and at that inſtant of time, make the Ship very faſt with thoſe ends of Cords or Cables (with which it was Swite or bound) to thoſe five, or more Tires of Beams, wherewith the foreſaid two Ships were imbreecht or grappled : And having well belayd or faſtned thoſe Cables, you muſt bale or take out a ſmall part of the water out of one of the two Ships, and then let it reſt ſo, till ſuch time as you have baled or taken a little more than that quantity out of the other Ship ; and then again take a little more out of the firſt Ship, and leave it ſo till you have taken another ſuch a quantity from the other Ship, and thus proceed gradually, till you find the Foundered Ship, water-born or looſned from the bottom : but being water-born (if it be in a Showle bottom, as was that at *Malamoccho*) you are to take out the ſaid water, equally from both the Ships, at one and the ſaid time, to the end the Ship may riſe evenly without ſwagging or ſhaking, and thus you are to proceed till you have taken all the water from the one & the other of the two Ships : In ſo doing, you ſhall ſee the two Shpis leaſurely and gently raiſe the Ship that was ſunk, ſo high above the Surface of the water, that you may commodiouſly free it, and diſcharge it of its lading, as appeareth in the following Figures. And if you would not keep the two Ships ſo long imploy'd, you may

warpe

warpe or towe the Foundered Ship at high-water to some place where it may lie a-ground : and by that means upon the Ebbe or Recession of the Tide, it will lie much more above water ; and then you may safely unfasten it from those five or more Tires of Beames, to which it was at first tyed, to hall it to a place of safety, as it was our purpose to do ; and this shall succeed as well in an ouzie bottom, as in a Stony, This though you may take notice of, that if the Cargo of this new Foundred Ship was such, that the things more grave than the water, did not much exceed the less grave, it would be easie to effect the recovery with two Ships, very much less than those which we have spoken of above ; yet neverthelesss it will be good prudence to take them rather bigger than lesser, that so they may exceed 200000 pounds in Power, rather than want one only ounce in Act ; especially in case you would in a deep place at the first motion hoist it by meer Force somewhat above the Surface of the water, for in that point alone it will require incomparably much more force, than in all the other operations.

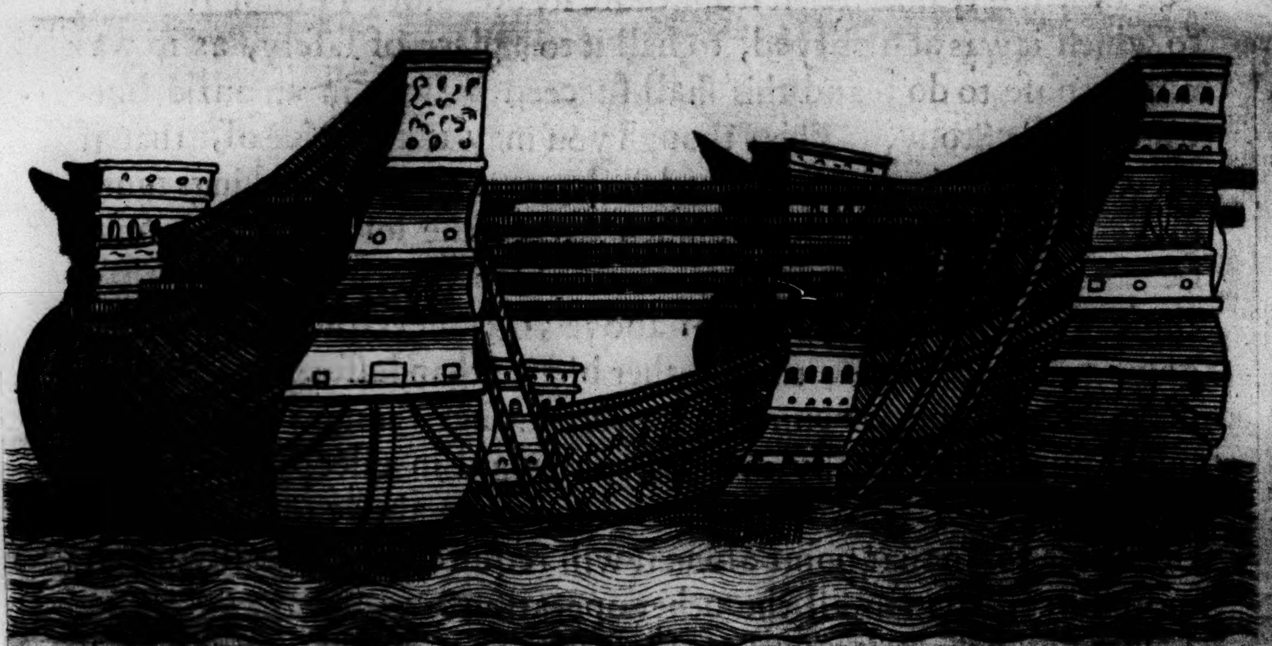
How you are to preceed, in case the Ship should be sunk in a place very deep, shall be declared in the seaventh Explanation. The Figures of this Explanation are these two that folllow.

The Figure of the two Ships filled with water, to raise the Ship that is sunk.



His

The Figure of the two Ships emptied as they lie, with the other Ship raiſed up above water.

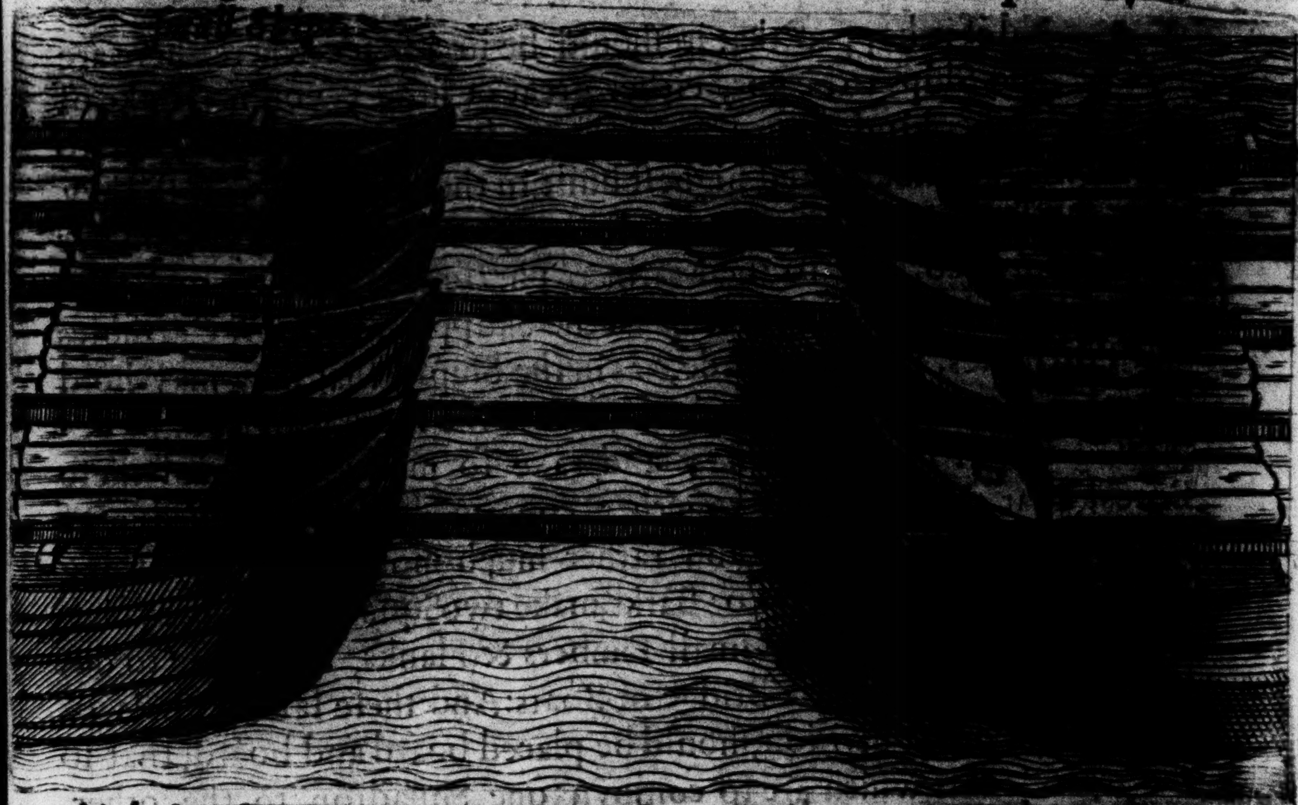


EXPLANATION III.

BUt if it ſo fall out, that you cannot on ſnch an inſtant, finde two Ships of the ſame Bulk with the Ship ſunk, you may take four ſmaller; provided, that all the four together hold twice as much burden as the Ship ſunk, and rather more than leſs. Which four ſmall Ships being all firſt cleer'd of their lading, and well ſtopt in all their Skuppers and Portholes (as was ſaid in the two) you muſt couple them with Beams and good Planks, by two and two, as you uſe to do with two Lighters, when you would make a Bridge of them: and theſe two pair of Hoys or Barkes thus coupled together, you muſt afterwards faſten one pair to another, with ſeven of thoſe Tires or Rows of thick and ſtrong Beams tripplicated, as was ſaid in the precedent Explanation; and place them at ſuch a diſtance one pair from another, as that you may leave berth or ſpace enough for the ſunk or foundered Ship to riſe between them, and ſomewhat more, (as was ſaid of the two.) And though this conjunction of the two pair of Ships, may be made three ſeverall wayes, yet I will have you make the two Poops or Hin decks of the one couple, to lie oppoſite to the two Poops of the other couple. And to make this conjunction, you are to place two Tires of thoſe great Beams along the upper parts of the ſaid Poops, ſo, that they may reſt in the inſide on thoſe leſſer Beams and Planks, wherewith each of thoſe two pair of Ships were coupled: and each of theſe Orders or Tires of Beames

Beames ought to be composed of three Beams conjoyned lengthwayes, as was said in the precedent Explanation; and make two of the Tires lie upon the Ships; and to those Tires, let that sunk Ship be grappled : and another Tire of the said Beams is to be placed in the midst between the one and the other couple; and two other Tires of the said Beams ought to be fastened upon the one and other side; that is, upon the Risings or Bends of those two couples of Ships; and that being done, there will be in all seven Tires or Orders of Beams, which seaven Orders of Beams ought conjunctly to be prolonged, on the one and on the other side, almost to the length of the Hull of each Ship, as in the Figure is represented : and

The Figurall Example how to recover a Foundered Ship with four small Ships.



this being done, you are to proceed, as hath been shewn in the two; that is, fill them top-full of water, and at low water, imbreech the Ship sunk very well, withall those ends of Ropes or Cables, that you did belay to those seven Tires of Beams : and when those Grapplings shall be well made fast, you shall at high water bale or free the water by little and little out of the Ships, one pair after another, till you feel the foundered Ship is disengaged from the bottom, and water-born, as was said in the two. And having seperated it from the bottom (if it be in a shallow place, as was that where the Ship was foundered neer *Malamoccho*) you are to proceed to let out the rest of the said water, but take it equally and gradually from the one and the other pair, that they may descend evenly, and without heeling, as was said of the two; and in so doing, the said Ship shall not only be hoisted up to the Surface of the water, but much

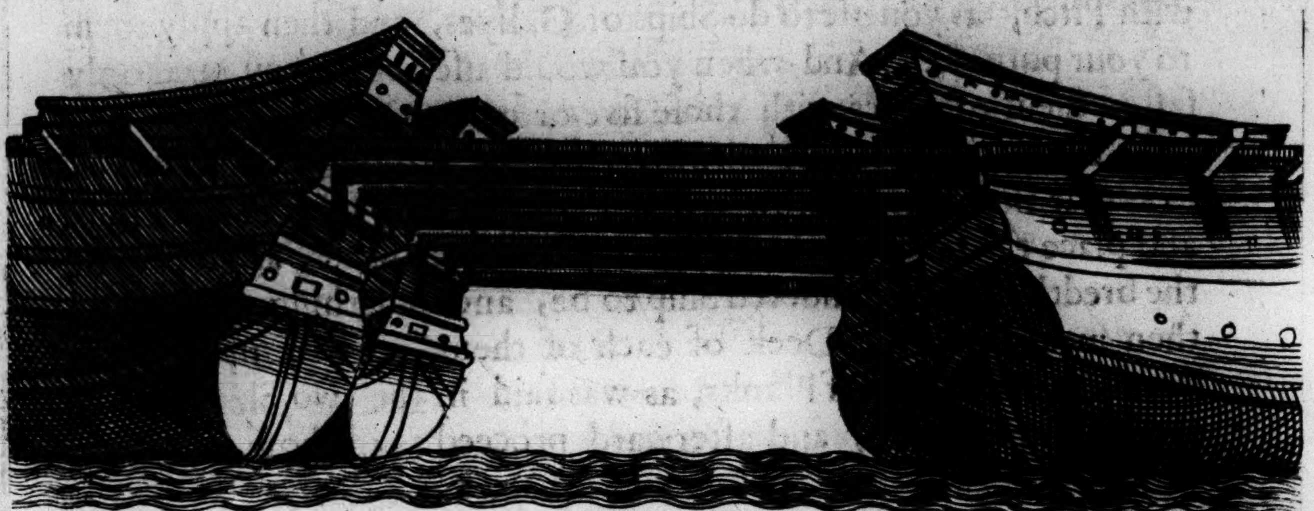
above the ſame; ſo that you may in that poſture free or drain it and diſcharge it of the Cargo. But if you cannot ſo long ſpare thoſe four Ships from other uſes, then you may at high-water tow it to ſome place, where running it on ground, you may at the ebbe of the Tide (for that then there will lie much more of it above water) ſafely looſe it from thoſe Beames, as was alſo ſaid in the precedent Explanation of the two Ships.

But in caſe the Foundered Ship ſhould chance to be in a very deep Sea, in the ſeventh Explanation (to be the briefer in this place) ſhall be ſhewn how you are to proceed.

EXPLANATION IV.

AND if it happen that it ſhould be in a place where there are no Ships to be got, either great or little; you may take of other kind of Pinaces, Barks or Barges, but endeavour to get ſuch as are floaty, and higheſt built in there Riſings, that ſo they may, at ſuch time as they are full of water, deſcend very far under water, (or according to the Mariners phraſe, may draw much water) and of theſe you muſt ſtop all the Skuppers, Hawſes, Cat-holes and Port-holes, that you finde, as in the Ships, that they may hold the more water, and conſequently draw the more water, or be depreſſed deeper into the ſame; and take ſo many couple of theſe Botes, that they may all together contain double the burden of the Ship to be recovered, and rather much more, than any thing leſs. And of all theſe Boats or Barks, make two Squadrons, conjoyning each Squadron with good ſmall Timbers & Planks, as you uſe to do, when you would make a Bridge of Boats: And theſe ſame Veſſels of the one and other diviſion, ſhould be placed board and board, that ſo the great Beams, which are to conjoyn one Squadron to the other, may bear upon the Riſings, Bends or Wales, of the ſaid Veſſels. And this being done, you are to couple theſe two Squadrons, to each other with thoſe thick and ſtrong Tires of Beams, mentioned in the former Explanations, which Orders of Beams ſhould be fixed between two & two of thoſe Botes, as is ſaid above, to the end, that they may bear or reſt upon the Bends of thoſe Boats; and place another Tire upon the outſides of both the Diviſions, upon the ends of the croſs ſmall Beams which hold the ſeverall Veſſels together: So that if the Squadrons conſiſted each of four Barks, the Tires of the ſaid Beams would come to be five, ; and if there ſhould be five in a Squadron, the Tires of Beams would be ſix, and ſo forwards; that is, the Orders of Beams, by this means, ſhall be alwayes one more than the number of Botes in each Squadron. But in the Ships you muſt obſerve another method, becauſe of thoſe two Orders, which are placed in each Poop; by which

The way to recover a Foundered Ship with many Barks or Wherryes.



which means in every two Ships to a Division (which in all make four Ships) there must be seven Orders of Beams, and in three Ships to a Squadron, there must be ten Orders of Beams, and in four Ships to a Squadron thirteen; and thus proceeding forwards to a greater number of Ships in a Squadron. And having understood the way of coupling many Barks or Wherryes in Squadrons; as also the manner how to joyn or fasten them to each other, and with how many Orders of Beams; you are to proceed in the rest, as in the precedent Explanations hath been demonstrated in showle bottoms, but the directions how to manage this affair in deep places, shall be declared in the seventh Explanation.

EXPLANATION V.

To remove this inconvenience of taking Ships or other Vessels; and of standing to lighten them of their Guns & lading, and of stopping their Loop-holes; you may in such a misfortune cause to be made two great Vessels, almost in form of * Chests without covers, the length of each to be equal to the Hull of a middle rate Ship, and the breadth equall to that of the same Ship at the Main-mast, and the height also the same with that of the Ship in the Bow, so that each of these Plat-forms or Chests, shall hold much more than a common Ship, and thus both will contain more than the double burden of such a Ship. And for the making of these Vessels, you must first make the Models in Carvel-manner of thick and strong Timber, with their Entertaces, Transomes and Knees, to hold their sides and ends together: and this done, spike down to them certain

*Of these Vessels Cardinall *Richlein* made use at the Siege of *Rochell* to shut up the Haven.

thick and strong Planks; and then cause them to be well graved and calked in the Seames or Strakes by a Calker, with Okum, and paid with Pitch, as you use to do Ships or Gallyes, and then apply them to your purpose. And when you would use them, you need only fasten them together with those five or more Orders of thick and lusty Beams, trippled lengthwayes, that is, prolonged both wayes, so as that they may lie athwart the Decks of the said two Vessels, and place the said Ships so far distant from each other, as you guesse the bredth of the Foundered Ship to be, and something more: And then make upon the Deck of each of them, that is, upon those Beams, a Plat-form of Planks, as was said in the two Ships of the second Explanation, and afterwards proceed as in those two Ships.

EXPLANATION VI.

AND incase you think the making of a couple of such great Modles or Vessels, as we mentioned in the foregoing Explanation, would be too great a trouble or expence; you may make two pair of such Chests, each of them but of half the bulk of one of the former: but if you judge these two pair too troublesome, you may make three, four, or more pairs; alwayes provided, that amongst them all they hold about twise the burden of the Ship sunk; and these Frames when you would use them, must be joyned together in two Ranks, with lesser Beams and Planks, as was said of the four Boats or Wherryes; and then fasten these two Ranks to each other at the requisite distance, with great and strong tripplicated Beams, as was said of the Ships, Barks and Boats; and then operate as you was to do with those: alwayes remembring in the freeing or emptying the said Vessels, to bale out the water by little and little first from one Rank, and then from the other; and so proceed interchangeably till you percieve that the Ship is clear of the bottom: and being disengaged, if it be in a shallow place, continue taking the water equally out of the one and other Division of Vessels, till all the water be drained out of them, as was required upon the former Explanations: but if it be sunk in a deep Sea, the next Explanation shall shew how you are to proceed; and that briefly.

EXPLANATION VII.

AND in case the said Ship newly sunk, chance to be in a very deep bottom; It will be necessary first to fix upon those two or four Ships, or upon those two Squadrons of Barks, Fly-boats or Wherryes, at least six or eight Capstains, Ship-Cranes or

or Windlasses, with their necessary Garnets or Pullies, requisite to such a weight: and you may easily accomodate these Pullies, to those Orders of great Beams, wherewith the said Vessels were conjoyned. And having prepared these Capstains, you are to proceed in all things, as hath been directed you in the precedent Explanations, excepting only in this, that whilst you are freeing the water alternately by degrees out of the two or more Ships, or from the two Squadrons of Barks, Fly-boats or Wherryes, as soon as you finde the Foundered Ship to be water-born or got clear of the bottom of the Sea, I would have you cease to take any more water forth of the said Ships, or lesser Vessels before filled; and I would have you with those Capstains, attempt to draw the said Ship that was sunk unto the Levell or Surface of the water, or to lie Horizontal unto it, which may be easily done, for that its ponderosity will be much diminished. And when you have drawn it to the Surface of the water, then I would have you discharge all the other water out of the two Ships, or the two Squadrons of small Vessels. And this second water, I would have raken equally, and at the same time, from the one and the other Ship, or from each Rank of Barks or Boats, as hath been said of the other. And thus those Ships or Squadrons of Boats shall hoist the said Foundered Ship, so high above the Superficies of the water, that you may free it of the water which was got into it, and unlade its Cargo, which was our purpose.

You must note, that all that hath been hitherto said of a Ship newly sunk, ought to be understood of all other kind of Foundered Ships, proceeding alwayes proportionately as was directed in that Ship. And again, I give you no Figure how you are to fit and fix the Capstains and Pullies, as being a thing common and manifest.

EXPLANATION VIII.

But if it so fall out, that the said Ship or other Vessel hath been sunk many Months; albeit that there might have been many matters in the Cargo of a lighter nature than water, yet you must suppose the case as if the Ship were as heavy as if it had been fil'd with Sand or Gravel; yea and much heavier, for many Reasons, as hath been alledg'd in the first Explanation. Therefore that you may not deceive your selves in the designed recovering of it, you would do well to double the Forces required to the recovery of a new sunk Ship; that is, you must take four Ships, each as big as the Foundered Ship, and combine these four Ships, as you were required to joyn the four small Ships in the third Explanation. And if you cannot procure them of that burthen, take eight lesser, provided that altogether they be quadruple in contence to the Ship to

be

to be recovered : and divide these eight lesser Ships or Barks into two Squadrons, of four in a Squadron, according as you was directed in the four Ships in the third direction. And if you cannot procure Ships great or smal, take so many pair of other Vessels, Fly-boats or Wherryes, that in all they may at least contain four times the burthen of the Foundered Ship : And reduce these Barks, Boats or Wherryes into two Divisions, as you are taught in the fourth Explanation : and in all other particulars, proceed according to the method prescribed in the recovery of the Ship newly sunk; and that as well in deep, as shallow places; that is, placing in a deep Sea upon the said Ships, or Squadrons of Boats, at least twelve or sixteen Capstains, which it will be easie to do, for that you will have a large space upon those Ships or ranks of Boats, as also there will not want room to fasten their Pullies to those Tires of Beams, which combine the said Ships or ranks of Boats. In all things else proceed precisely according as you have been directed in the second, third, fourth, fifth, sixth and seventh Explanations.

This indeed must be granted, that incase the said Ship long sunk, should be in a Stony bottom, or where she hath a great current, the which current suffereth not any great bed or shelves of Mudd to gather about the said Ship, it may then easily be got clear of the bottom, with the same Forces as were imploy'd in that newly sunk, to recover it; and also may as easily be drawn to the Surface of the water : But whether you can raise it with part of its Hull above the Superficies of the water, is a thing much to be doubted; yet if it should prove so upon the Experiment, namely, that you cannot elevate its Hull above the Surface of the water, you may in such a case hall it at high water to shore, or to some place where it may lie a-ground, whereby at the retreat of the Tide, it will lye with part of its Hull above water, so that you may commodiously clear it of the imbibed water and Cargo.

EXPLANATION. IX.

AND to the end that this invention may be of generall use for the recovery or raising any kind of Collossus, that may happen to be sunk, to wit, of all Species of Solid Bodies, whether of Stone, Iron, Pewter, Brass, Lead, Silver or Gold (as you may have many occasions voluntarily to sink them in time of war, to preserve them) and then that you may know how to get them up again, you must observe this Rule : If the Solid long time submerged were of Brick; so soon as it is imbreecht, you must take so many couple of Ships, Barks, Hoyes or Wherryes, that the sum of their contents put together, may exceed the Square of the Solid Area of the submerged Solid : and if the Solid so long sunk were of Marble

ble, the Solid Content of all the *Vacua* of those Ships or Vessels added together, must not be less than Septuple to the Solid Content of the submerged Body; namely, seven times as much. And if that long sunk Solid chance to be of Iron; you must make the Solid Content of all the *Vacuum's* of those Vessels to be no lesse in the Aggregate than 12 $\frac{3}{4}$ times as much as the Solid Content of that submerged Solid: and the like must be done, if the submerged Solid be of Pewter, for that Iron and Pewter differ not much in Gravity. But and if the drowned Solid be of Copper, it is requisite that the Solid Content of all the Vessels Cavities in sum, be no less than thirteen times as much as the Solid Content of the said Solid sunk. And if the submerged Solid were of Lead, the Solid Content of all the *Vacua* of those Ships, wherewith you would recover it, should be no less than twenty times as much as the Solid content of the drowned Solid, and rather more than less; and almost the same proportion ought to be observed, if the submerged Solid were of fine Silver, for that Lead and pure Silver differ not much in Gravity: truth is, that Lead is somewhat more weighty than Silver, but not much.

But if the Solid which was sunk, should chance to be of pure Gold, you must for its recovery take so many couple of Barks or Boats, that the Solid Content of their *Vacua*, taken in aggregate, may be no less than 34 times as much as the Solid content of the said Golden Solid submerged. And that you may the better understand me, I will put an Example, that you were to recover or raise out of the water, a Solid Body resembling a great Tower, which I imagine to be in length an 100 Paces, and in breadth 10, and in thickness also ten: and I suppose that it is all one Solid, that is to say, not hollow within. And first we put the case that this Tower were made of Brick. Now because the Solid Content of this supposed Solid would be 10000 cubical Paces: therefore in this case, if you would recover this same Body, that is, not only loosen it from the bottom of the Sea, but also raise it a good height above water, it will be requisite, as is said above, to take so many pair of Ships, Barks, Boats, or other Vessels, (as hath been shewn in the 5 and 6. Explanation) that the Solid Content of all the *Vacua* of them put together, be not less than four times the said sum of 10000 cubick Paces; that is, it must not be under 40000 cubicall Paces, as was above determined. And so if it happen that the said submerged Solid should be all of Marble, the Solid Content of all the Vacuities of the said Ships, ought not to be less than 70000 cubicall Paces, namely Septuple, as was before concluded. And thus if the sunk Solid were all of Iron or Pewter, the aggregate of all the Solid Content of all those *Vacuums* put together, must be rather more than

less

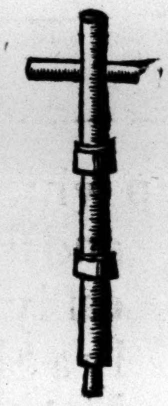
less then 126666⅓ cubical Paces. And in case the Solid were all of Copper, the Solid Content of the said *Vacua* ought to be about 130000 cubick Paces. And likewise if the Solid were all of Lead or Silver, the Solid Content of all the said *Vacua* is to be no less than 200000 Paces cubical. Lastly, if such submerged Solid be propounded all of fine Gold, the sum of those Cavities ought to be no less than 340000 cubick Paces.

The manner how to proceed in the recovery of those severall kinds of Solids, is to be understood to be like to that which was prescribed in the recovery of the Ship: and that as well in deep, as shallow waters. And the greater number of Ships or Boats are required to opperate in the recovery of the said submerged Solid in a deep Channell, so much the more room must yon take upon the one and the other Squadron, for to be able to pitch such a number of Capstens as shall be needfull, and more if occasion be. Yet you must observe, that in the taking the water alternately from the one and other Squadron; when you perceive the said Solid to be disengaged from the bottom, you are to forbear taking out any more from either of them; as was appointed touching the Ship, in the seventh Explanation. And make use of as many Pullies as you shall see cause for, not only to lift it to, but also to draw it above the waters Surface: and that if notwholly, yet for the greater part: and when it is lifted as high as is posible, then take the remaining water by equall measures, out of the one and other Squadron, or Rank of Ships; which being done, it shall be hoisted so high out of the water, that you may put under it as many Lighters or Flat-boats, as shall be sufficient to bear it up, and to carry it to any place, as occasion shall require.

EXPLANATION X.

ALbeit *Vitruvius*, *Vegetius* and *Valturius* do teach diverse and sundry wayes to carry water up on high; many whereof may stand us in much stead in this our Invention, for the commodious filling and emptying all the severall kinds of Vessels spoken of above; of which also, many are very well known and familiar to every one; to wit, with Bur-pumps, Chain-pumps, common-pumps, and many others: yet neverthelesse to fill the said Ships or other Vessels with water, with great facility and dexterity; I judge this more expedient than any of them; namely, to make a Hole in the bottom of each of those Ships or other Vessels, of two or three inches Diameter at least, and for every Ship to appoint a Boome or long tapered Pole like a Plugg or Tapp, so that being thrust into the said Hole, it will stop it so close, that unless you consent thereto, no water

water can enter in thereat, and this Pole is to be somewhat longer than to reach from the Keel to the upper-deck of the said Ship; and near the other end, put another piece of a Pole cross wayes; that you may be able by means of that to rule it; namely, to pull it up, when you would unstop the Hole, to let in the water that should fill the Ship, and to thrust it down when you would stop the Hole that no more water may enter; and this same Pole should pass through two Rings, fixed in the Hold of the Ship, which are to keep the said Pole directly over the Hole, that if you would stop it, the Plugg or Spiggot may not go besides the Hole, when you thrust the Pole downwards. And that I may be the better understood, I have here below drawn the same Pole, with its Tapp or Plugg at the end. And when you go about to recover any Ship, you must stop the said Holes, till such time as the said Ships are carried and fitted upon the place, as is shewn above. And when you would fill them with water, it is but withdrawing the said Poles, and opening the Holes; and fasten them at that stay, till you have a mind to stop the Holes; and then look downwards, and observe when the Ships are as full as they can swim, or when they are full enough, which will be in a very short time: and then let down those Poles, and stop the Holes very close. And when they are as full as they need, in the ebb of the Tide, combine the Ship with the Pullies, to those five or more Orders of Beams often mentioned: and then draw out the water with Pumps by little and little, and one while out of one, and another while out of the other Ship, as was appointed in the second Explication: and in all other particulars proceed, as was also there directed. But if the Gravity of those Vessels, causeth them not to fill fast enough, you must fill them at the top, that is by baling in water by the Deck (I mean the said Poles being first thrust down) to make the said Vessels to descend faster, and to raise the Matter submerged with more Force; many other new wayes might be shewn, as well to empty, as to fill these Vessels; but for the present this shall suffice.



EXPLANATION XI.

If you would attempt to recover a Ship or other Vessel by the wayes here prescribed: you must go about the same, when the Moon is in the Auge of the Excentrick, for at that time the Sea ebbeth and floweth more than at any other time in the Moneth; and this happens in her Conjunction and Opposition, which is a matter of great avail in these operations: and herewith we conclude this our first Book.

i. e. At a spring-tide, which is greatest the third day after the full and change.

 The

THE

Industrious or Troublesome

INVENTION

OF

Nicholaus Tartalea:

BOOKE II.

In which are taught, some artificial wayes of *Diving* and staying long under Water: whereby one may easily descend to the Bottom, to finde out, not only a Ship sunke, but also, any other small thing of Value: And the place being darke, many wayes are shewn how to enlighten it: And the thing sunk being found, severall wayes and means are prescribed how to imbreach them, as well in a Deepe, as Shallow Channel.

EXPLANATION I.

HAving understood, *Most Serene Prince*, from sundry Sea-men, that there are many now adayes, who without any particular Artifice or help, do upon occasion dive and continue a long time under Water, and in places very deep; I had thought to have added nothing touching the way of Artificiall Diving, and staying under water, to seeke and finde out a Ship, Boate, or other thing of Value submerged, and that for two Reasons. First, Fearing that I should be derided by those kinde of men, it being to them a superfluous thing to go about to do those things by Art, which they know how to execute without any artificiall help.

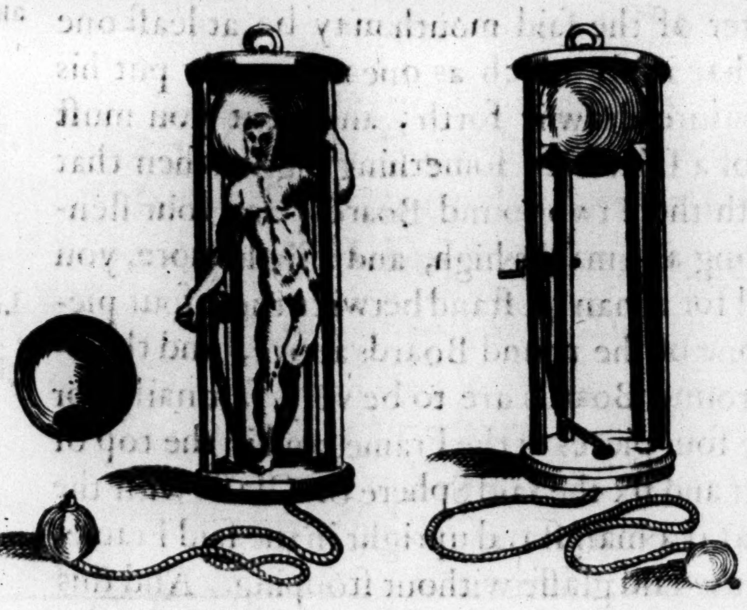
Se-

Secondly, doubting, by reason of my small experience in Maratine Affairs, to incurre some Solecisme: but there coming into my mind an excellent expression of a famous Philosopher of this Renowned City; who upon a time perswading me to write something that was new, and I having answered (it being incident for humaniy to erre) that I was afraid least my so great desire to publish my sundry new Conjectures, might run me into some fantastical conceits, that might make me become the subject of vulgar discourse, this excellent person replied: That if Nature should forbear her operations for fear of producing sometimes some monstrous things, the worlds destruction would ensue, for that they onely are free from erring who do nothing, whose speech hath emboldened me to speak of a point, which I never thought to have medled with; namely, To declare some of my conjectural wayes of artificial diving, and continuing under water, to seek out any thing that was sunk in the same, though in places very deep. And I judge these the most expedient that can be devised: and because these and the like wayes may be varied into several forms, and sorts, one more ingenious, and artificial than another; the prettiest, and most ingenious is this, I would have you get, made at *Murano*, a hollow Globe of Transparent Glasse, the diameter of which I would have to be at least two foot, with a round mouth, that the Diameter of the said mouth may be at least one foot, or wrather more; that is, so much as one may easily put his head therein, and at pleasure draw it forth; and next you must make two round Boards of a Diameter something bigger then that of the said Globe, and with these two round Boards, and four slender pieces of Wood, as long as a man is high, and a little more, you must make a little Modell for a man to stand betwixt these four pieces of Wood; and with one of the round Boards above, and the other beneath; and these round Boards are to be very fast nailed or otherwise fastened to the four pieces of the Frame, and in the top of this Machine, you must fit and fix the said Sphere of Glasse with the mouth downwards, so, that if a man stand upright in the said Frame, he may hold his head in the said glasse without stooping. And this being done, take neer upon as much Lead as all this Machine weighs, and make it into a round figure, of the compasse of the round Boards, and then fasten and nail it to the bottome of the said Modell, namely, underneath the lowermost Board on which your feet stand when you put it into the Water: And then, (or before) make an hole as big as a Shilling in the Centre of this Lead and Board, passing through them both; and this same Lead will be able to draw almost all the Machine together with him that shall be therein under Water. Truth is, that the Experiment requireth that the said Lead be so limitted that it may be able to draw the Machine

A Place near to *Venice*, where the famous Glasses are made.

Like the Frame of an Houre-glase.

chine and person in it under Water, but so, that the supreme or upper part of the same, that is the uppermost round Board, may stay at the Superficies of the Water; that is, if the Lead chance to be so ponderous, that it cause the Engine to sink leisurely to the bottome, you must take away some of the said Lead; and on the contrary, if it chance that the Lead be not able to draw it all in that manner under Water, so as to make the said upper round Board to lye and stay exactly level with the Surface of the Water, but that a part of it rests visible above the Water, you must encrease the said Lead so, that the upper Board may lye and abide precisely, as was said before, in the Surface of the Water: and when you have thus adjusted the said Lead, I would have you take a Ball or Bullet of Lead weighing two or three pounds, (that is to say of such a weight, that it may be sufficient to make the Machine and person diving to descend to the bottome as oft as it is interposed, or added,) with an Iron Ring in the said Ball, to which bend or fasten a Rope as long as the said Water is deep, in which the Diver is to descend, and somewhat more; and reeve or passe the other end of the said Cord through the hole made in the Board and Lead through the bottom of the Model; and fasten that same end of the Cord in a place of the Machine, so, that the Diver may take it, and draw it, or slack it as he pleaseth: and this being done, the said Machine will be finished. And that you may better understand it, I have here inserted it graphically: yet I should have told you, that for many reasons you should in the beginning have fastened a Ring in the Centre of the upper Board, on the outside, to tye a Cord to the same as occasion serveth.



EXPLA

EXPLANATION II.

HAving understood the manner how to make this same Engine, it remains to shew how it is to be used; And for your direction therein, I say, That he that would dive or go under Water to seek any thing that was sunk, should carry the said Machine to the place where he resolves to descend, and first to let that Ball of Lead with the Line go to the bottome, and then to put in the Machine it self, which by means of its heavy bottome of Lead will rest upright in the Water, with almost all the Globe of Glasse above Water, in such sort, that he that would may easily enter into the same: yet you must be dexterous in going into it, that you do not much sway the Machine sidewayes, for that, if it lye too oblique the Water will enter into the Globe of Glasse, and drive the Aire thence that was in the same, or at least in part, but holding it upright when you enter the same, the Water shall keep in the Aire on all sides, whereby the water will be kept from entring. And therefore if he that shall enter into the said Machine, do nimbly thrust his head into the said Globe by the hole thereof, he shall finde it quite filled with Ayre; in which place he may breath for verry many Respirations, without the least obstruction from the Water: And because this Machine will stay with its upper end level with the Waters surface (the affixed Lead having been so limited) therefore desiring to descend to the bottom, the Diver should hale the Ball and Line upwards, which was sent before to the Bottom, in haling of which the said Machine will descend as much under Water as he hales the Corde; and if he continue haling it, till there be none of it left, he shall descend to the Bottome; and in the descent, and after that he shall be got to the bottom, he must look round about him through that transparent Globe for to finde out the thing he seeks, and seeing it, he may many wayes with ease transferre himself thither without rising again to the top; And when he would return upwards to the toppe of the Water, he needs do no more but slacken that corde fastned to the Ball of Lead, for thereupon the Machine shall begin to rise upwards, and letting the said Corde goe, it shall not stay till the Machines upper parte arrive at the surface of the Water; and being ascended thither, the Diver may come out thereof, and swim to the top, and provide himself afterwards of such things as are necessary for embreching the said Ship or other matter sunke: And in case the Diver cannot swim, it will be necessary to fasten a Corde to the Ring placed in the Centre of the upper Board, and thereby to draw the Modell above the Surface of the

Water; but knowing how to swim, he may enter, ascend, and descend of himself, without any help.

EXPLANATION III.

BUt if you chance to be in a place where you cannot procure the said Globe to be made of Glasse, it may be made of Wood; but then you must make therein great Sights, or Eye-holes of clear Glasse of each side to look four severall wayes; and pay it without, and also within if you see cause with Pitch. And if you cannot get such a Ball of Wood, you may make shift with a little Cubicall Chest or Boxe, like one of those Chests wherein they plant Ceaders, which must be well joyned graved and pitch't, with four such Sights of Glasse as before, namely one upon every lateral flat or plain, so placed, that the Diver may see through them every way, and be able to look downwards, it would be good to make the Box somewhat narrower towards the mouth, that so the four laterall Planes may look somewhat sloping: and in the entrance, descent, ascent, and coming forth, you are to use the same Rules as before; aud if you have a desire to descend faster, you must make the Ball of Lead somewhat heavier, that was tyed to the end of the Corde, and this done the Machine shall descend faster to the bottom upon halling the said Corde and Ball; and when you vere or let loose the Cord, the Engine will re-ascend, but according to its former speed: But if you would also make it swifter in its ascent you are to proceed quite contrary, that is, you must somewhat diminish the Lead, which is under the Base of the frame; and the more you diminish the said Lead, the swifter shall it be in ascending. But you must remember withall to encrease the Ball of Lead, so that it may be able to draw the said Machine to the bottome speedily or leisurely according as occasion requires.

EXPLANATION IV.

BUt if there be any likelihood of any obnoxious Fish in the place where the Diver is to descend, that may hurt him, being quite naked; though that in the former kind of Machine with four pillars you may se u e him with a wire Grate, made in the manner of doors to the same, yet to the end that you may know that this Invention may be vari d sundry ways; you may in this case have a Globe of transparent gl s made at *Murano*, of such a bigness, that a man standing on his feet, or else sitting, may be contain'd therein, having amouth or round hole of capacity sufficient for a man, commodiously to enter and goe out thereby, and somewhat larger: & then coffin or enclose the said Globe between

between two round Boards of somewhat a greater Diameter than the Globe, with four pillars, as in the ensuing figure doth graphically appear. But in the round Board which is put over the hole or mouth of the said Globe, you must also make a round hole somewhat narrower than that of the Globe, but yet big enough for a man to passe in and out thereat. Afterwards under this round Board so bored, you must place and fix another round bored piece of Lead of such thicknesse, as that it may be able to draw the said Ball or Globe of Glasse, together with the Diver in such manner under Water, that the upper round Board do rest in the Surface of the Water, namely, that it may not be so heavy as to sink the Globe and Diver to the bottome, but only to retain it beneath the Surface of the Water, which by tryal may be easily proportioned, namely, by adding or taking away Lead from the Base, according as occasion shall require. Next you are to frame a seat for the Diver to sit commodiously in the said Ball or Globe, and next fasten a Ball of Lead to the end of a Rope, as many fathom long as the water is deep into which you would descend, and somewhat more, as was said in the preceding Explanation. And that Ball of Lead should be of such bignesse, that applied to the said Model, it may be sufficient to make it descend to the bottome leisurely, or swiftly, as he seeth cause who is to dive. And make an handle or peg in the said Globe whereat to fasten or belay the other end of the said Rope, and to draw it easily upwards, or let it loose at the pleasure of him that is within, and this may be easily done by joyning and fastening four pieces of wood upright in the mouth or hole of that bored Board and Lead, which shall be about the mouth of the said Globe; and that I may be the better understood, I will give it you in figure with the Diver sitting therein.

If you would descend to the bottome of some deep water by help of this Machine, you are to proceed according to the directions given in the precedent Explanation.

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EXPLANATION V.

IN case you should be in a place where you could not have such a Globe made of Glasse, you may procure one of Copper or Lead, round in fashion of a greater * Churne, wide in the bottome and narrow in the mouth, and at least five foot high, and four foot broad. It may indeed be made quadrangular, that is, so that the mouth be at least three foot square every way, and the bottome at least four foot every side, and not under five foot high, and this same vessel, making it of Lead, must be so contrived, or proportioned, that the corporeal or solid *Area*, or Content of its interiour vacuity, or space, be about noncuple to the solid *Area* of the Lead, which is imployed in making the said Vessel; that is, make the Lead of such a thicknesse, that the Vessels vacuity may be nine tenths of the solid Area of all the whole Frame, which may be easily done by any one that is not ignorant of practical Geometry: and this Vessel being made, you should place or set therein four great Eye-holes or Sights of transparent or cristaline Glasse, so placed as to see any way as you shall need or desire: and furthermore, in the framing of this same Vessel, you must make some provision for the setling or staying your feet, and to sit down, and likewise you must make a Pulley to hall the Ball of Lead up, or let it down, which is fastened to the end of the long cord, as was said in the two precedent cases. And moreover, in the making of this Vessel, you are to fasten four Rings of Iron to the bottome without, namely, to the four Angles, it being Quadrangular; (and being round, let them divide the Circumference into four equal parts) and betwixt these four Rings, you must place a square or round Deal Board. And this Vessel thus modellized shall be so contrived, that putting it into the water with the mouth downwards, with him in it who is to Dive, it shall but just stay in the Surface of the water with that bottome of wood; but if it chance that it shall not stay at the Surface of the water by help of that bottome of Board, but that it will descend, you must upon that bottome fasten another, or two, or more square or round Boards to the four Rings, in such wise, that by means of the said Boards it may be reduced to such a quality, that it may rest with the said round Boards in the Surface of the water, and descend no farther. Having with judgement and experience provided all these things, and the Diver being desirous to descend of himself, and likewise to return to the top when he pleaseth, this may be performed with that Ball of Lead tied to the end of that long Rope, as hath been said in the precedent Explanations, that is, to send the Ball first to the bottom in the place where the Diver would descend, and then to enter into the

* *Brenta*, a Vessel in which they in Italy carry Grapes to the Press.

Machine,

Machine, and to settle himself therein; and then to pull the Ball upwards, which should be of that Gravity, that it may be apt to make such a Vessel or Machine descend together with the Diver; and if the Machine chance to be justly contrived, as hath been said above, I hold that a Ball of five or six pounds may be sufficient to make it descend nimbly upon the pulling of the Cord, and lifting the Ball from the bottome, and continuing to draw the said Cord, as long as there is any remaining, he shall arrive at the bottome; and whenever he would return upwards, he needs but only vere or slacken that Cord, and letting it all go he will not cease ascending till the Machine attains with its top (covered with those square or round Boards) unto the Surface of the Water, as hath been said of the others. I will not stand to shew you the many particularities which might be inserted for the transporting your selves from one place to another, keeping at the bottome, that is, without returning to the top, for that they are almost infinite, but it shall suffice to let you know, that he may easily do it, carrying with him a long Hitcher, or a Boom, or a Spike with a Hook at the end.

Many other particulars there might be insisted on, and especially how many may simply (that is, without any of the foresaid Machines) go to the bottome, and stay for many hours under Water, which, besides the many profitable conclusions that might from thence be inferred for Diving in indifferent depths, being accompanied with the helps prescribed in the foregoing Explanations, they would be much to the purpose, for that the Diver being once conducted with the Machine near unto the thing sunk, he might come out of the said Machine, and go and stay for a long time about the same, to fasten, or prepare those things that are necessary for the raising it: And farthermore, there is something to be said, when the thing sunk is in a muddy or dark Water, how the Diver may in sundry wayes, kindle there a great and flaming light, which flaming fire, besides that it would make him discern the thing sunk, it would also secure him in his going forth of the Machine from any devouring Fishes, for that all such as should chance to be near that place would be affrighted at such an unusual spectacle, and would make far from it. I might also shew many wayes to embreech and grapple a Ship when it is found, as well in deep as shallow Channels, which particulars I shall reserve for another time.

I will not stand to shew how this kind of Diving Machine might be made of Boards, and that in sundry fashions, well calked and pitcht, with four Lights or Sights, fastening about the mouth of the same as much Lead as should be necessary, forasmuch as by what hath been spoken in the third Explanation, it is sufficiently manifest.

A
SUPPLEMENT
OF THE
n dustrious or Troublesome
INVENTION
OF
Nicholaus Tartalea:

In which is shewn a general and safe way to imbreech Cables, and hitch Grappling irons to any Shi p that's sunk, aswell in a deep as shallow Bottome, provided you know the exact place where the said Ship is. Together with another new way of raising or recovering the same.

Whereunto is, last of all, added some new ways to conduct a Light, or Flaming Matter, unto the Bottome of the Water, to enlighten, upon occasion, any dark Bottome, for the discovery, not onely, of a Ship or Bark, but also any small thing of value that is sunk, and that in the night as well as in the day.

To the Most
Illnstrious and most Serene
PRINCE
Francesco Donato,
Duke of
VENICE.

HAving not long since, Most Serene, and Most Illustrious Prince, published under the Glorious Name of your Highnesse, sundry and diverse Ways to raise a Ship sunk, with its Cargo in it (when once it

it is Grappled) I must confesse I was not then sollicitous to find a way to imbreach or grapple the said Ship (though it is necessary to be known) and the cause thereof was, for that I concluded that amongst Mariners there were a thousand means to effect it, and I was loath to enquire after such things as are commonly known to many, although I be ignorant of them; but delight to search into those things which none else can do. Now, having been since told and assured by many, that Mariners, and all other persons of ingenuity find far greater difficulty in imbreaching and Grappling such a Ship, than they do, (when once they have hold of it) to raise the same: I understanding the same, presently deliberated upon some way that should be general and secure, and to adde it in the end of my Treatise, that so it might not, for want thereof, be vain and useless. And thus; of many that I have found, that which to me hath seemed most universal and easy to be explained by writing; I have here subjoined, together with another new way to recover the said Ship: and the manner how to illuminate the bottome of a dark Water, but still under the Illustrious Name of your Serene Highnesse, at whose feet I once more humbly throw my self.

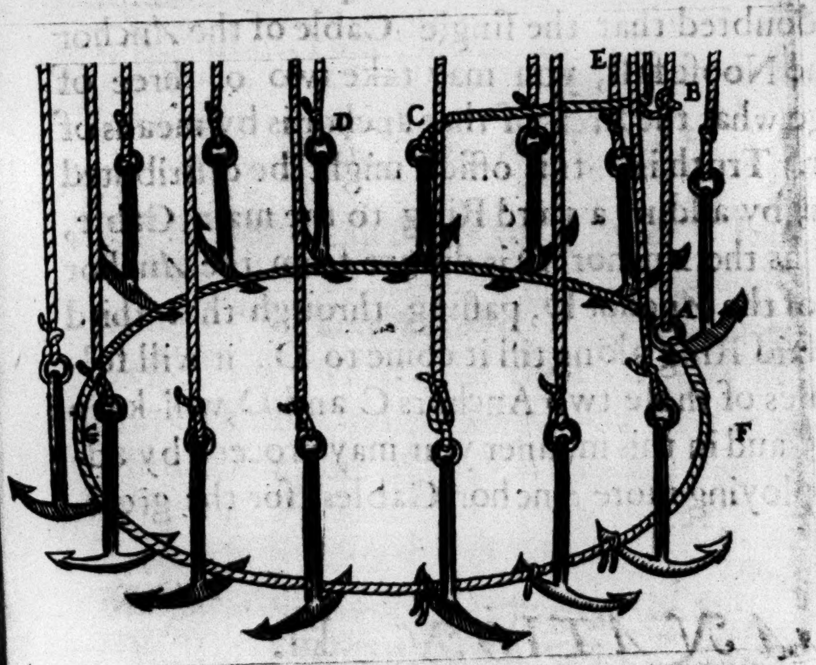
NICOLO TARTAGLIA.

A Supplement.

EXPLANATION I.

TO hitch therefore, and sling, or grapple fast a laden Ship that is sunk, being in a scowle bottome, as was that broken up near to *Malamoccho*, you are to take a very strong Sheat-anchor Cable, of such a length as is sufficient for the Uses hereafter to be understood, and at one end of such a Cable you are to seiz or fasten very well a thick and strong Iron Ring, big enough for the other end of the Cable to passe through with ease, and make thereof a running Parbunckle: and then, near to this Ring (that is under this Cable at the place where it shall be bent to the Ring) you must seiz or fasten one of the Flooks of a thick and strong Anchor, and about three fathoms space from that first Anchor hitch the Flook of another second Anchor into the said Cable, seizing or fastening it that it stir not: and about two fathoms distance from this second Anchor, seiz, as before the Flook of a third Anchor, and so two fathom from that a fourth Anchor; and so proceed, placing in that manner as many Anchors as suffice to go round the Hull of the said Ship under its Wails, and rather lesse than more, to the end the last Anchor may be no hinderance to the running of the Parbunckle at the Ring at such time as it is to be roused or vered, that is, to be drawn or let slip. The truth is, that in the part of the Cable marked E, in the Figure following, and in the opposite part marked G (which parts you are to place so that they may fall one at the Stem, the other at the Stern) no Anchor is to be placed, but you must leave at least three fathom interval betwixt those Anchors at G, as was required to be done betwixt the first and second at E. And then form the said Running Parbunckle, that is, reeve the other end of the Cable through the Ring of Iron; and, that being made, you are to place many persons upon Flat-bottome Boats fastened in an Oval Figure round the place where the Ship lyeth: and then vere or slacken the Parbunckle, but in an Oval Form, to that widenesse, that it may at four or five foot distance, inviron the foundered Ship: and this done, you must let all the Anchors, together with this Girdle or Parbunckle, (being kept at that widenesse) gently and equally fall to the bottome of the Sea, keeping the Ship in the midst of the Ovall: and when you perceive all the Anchors descended to the bottome, you must vere there several Cables, that they may sink deep into the sand or Ouze; and then after this you must

muſt draw, and bring them by degrees cloſe underneath the Hull of the Veſſel, and then hall or ſtrain hard the end of the Sheat Anchor Cable which was reeved through the Ring; and begirt the Hull of the Ship therewith, as with a Girdle (and to ſtrain it very taught, it would not be amiſſe to make uſe of a Capſtan) and when this Girdle is drawn to its due exactneſſe, to the end it may not ſlip (in the elevation of the Ship) faſten to that part which you hold above Water another Ring of Iron, and paſſe through this Ring one of the Anchor-Cables that is on the ſame ſide as the firſt Ring is on, and almoſt as far from the ſaid Ring, as the ſecond Ring is diſtant from the firſt; whereupon making this ſecond Ring to ſlip along the ſaid Anchor Cable, and then in the Elevation halling the ſame, it ſhall make the ſaid Girdle taught under the ſaid Ship: and that I may be the better underſtood, I have here underneath repreſented the ſaid Girdle pul'd together in an Oval Figure as it is to lye under the Rake of the Ships Hull with fourteen Flooks of fourteen Anchors under the ſame (except in the part inked E, and in its oppoſite part G,) well ſeaſed; of which Girdle, or Parbunckle, the firſt Ring ſhall be A, through which the Sheat-Anchor Cable paſſeth, namely, the Cable A B, to which Cable was faſtened a ſecond Ring in the point B, through which ſecond Ring, (to the end the Girdle might not ſlip) we will reeve the Cable of the Anchor C; which Anchor C we ſuppoſe to be ſomewhat farther from the Ring A, than the ſecond Ring B is from the firſt Ring A, and then make the ſaid Ring B to ſlip along the Cable of the ſaid *A*nchor C, till it come to the point C. *A*nd thus the Ship ſhall be ſecurely and ſtrongly grappled and begirt. *W*hich done, proceeding as we directed in the firſt Book of our *Induſtrious Invention*, you will execute your purpoſe; That is, when the two or more coupled Ships ſhall be full of water, at the ebbing of the Tide you are to faſten and belay to thoſe Tires of Beams that couple the ſaid Ships, all thoſe fourteen Cables, taking a little more care in tying, and belay-



ing that of the Anchor C, which will keep the Girdle from slipping in the Elevation.

But if you doubt that that single Cable, to which the Anchors are fastened, is not sufficient for so great a weight, you may above that, place another with a Ring also, through which (as before) the end of it may passe, by that means begirting the Ship with two of those Girdles, and observing the same Rules you may take three or four of those slipping sheat-anchor Cables, each with its Ring wherein to run in the manner of a Noose. And when the said new Girdle is pulled strait and close to the Ship, fasten to the said Cable, (or to each of them if you use more) another second Ring, to gird and hold the said Noose fast, that it slip not with the Cable of the Anchor C, or with more of those Anchor-Cables if there be occasion.

And in case that those fourteen Cables be thought insufficient to bear so great a burden, you may take twenty or thirty of them, or as many as you please, tying them closer to one the other, under the running Cable, and make half of them to be placed on one side, and the other half on the other side of the said Ship.

And if again it be doubted that the single Cable of the Anchor C s not able to hold the Noose fast, you may take two or three of them, for you may judge what the stress of that anchor is by means of the height of the water. Truth is, this office might be distributed amongst more Anchors, by adding a third Ring to the main Cable, as far from the second, as the Anchor D is distant from the Anchor C, so that the Cable of the Anchor D, passing through that third Ring, and slipping the said Ring along till it come to D, it will follow that those two Cables of those two Anchors C and D, will keep the Parbunckle straight; aud in this manner you may proceed by adding new Rings, and imploying more Anchor-Cables, for the greater security.

EXPLANATION II.

THe same method may also be observed when the Ship is in a deep place, provided that the depth exceed not the length of the Hull of the Ship, because then there may be alwaies found some one or more Cables sufficient to reeve through the second and following Rings of the Main Cable to secure the Noose from slipping, or growing slack, as in the preceding declaration hath been said. But if it chance that the depth of the place be far greater than the length of the Ship, you can no longer secure the Noose with that second Ring, but must find out some other way, and though there might be many found out, I shall instance but in this one.

After

After you have strained, drawn the said Girdle as taught as you can, you may take the Cable thereof, and the Cable of the anchor next adjoyning on the same side that the first Ring is on (namely, the Cable marked F,) and twist and wind them together, and then reeve the single Cable of the Girdle *A B*, through the Ring of a Sheat-anchor, (without its Cable) and let the anchor slide downwards along the said Main Cable, which by reason of its weight will run almost close to the Ring *A*, of the Main Cable, pressing the twist of the two Cables close at *A*; and this done, once more twine or twist a little the two former Cables, namely the Sheat-anchor-cable B, and the lesser Cable F, and then sease those two Cables severally to the Orders of Beams, that is, one to one Order, and the other to another at some distance from the former, to the end they drive down the twisting near to the Ring of the anchor: which twisting will keep the Noose from slipping or opening in elevating the Ship. And if there be any occasion to use a Capstain (as was said in the seventh Explanation of the first Book) you must always take care to strain these two Cables equally, and much asunder, which doing, the Girdle shall be kept strait. Many other ways might be shewen for to keep the said Grand Cable from slipping, but esteeming them superfluous, I omit them.

EXPLANATION. III.

He that is desirous to recover a foundered Ship laden with Fraight, by other ways than those prescribed in the first Book, namely, without standing to fill those two or more Ships, or other Vessels with water, and then to empty them, may only by force of Capstains or Cranes easily effect the same in the manner following, (still making use of the Parbunckle and flooks of anchors explained in the first Explanation of this) namely: By taking from their anchors Rings all their Cables, except that which is to make fast the Main-Cable Noose that begirts the Ship, and in their places make fast to each Ring a strong Pulley or Block, in such sort, that all the said Pulleys or Blocks have equal number of Shivers, or wheels, and those as many as you can make them: and through these Shivers or wheels reeve their proper and convenient Cables or Ropes, incatenating each Pulley with its superiour; and this done, make two squadrons of Barks, or Lighters, or Flat-boats, according to the method laid down in the fourth Explanation of the first Book, collated and bound together with those Tires of thick and strong Beams tripled, and with a great and spacious platform of thick Planks upon each squadron, and upon those two spacious platforms place as many Capsters or Ship-cranes as you shall judge necessary for such a weight,

weight, and rather much more, then ever so little less, and then let fall the said Anchors leisurely, with the Girdle opened in an Oval Figure, untill they come to the bottome of the Sea, so that the Girdle do encircle or surround the foundered Ship. And having once begirt it carefully, approximate all the Anchors with the Girdle to the Hull of the Ship, and then sharpen or make taught the Girdle-cable by halling it hard and streight to the Ships hull, and when it is drawn close, belay it that it may not slacken, with that single Anchor-cable, or more, according to that secure way spoken of but now, or by some other that shall seem more expedient, (for many more, if one think thereon may be found:) and this being done, seek to loosen the Ship by degrees from its bed of Ouze, a little on one side, and a little on the other with the aforesaid Capsters, and, being once water born, then draw it upwards equally on both sides, and proceed in this manner till such time as you have hoisted it sufficiently above the Waters surface, and then pump out the Water, and unlade its Cargo.

EXPLANATION IV.

Having in the second Book shewn several ways of Diving under Water in search of things sunk, in this place I have thought fit to add, in case that some little thing of value should fall into a Water in some shady place, and where its bottome is obscure and dark, a way how to conveigh a Light thither that may give light enough for the discerning of that little thing, provided that it be not buried in, or covered with the old Ouze. Now to perform this, and that with expedition, we may in small depths take one of those brass Buckets or Pails, which are used in carrying and keeping of Water for household uses: and those of them that are shaped long and deep, with feet shall be better then those that are made round and shallow, without feet; and the bigger and higher it is, so much the better it shall be. And having made choice of such a Bucket, you are to fasten to the Ears of it two small Ropes of about two yardes apiece, in such a fashion, as that they may one cross the other at the mouth of the Bucket, making upon it a perfect cross, and that the Knot of the Ropes may be in the midst of the Buckets bottom without, making of the ropes a Hoop over the bottome whereat to fasten another Rope of greater length; so that the Bucket being held by that last Rope may come to hang with its mouth perpendicularly downwards. And this done, fasten as much Lead to the two Eares of the Bucket as may just make it sink to the Bottome, and then set and fasten a little Wax candle lighted in the intersection that those two Ropes make over the mouth of the Bucket, that is, in the centre

of that perfect cross; so that the candle with its light may be within, and near the bottome of the said Bucket. This being done, let down the Bucket, with the candle in it gently unto the bottome, which doing, you shall see the burning candle clearly enlighten the bottome of the Water. And this Bucket you may remove from place to place, without drawing it upwards. The truth is, that this candle will not long continue burning, but will serve for a little while, and when it shall go out of it self, it may be drawn up, relighted, and let down, as occasion requires: but the greater that the Bucket, and the lesser that the candle shall be, so much the longer time shall it keep its light under Water: and therefore if the said bottome were very deep, it would be requisite to perform that effect with so much a greater Vessel, as a great Caldron, but yet of Brass, or by that means the candle shall continue longer lighted.

EXPLANATION V.

But in case that a Ship or Bark were foundered in some spacious and profound Gulph, and that the exact place where it sunk were unknown, and that the bottome of the said spacious Gulph were very obscure, it is manifest that so little a light as that spoke of in the precedent Explanation would hardly serve. And therefore if you would convey thither one much bigger, you may do it severall wayes, of which one is this. Take nine ounces of refined Saltpeter, six ounces (Greek weight) of Brimstone that is clear and transparent, three ounces of Camphire refined, and one ounce of Mastick; and beat all these things severally, not very small; and when you have beaten them, mix them all together in an Earthen Pan; and when they are well mingled, put thereto three pounds of common Gunpowder, and then remingle them very well together; and afterwards put therein four ounces of oyl of Stone, and mix all very well; and this done, take a Cartredge thereof, and give fire to it; and if it burn too slowly, put a little more Gunpowder to it, but if it burn too vehemently and suddenly, add thereto more oyl. Put this Composition, after this, into a little Bag of double Canvis; of such a widenesse, that when all the mixture is out, therein it may be as broad, as high, and cram the Composition hard down into the Bag; and then with very good Packthread sew up the mouth of the Sack, cutting away the superfluous Canvas. Then winde a good hempen cord round about it very hard every way, reducing it to the form of a round Ball, and after it is very well bound and swathed about many severall times, you must melt Brimstone into a great Vessel, and when it is melted, roll the said Ball therein so, as that it may be covered all over with a crust of Brimstone. And this being done

affix a piece of Lead unto the Ball by an iron Wire, and make it very fast, and frame in the top of the Ball a Bow or Noose with the said Wire, and to that fasten a long Rope, and then in the opposite place where the Lead is fixed, make an hole with an iron rod into the middle of the Ball, and stop that hole with a little fine Gunpowder, holding it suspended by the Rope: and when you would have that Light descend into the bottome of the Sea or Gulph, goe to the place, and give fire to the little hole, and when it is inkindled, let down the Ball and Lead, lengthwayes, almost to the bottome, where he shall be that would find the thing sunk, and you shall find that the said fire will illuminate very much round about the said bottom, and shall last a long time, and more or less, according to the hole made in the Ball. 'Tis to be noted, that the Ball is to be held over the head of him that diveth, for that the smoke proceeding from it will much obscure the Waters above it, so as that it will give Light only downwards; and this fire will be a dreadful sight unto the Fish, so that they will fly from so new a spectacle.

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affix a piece of Lead unto the Ball by an iron Wire, and make it very faſt, and frame in the top of the Ball a Bow or Nooſe with the ſaid Wire, and to that faſten a long Rope, and then in the oppoſite place where the Lead is fixed, make an hole with an iron rod into the middle of the Ball, and ſtop that hole with a little fine Gunpowder, holding it ſuſpended by the Rope: and when you would have that Light deſcend into the bottome of the Sea or Gulph, goe to the place, and give fire to the little hole, and when it is inkindled, let down the Ball and Lead, lengthwayes, almoſt to the bottome, where he ſhall be that would find the thing ſunk, and you ſhall find that the ſaid fire will illuminate very much round about the ſaid bottom, and ſhall laſt a long time, and more or leſs, according to the hole made in the Ball. 'Tis to be noted, that the Ball is to be held over the head of him that diveth, for that the ſmoke proceeding from it will much obſcure the Waters above it, ſo as that it will give Light only downwards; and this fire will be a dreadful ſight unto the Fiſh, ſo that they will fly from ſo new a ſpectacle.

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